Coupled Mode Theory Based Modeling of 2D Cylindrical Integrated Optical Microresonators

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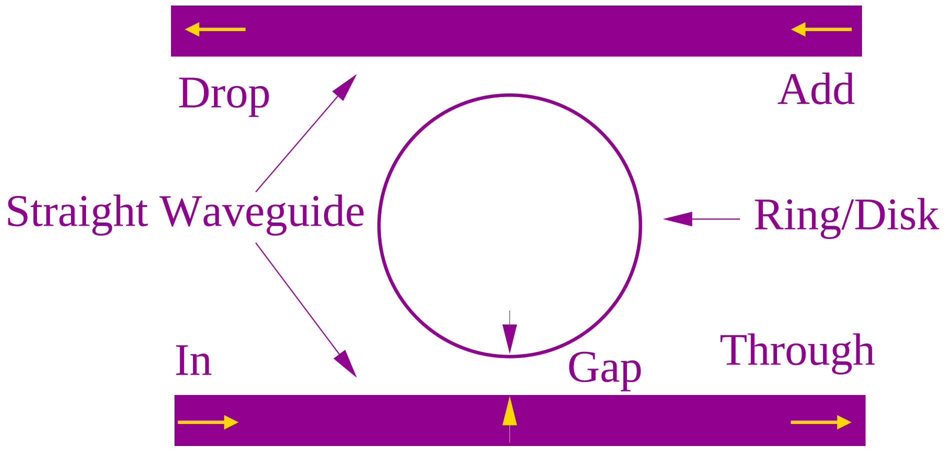
Microresonators

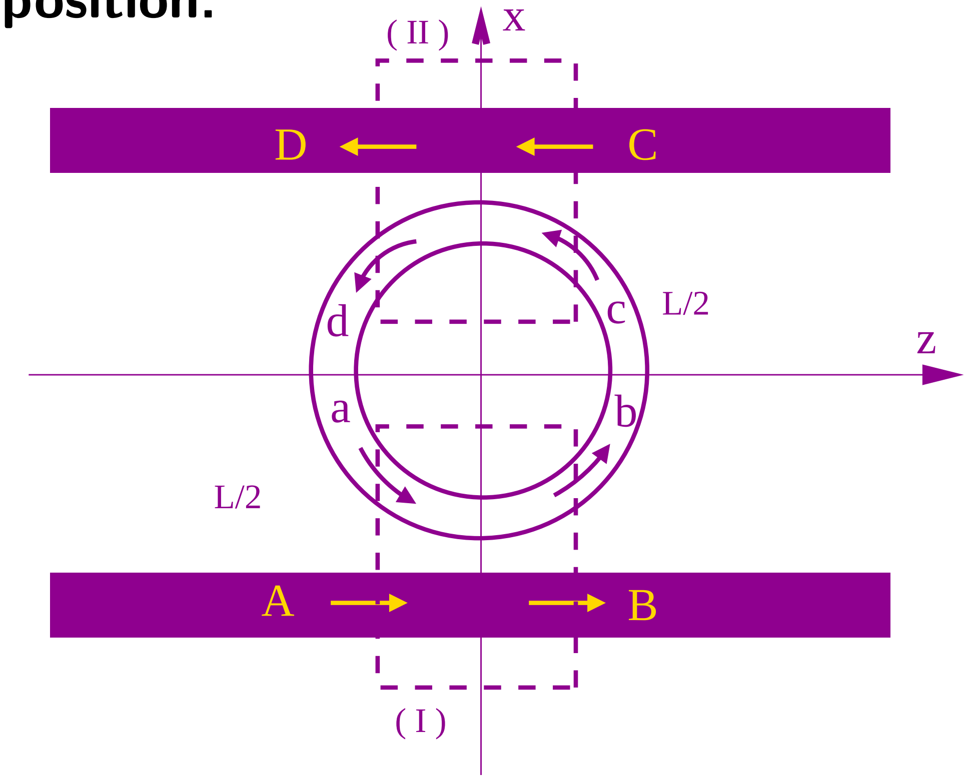
- Two dielectric waveguides, coupled to cylindrical /disc/ring shaped cavity



- Interested in the modeling, simulation and analysis of 2D integrated optical microresonators as a *channel dropping filter*.

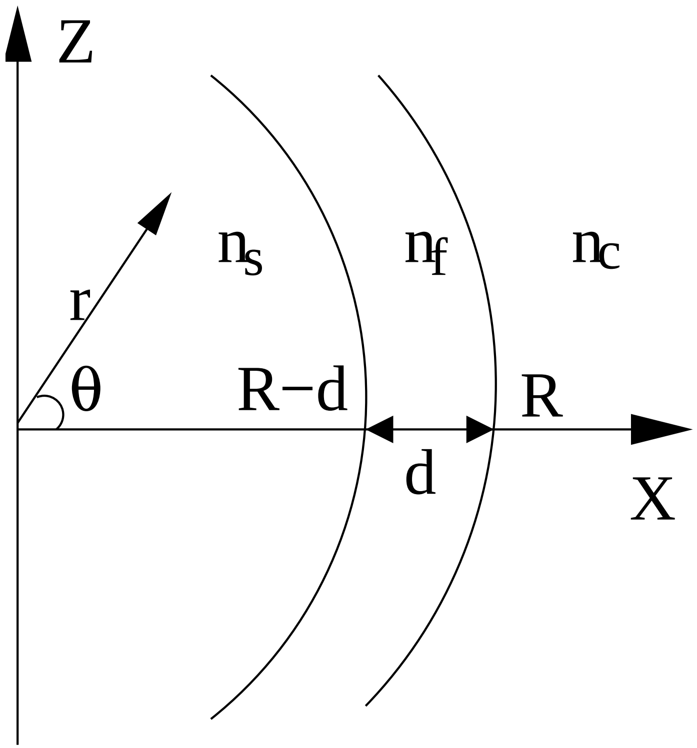
Microresonator Model

- **Functional decomposition:**



- **Advantage:** Modeling of the microresonator reduces to the modeling of a bent waveguide and the modeling of the bent coupler.
- **Approach:** Spatial Coupled Mode Theory
- **Requisite:** Essential to know bent modes.

