Dual-Spacecraft Formation Flying in Deep Space: Optimal Collision-Free Reconfigurations

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I. Introduction

The distributed space system architecture has been identified as a new paradigm for many of the future NASA (Earth and deep space), U.S. Air Force, U.S. Navy, and commercial satellite space missions. Examples where the distributed space system technology will have a tremendous impact in the conceptualization and the development of the future space missions include space-borne optical interferometry, three-dimensional stereo imaging, robotic planetary explorations, and atmospheric monitoring and control. At the same time, the distributed spacecraft framework poses a unique set of challenges in the areas of planning, guidance, and spacecraft control. The most intuitive issues are in fact in the area of planning: For example, how a multiple spacecraft system should reconfigure itself while avoiding spacecraft collisions, a problem that is absent in the single spacecraft case. In this general venue, much effort has recently been devoted to devising conflict resolution algorithms for collision-free reconfigurations of multiple ground and aerial vehicles. These studies adopt a variety of approaches including game theory, semidefinite programming, potential fields, and mixed-integer linear programming. In this paper, we consider the optimal collision-free reconfiguration for a dual-spacecraft system as a state constrained optimal control problem. It is often the case that such a problem formulation and the ensuing necessary optimality conditions do not lead to efficiently computable optimal control forces. However, we provide a solution to the dual-spacecraft optimal collision-free reconfiguration by relying on this approach.

The organization of the paper is as follows: In Sec. II, the mathematical formulation of the optimal collision-free dual spacecraft reconfiguration is presented. This is then followed by the characterization and the reparameterization of the optimal state trajectories and control forces via an optimal control formalism. Simulation results in Sec. III highlight some of the key features of the proposed framework for the dual-spacecraft reconfiguration problems.

II. Problem Statement and Parameterization of the Optimal Control

The problem considered in this paper is as follows: The initial inertial positions and the desired terminal relative position of the two spacecraft are given. We would like to specify the control forces that steer the dual spacecraft to the given terminal relative position, avoiding any collisions and requiring the minimum energy.

We note that the desired inertial positions for the two spacecraft are not necessary specified. In other words, the dual spacecraft is allowed to assume an inertial terminal position that is consistent with the specified relative terminal position. Moreover, the path that each spacecraft has to traverse to achieve the required reconfiguration is not explicitly sought for, that is, the problem considered is not a guidance problem. Let us proceed now by denoting the translational inertial states of the two spacecraft and as and . Furthermore, let the respective control forces on the dual spacecraft be and . The optimal collision-free reconfiguration problem can thereby be stated as

\[
J := \int_{t_0}^{t_f} u(t)^T u(t) + u(t)^T u(t) \, dt \rightarrow \min
\]

subject to the dynamic constraints,

\[
\dot{x}_i(t) = A x_i(t) + B_i u_i(t), \quad \dot{x}_j(t) = A x_j(t) + B_j u_j(t)
\]

initial inertial and final relative positions, , and , and the parameter , , collision avoidance constraint,

\[
\|C(x_i(t) - x_j(t))\| \geq \rho_j, \quad \text{for all } t \in [t_0, t_f]
\]

where \(\| \cdot \|\) denotes the 2-norm. In Eq. (2) and (3), one has \(A \in \mathbb{R}^{6 \times 6}\), \(B_i, B_j \in \mathbb{R}^{6 \times 3}\), and \(C \in \mathbb{R}^{6 \times 6}\), where

\[
A = \begin{bmatrix}
0_{3 \times 3} & I_{3 \times 3} \\
0_{3 \times 3} & 0_{3 \times 3}
\end{bmatrix}, \quad B_i = \frac{1}{m_i} \begin{bmatrix}
0_{3 \times 3} \\
I_{3 \times 3}
\end{bmatrix},
\]

\[
B_j = \frac{1}{m_j} \begin{bmatrix}
0_{3 \times 3} \\
I_{3 \times 3}
\end{bmatrix}, \quad C = [I_{3 \times 3}, 0_{3 \times 3}]
\]

and \(m_i\) and \(m_j\) denote the mass of spacecrafti and j, respectively. We note that a double integrator has been employed to represent each spacecraft dynamics, effectively ignoring the orbital forces. This simplification has been adopted in view of the following observation. Consider a deep space Earth-trailing dual-spacecraft formation flying mission, with its two spacecraft only a few kilometers apart. Assume furthermore that the masses of the two spacecraft are on the order of a few hundred kilograms. Then when the linearized Hill’s equation (see Ref. 16) is used, it can be shown that the differential orbital force between the two spacecraft is on the order of \(10^{-23}\) N. Because the reconfiguration scenarios that we are interested in occur on relatively short timescales (as compared to, for example, 365 days), ignoring the differential orbital forces in this work is well justified.

Let us first reparameterize the optimal control problem Eqs. (1–3) in terms of the relative state of the two spacecraft \(z(t) := x_i(t) - x_j(t) \in \mathbb{R}^6\). With the combined control force \(u^{(c)} := u(t) + u(t) \in \mathbb{R}^6\), the reparameterization optimal control problem takes the form

\[
J := \int_