Modeling of a Bent Waveguide



- Field ansatz: *No field variation in y-direction*

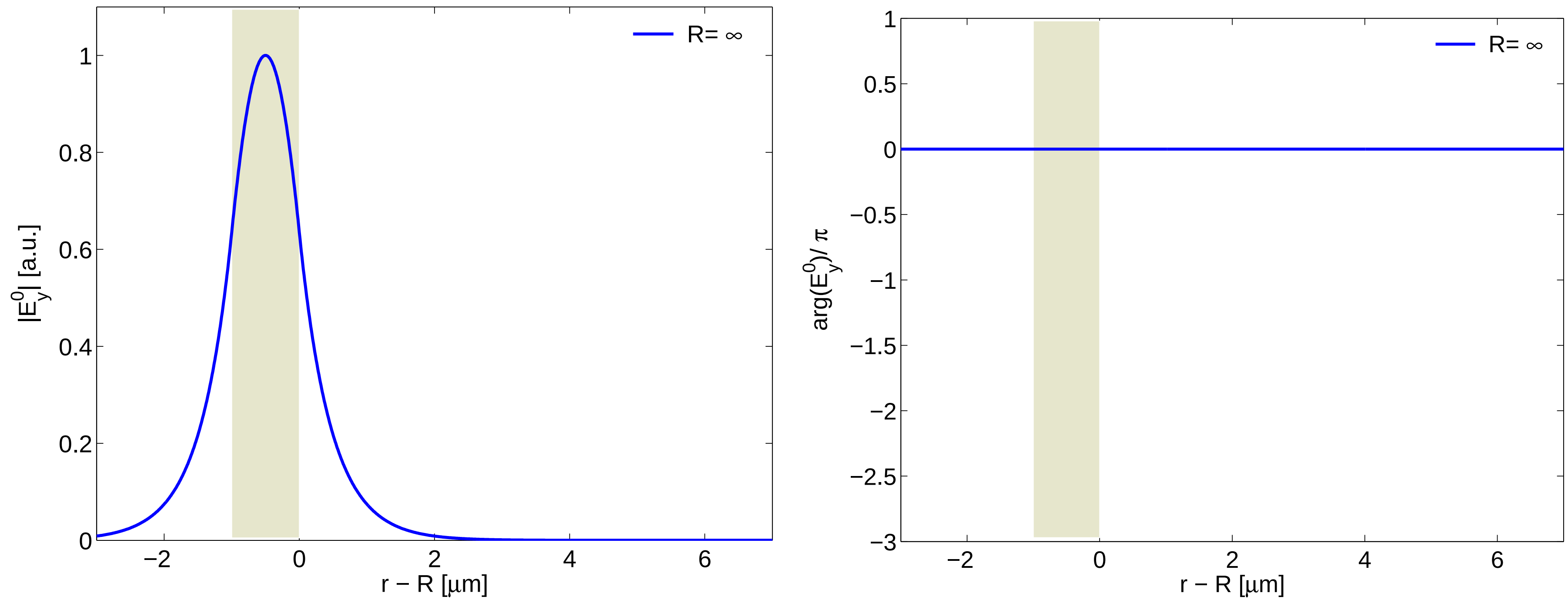
$$\mathbf{E} = (E_r^0(r), E_y^0(r), E_\theta^0(r))e^{i(\omega t - \gamma R\theta)}$$
$$\mathbf{H} = (H_r^0(r), H_y^0(r), H_\theta^0(r))e^{i(\omega t - \gamma R\theta)}$$

- Propagation constant (γ) is complex. $\gamma = \beta - i\alpha$
- Enough to know E_y^0 (TE) and H_y^0 (TM)
- *Bessel equation*: $(\phi = E_y^0$ or $H_y^0\ )$

$$r^2 \frac{d^2\phi}{dr^2} + r\frac{d\phi}{dr} + (n^2k_0^2r^2 - \gamma^2R^2)\phi = 0$$

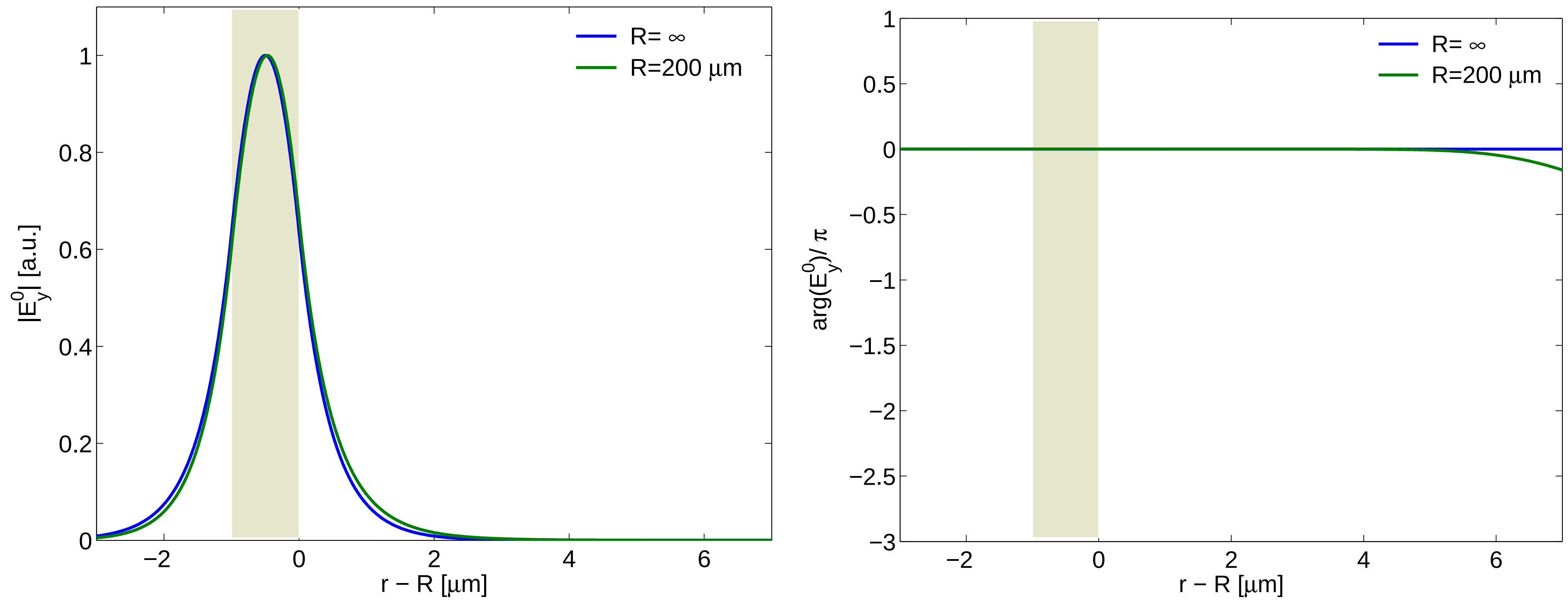
- Piecewise field ansatz + Interface conditions ⇒ Dispersion equation
- Dispersion equation + Complex order Bessel functions ⇒ Analytic Bend Modes

Bent Mode Profile



Absolute value and phase plot for E_y^0. $(n_s, n_f, n_c) = (1.6, 1.7, 1.6)$, $d = 1$, $\lambda = 1.3$.

Bent Mode Profile



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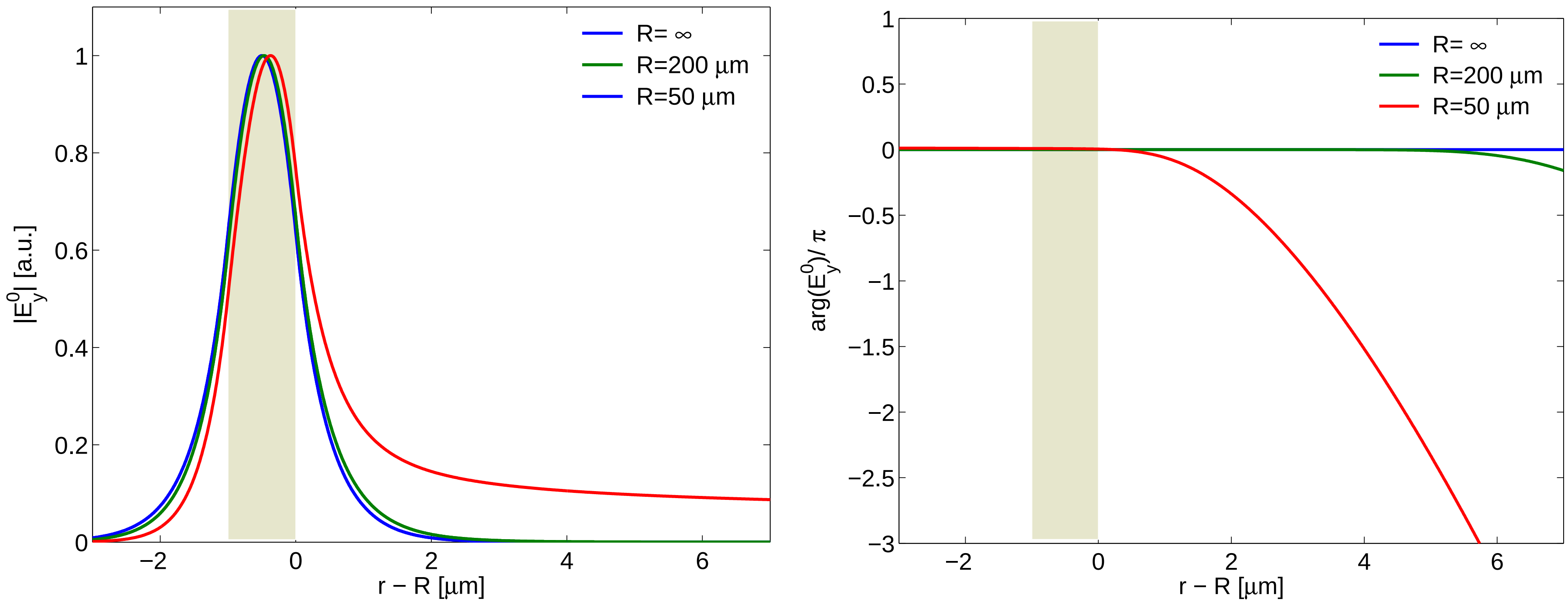
Bent Mode Profile



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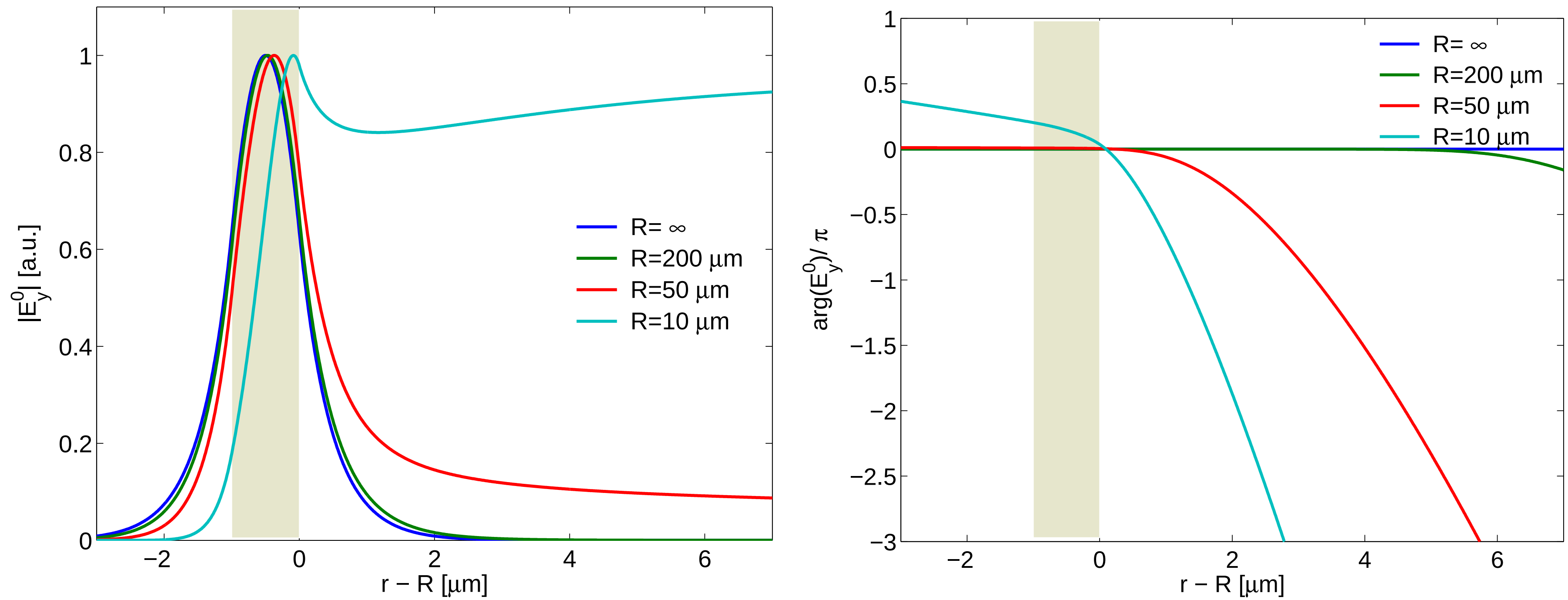
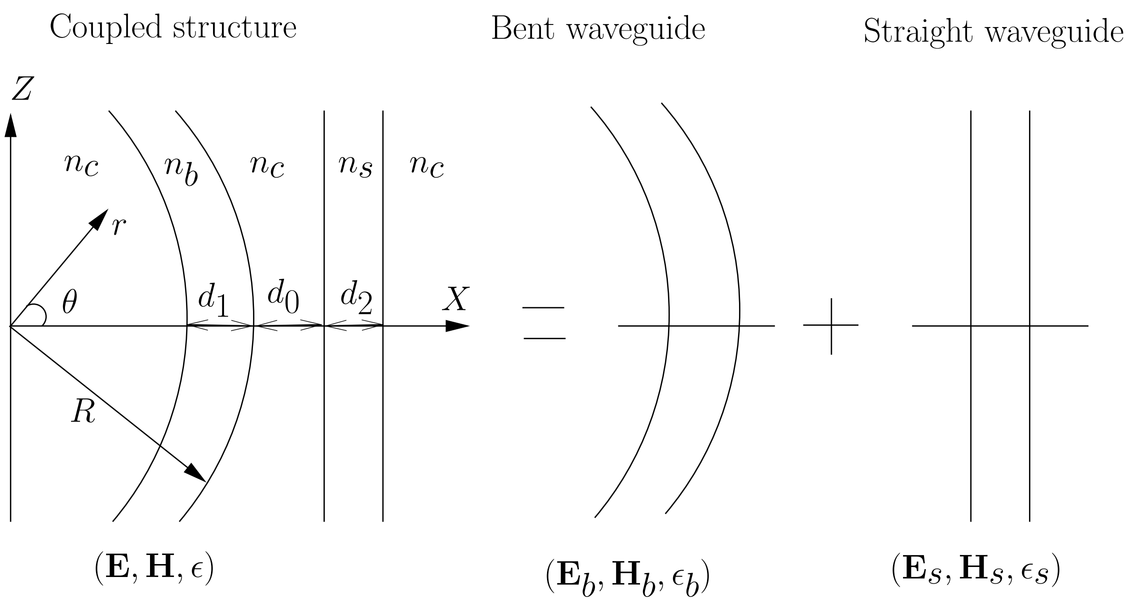
Bent Mode Profile



Absolute value and phase plot for E_y^0. $(n_s, n_f, n_c) = (1.6, 1.7, 1.6)$, $d = 1$, $\lambda = 1.3$.

Modeling of a Bent Coupler

- Forward propagating modes, no back reflections, all elements are linear



Coupled field Ansatz: $$\begin{cases} \mathbf{E}(x,z) = C_b(z)\mathbf{E}_b(x,z) + C_s(z)\mathbf{E}_s(x,z) \\ \mathbf{H}(x,z) = C_b(z)\mathbf{H}_b(x,z) + C_s(z)\mathbf{H}_s(x,z) \end{cases}$$

where $C_b(z)$ and $C_s(z)$ are unknown amplitudes.

Coupled Mode Equations

- **Lorentz Reciprocity Theorem** (LRT):

$$\int \nabla \cdot (\mathbf{E}_p \times \mathbf{H}_q^* + \mathbf{E}_q^* \times \mathbf{H}_p)\ dx \quad = \quad -i\omega\epsilon_0 \int (\epsilon_p - \epsilon_q)\mathbf{E}_p \cdot \mathbf{E}_q^*\ dx$$

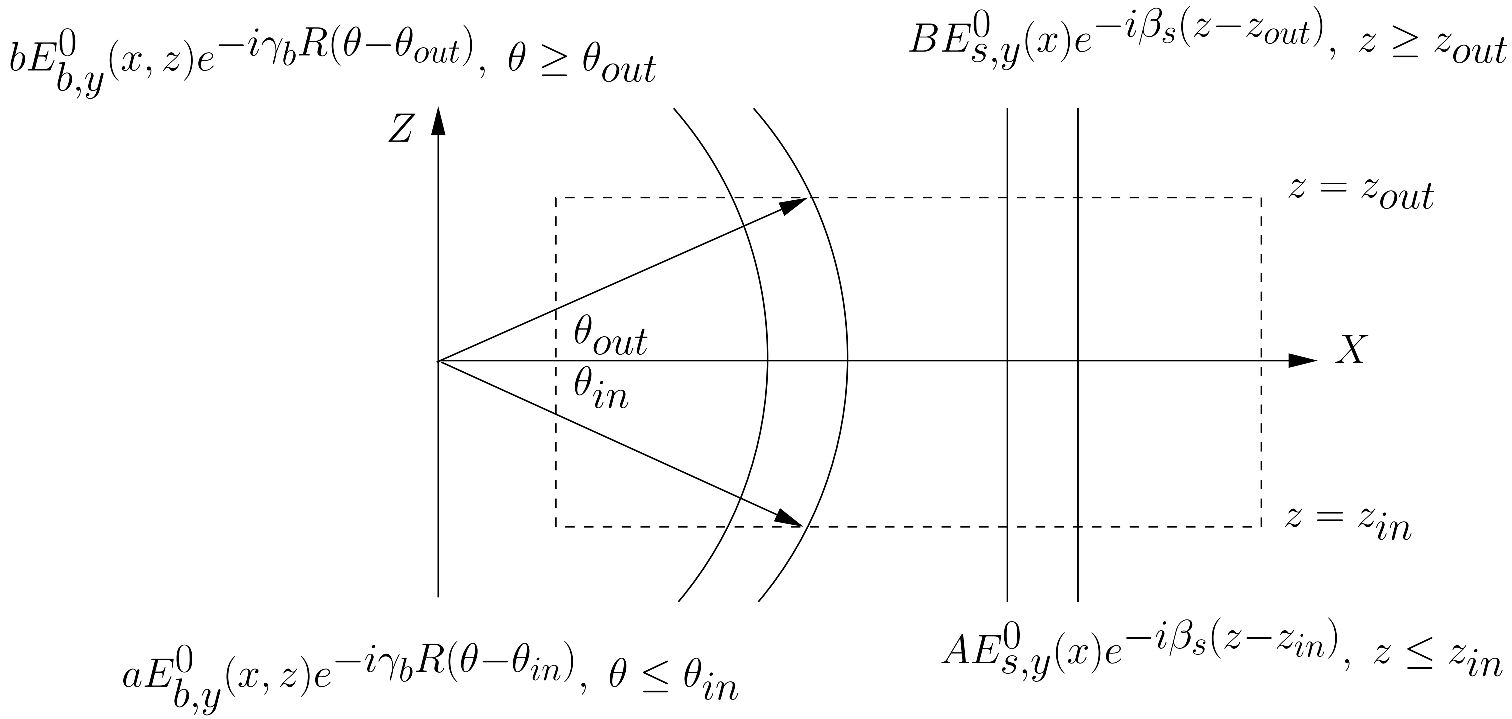
- **Coupled Mode Equations (CMEs)**:

$$\begin{bmatrix} \langle \mathbf{E}_b, \mathbf{H}_b | \mathbf{E}_b, \mathbf{H}_b \rangle & \langle \mathbf{E}_s, \mathbf{H}_s | \mathbf{E}_b, \mathbf{H}_b \rangle \\ \langle \mathbf{E}_b, \mathbf{H}_b | \mathbf{E}_s, \mathbf{H}_s \rangle & \langle \mathbf{E}_s, \mathbf{H}_s | \mathbf{E}_s, \mathbf{H}_s \rangle \end{bmatrix} \begin{bmatrix} d_z C_b \\ d_z C_s \end{bmatrix} = -i\omega\epsilon_0 \begin{bmatrix} \int \delta\epsilon_b \mathbf{E}_b \cdot \mathbf{E}_b^* dx & \int \delta\epsilon_s \mathbf{E}_s \cdot \mathbf{E}_b^* dx \\ \int \delta\epsilon_b \mathbf{E}_b \cdot \mathbf{E}_s^* dx & \int \delta\epsilon_s \mathbf{E}_s \cdot \mathbf{E}_s^* dx \end{bmatrix} \begin{bmatrix} C_b \\ C_s \end{bmatrix}$$

where $\langle \mathbf{E}_p, \mathbf{H}_p | \mathbf{E}_q, \mathbf{H}_q \rangle := \int \mathbf{a}_z \cdot (\mathbf{E}_p \times \mathbf{H}_q^* + \mathbf{E}_q^* \times \mathbf{H}_p) dx, \qquad \delta\epsilon_i := \epsilon - \epsilon_i$

- Solve it using Runge Kutta method of order 4.

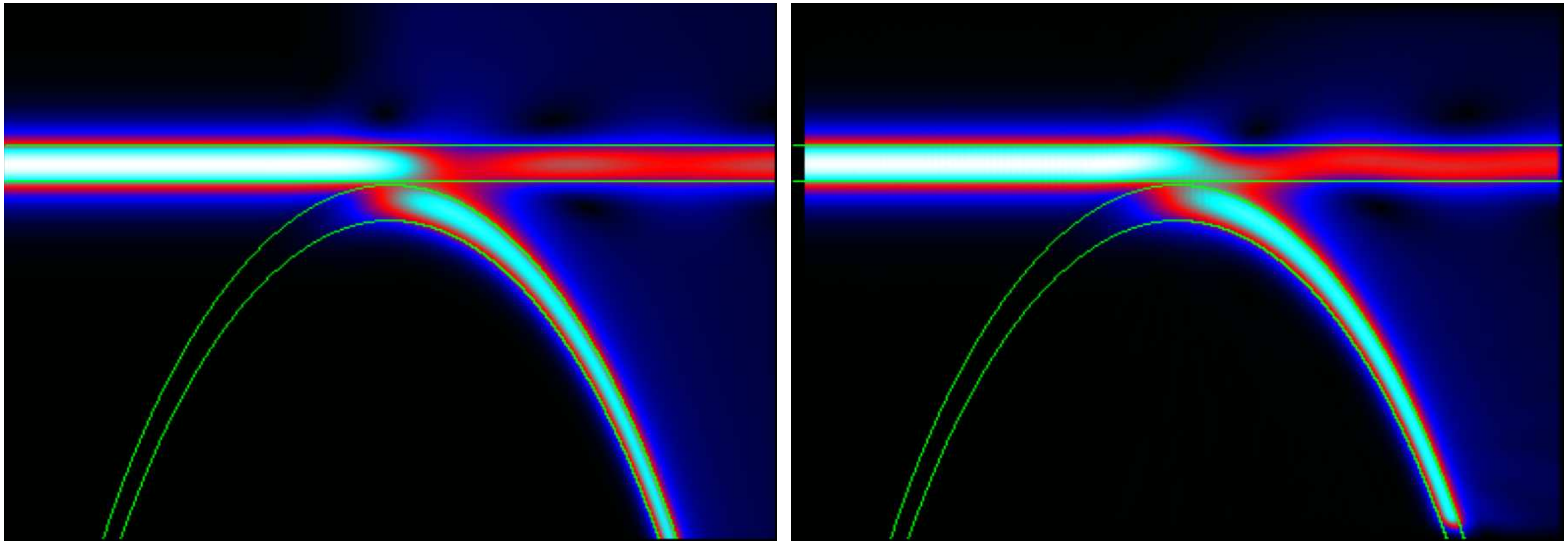
Scattering Matrix



$$\begin{bmatrix} b \\ B \end{bmatrix} = \underbrace{\begin{bmatrix} \rho & \chi \\ \kappa & \tau \end{bmatrix}}_{\text{scattering matrix}} \begin{bmatrix} a \\ A \end{bmatrix}$$

Comparison with FDTD

- Plots of absolute value of $\mathbf{E}$.

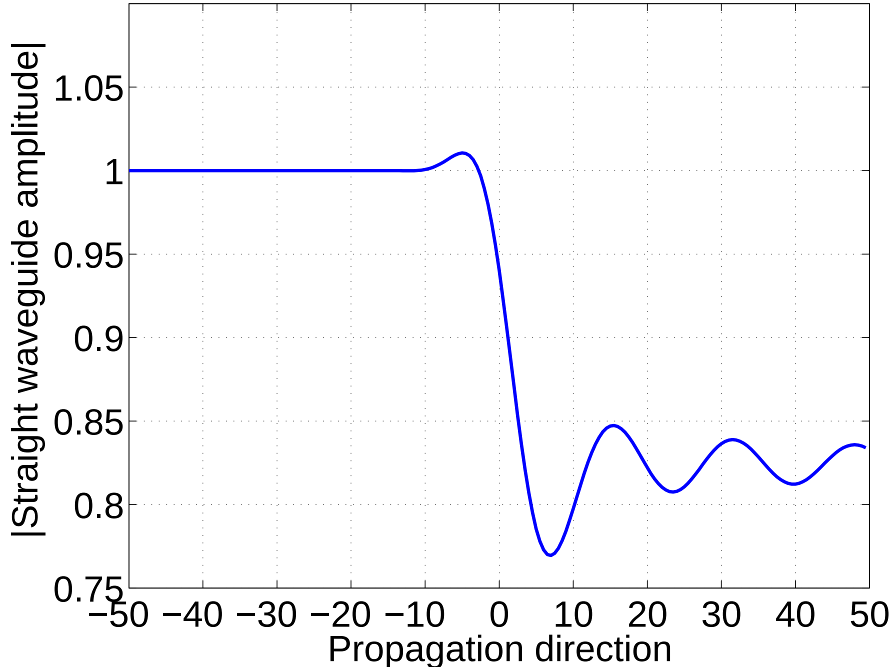


CMT FDTD

$n_b = n_s = 1.6$, $n_c = 1.45$, $d_1 = d_2 = 1\ \ \mu m$, $\lambda = 1.55\ \ \mu m$, $d_0 = 0.5\ \mu m$ and $R = 30\ \mu m$

Oscillating Straight Waveguide Amplitude

- Beyond bent waveguide region, amplitude of the straight waveguide field oscillates.

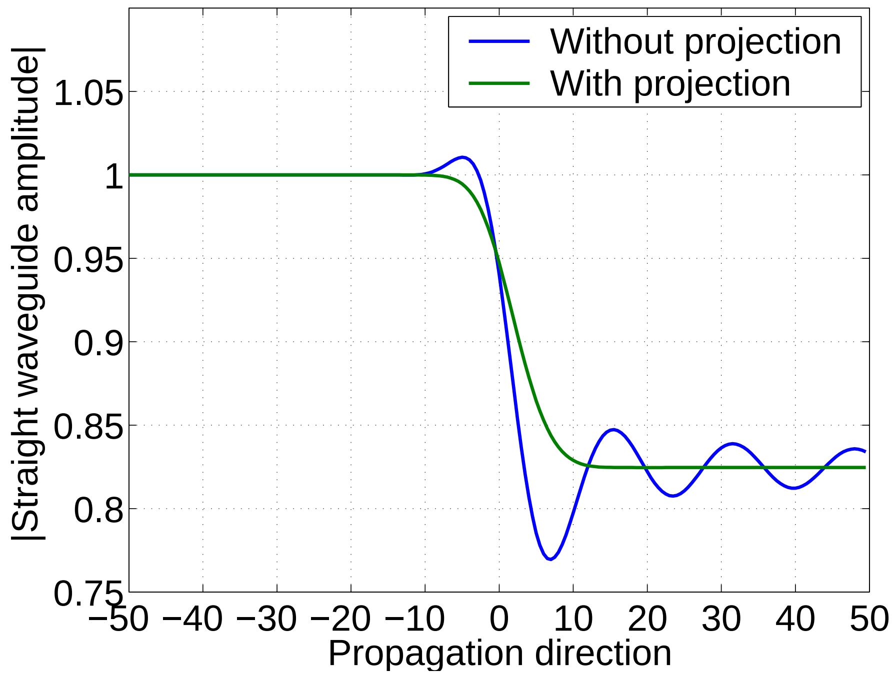


$n_b = n_s =$ 1.6, $n_c =$ 1.45, $d_1 = d_2 =$ 1 μm, $\lambda =$ 1.55 μm, $d_0 = 0.5\ \mu m$, $R = 30\mu m$

Projection

- Amplitude of the projection of coupled field on the straight waveguide field is stable.

$$G(z)|_{proj} = C_b(z)\frac{\langle \mathbf{E}_b, \mathbf{H}_b | \mathbf{E}_s, \mathbf{H}_s \rangle}{\langle \mathbf{E}_s, \mathbf{H}_s | \mathbf{E}_s, \mathbf{H}_s \rangle} e^{-i\beta z} + C_s(z) e^{-i\beta z}$$



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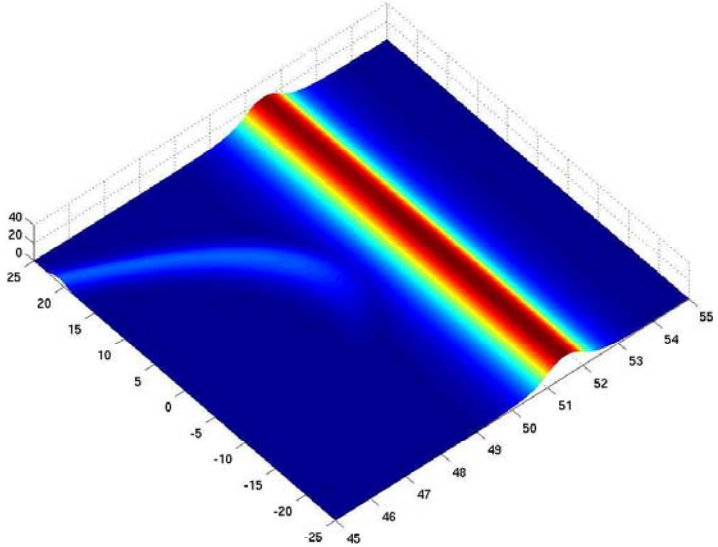
Projection operations are incorporated in the scattering matrix.

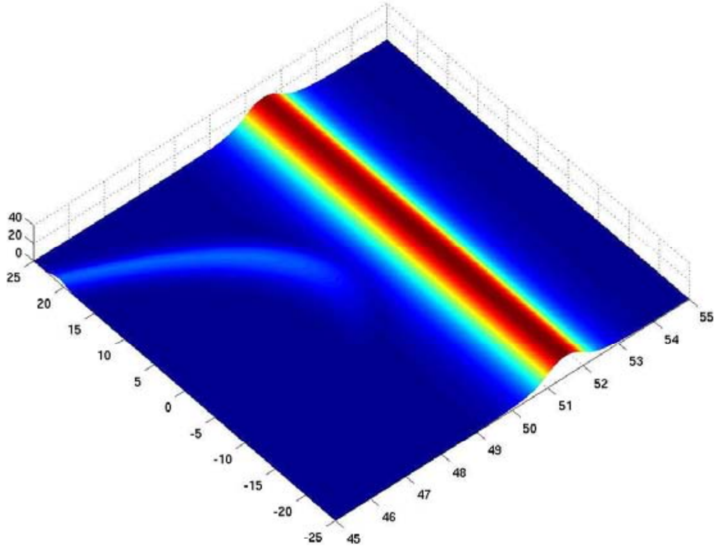
Separation distance = 1.0 μm Separation distance = 0.8 μm



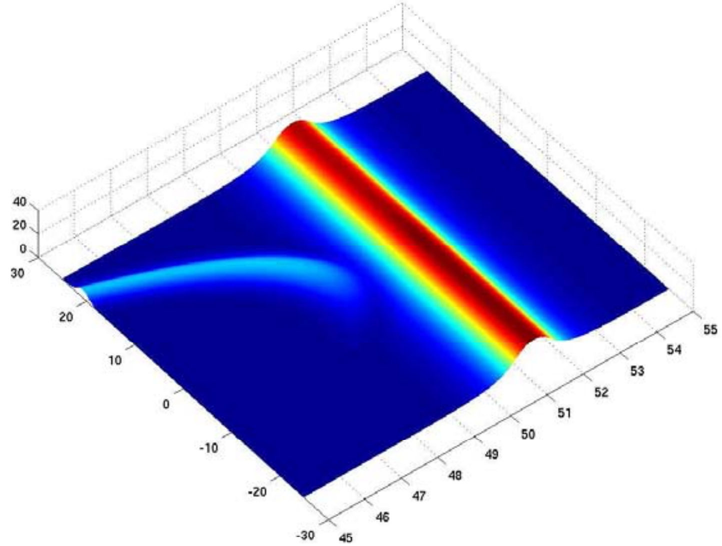
Separation distance = 0.6 μm Separation distance = 0.4 μm

$n_b = n_s$= 1.6, n_c=1.45, λ= 1.55 μm, $d_1 = d_2 = 1$ μm, $R = 50 \mu m$.

Separation distance = 1.0 μm



Separation distance = 0.8 μm



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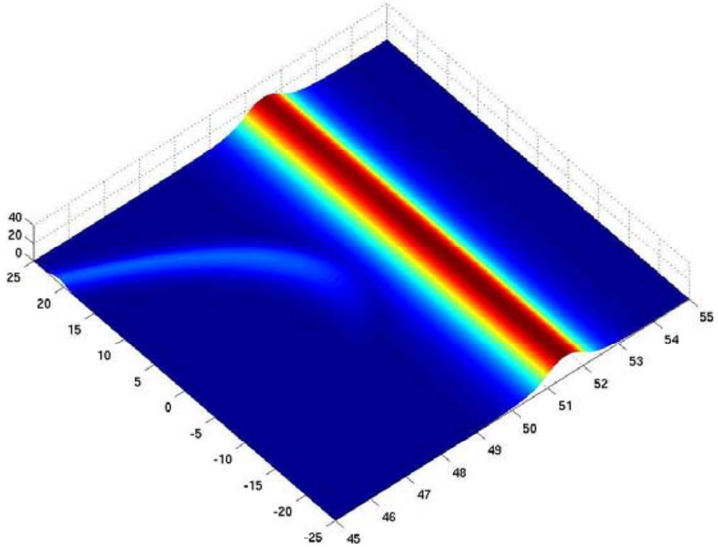
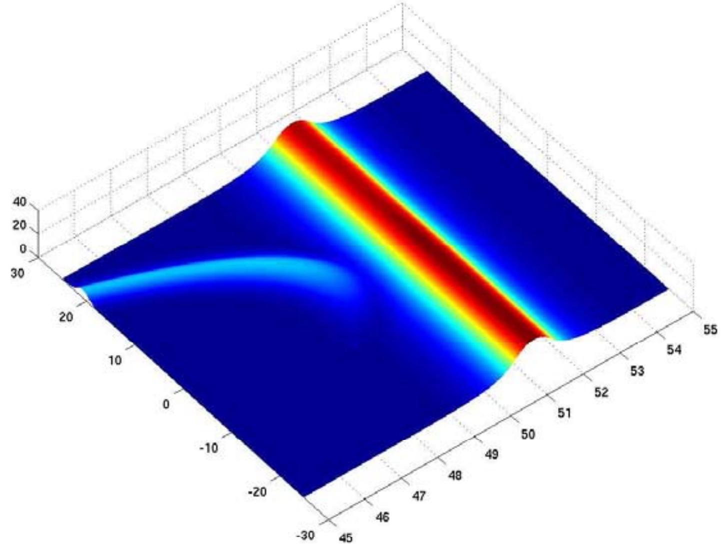
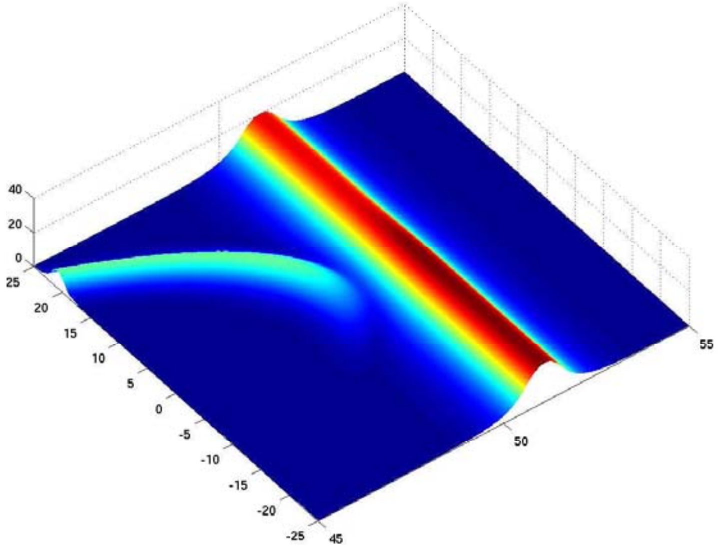


Separation distance = 0.8 μm





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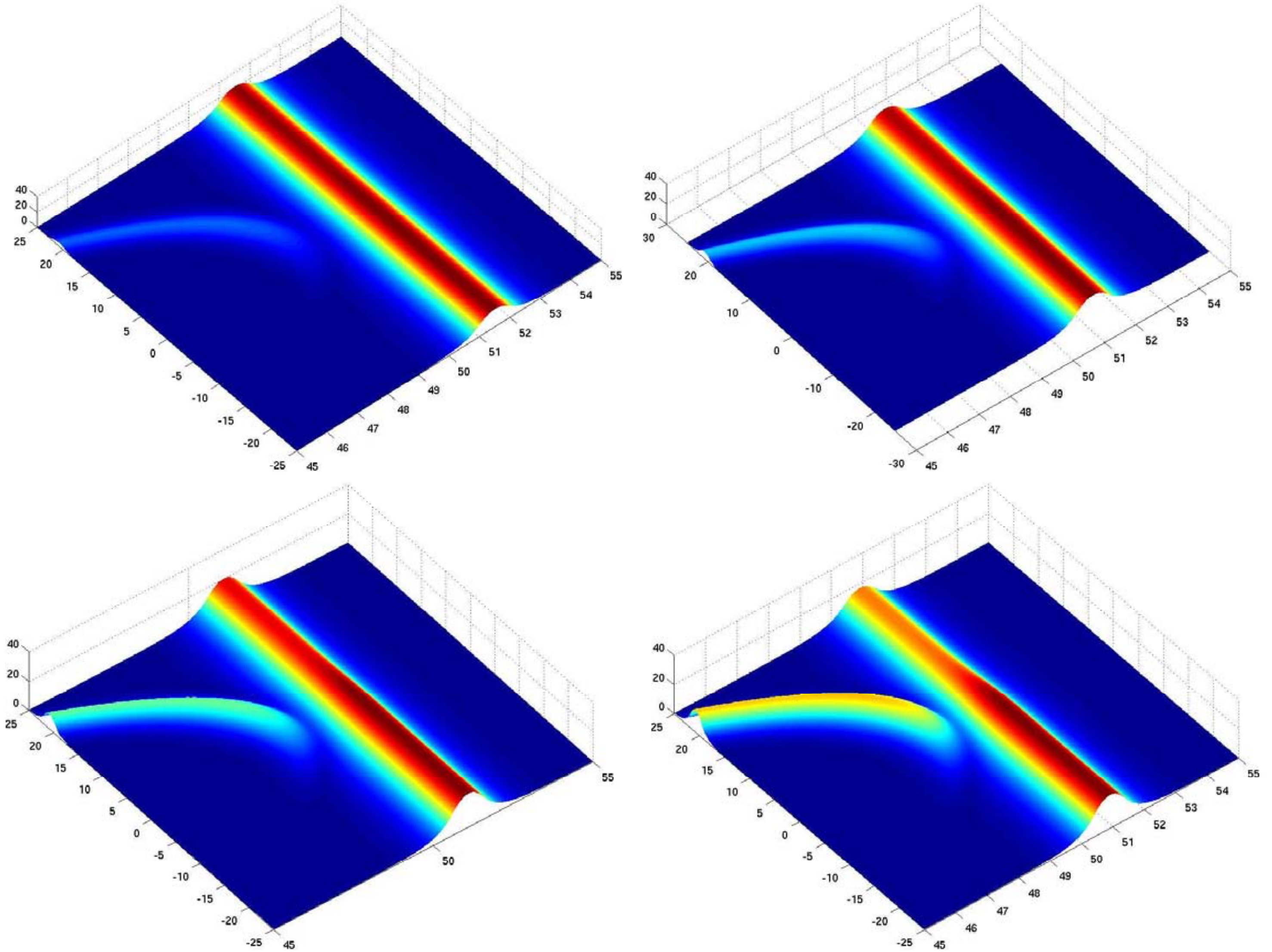

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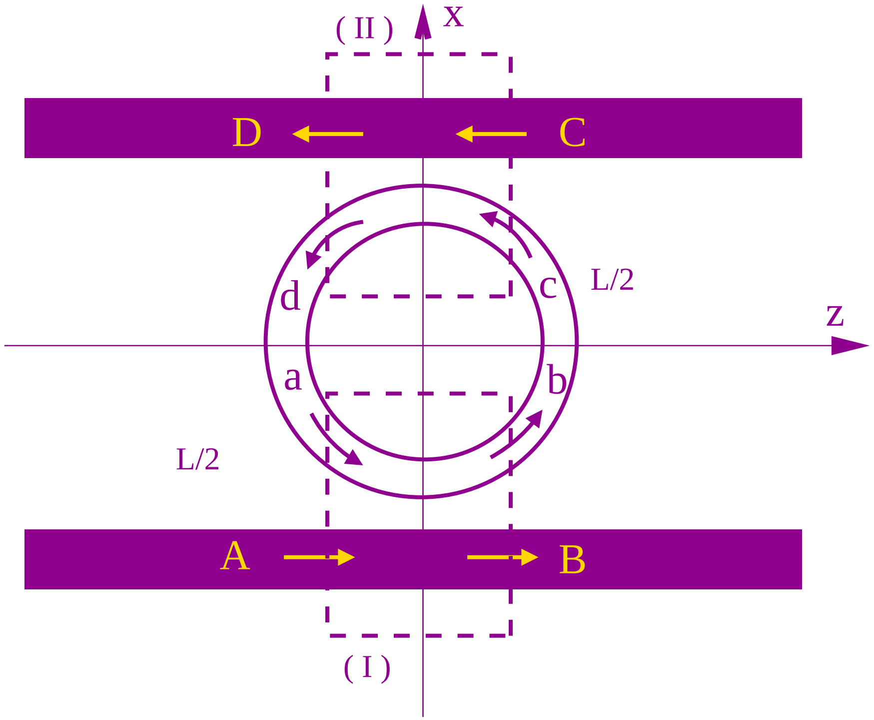
Modeling of Microresonators

- Negligible back reflections, negligible interaction outside coupler.
- $A, B, C, D,\ a, b, c, d$ denote the amplitudes of the guided modes



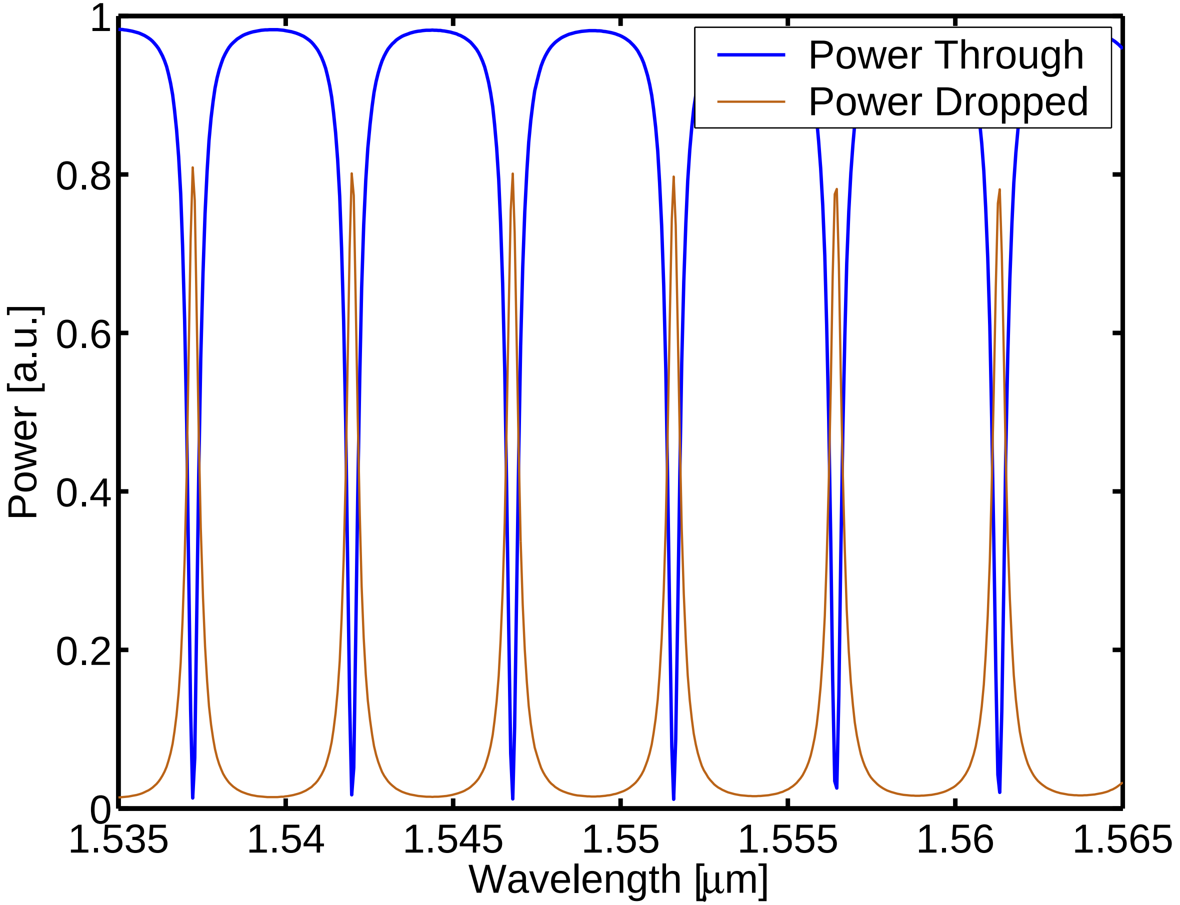
- Port Eqs:

$$\begin{bmatrix} B \\ b \end{bmatrix} = \begin{bmatrix} \rho & \chi \\ \kappa & \tau \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} \quad \begin{bmatrix} D \\ d \end{bmatrix} = \begin{bmatrix} \rho & \chi \\ \kappa & \tau \end{bmatrix} \begin{bmatrix} C \\ c \end{bmatrix}$$

$$a = d \exp(-i\gamma L/2) \quad c = b \exp(-i\gamma L/2)$$

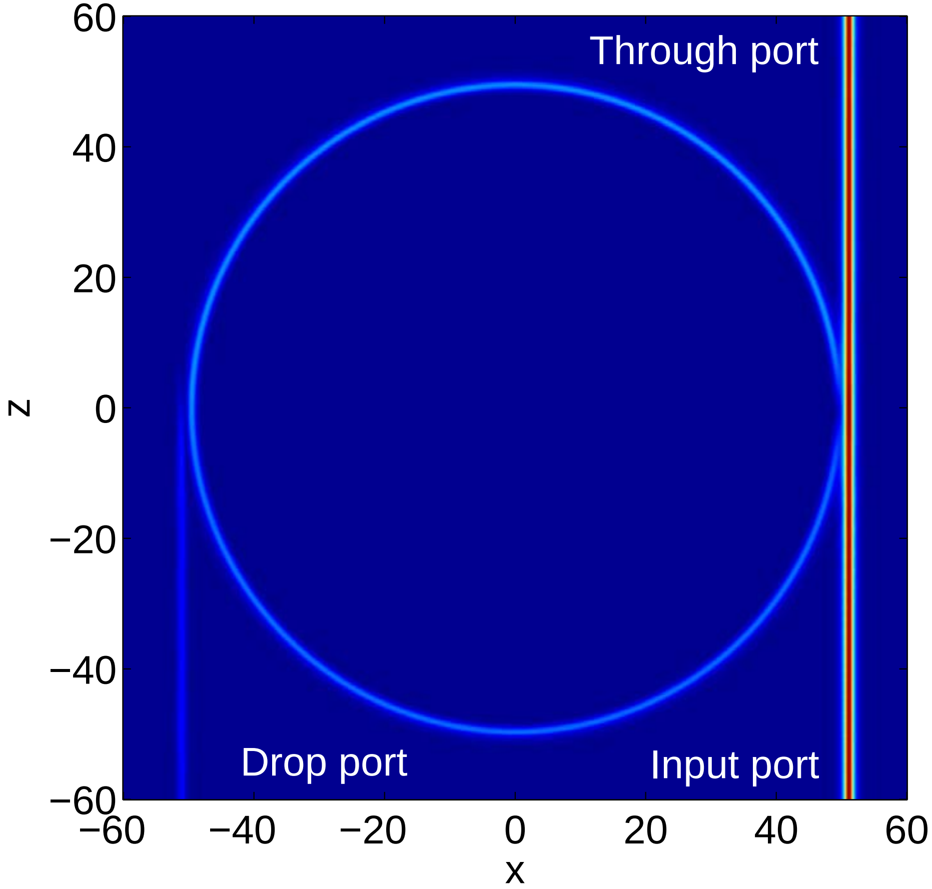
- $A = \sqrt{P_{in}}, C = 0$, solve above eqs. for $P_T = |B|^2$ and $P_D = |D|^2$

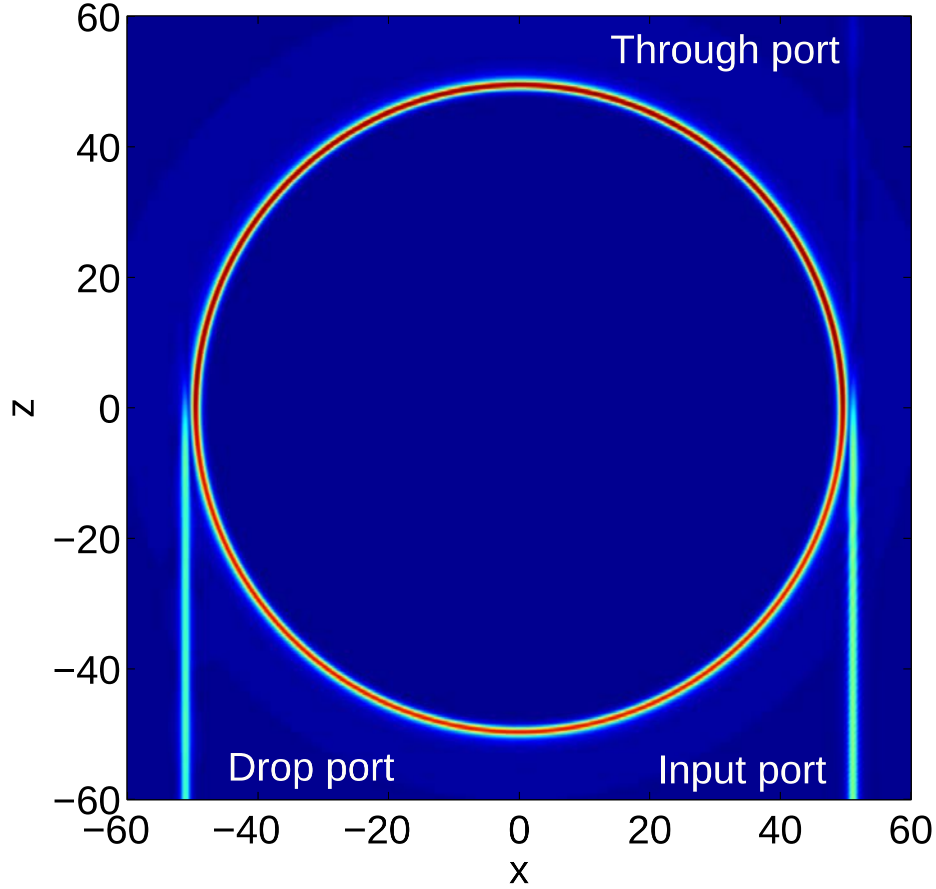
Microresonator Simulations



n_b=n_s=1.6, n_c=1.45, R= 50 μm, d_1=d_2=1 μm,, d_0=0.6 μm

- Absolute value of $\mathbf{E}$ field:





Off resonance

λ=1.54678 μm

$P_T \approx$ 98 % ,$P_D \approx$ 1.5 %

Resonance

λ=1.54925 μm

$P_T \approx$ 1.1 %, $P_D \approx$ 80 %

Conclusions

- $\surd$ Here we presented *spatial coupled mode theory* modeling of 2D microresonator.
- $\surd$ For this approach, it is essential to have access to the analytical representation of the bent modes. We had developed semi-analytical bent mode solver.
- $\surd$ Then we modeled a Bent -Straight waveguide coupler using a spatial coupled mode theory. The results of this model agree very well with the FDTD results.
- $\surd$ Using the scattering matrix of a Bent -Straight waveguide coupler a microresonator is modeled.

Acknowledgment

We are thankful to Brenny van Groesen, Hugo Hoekstra for useful discussions.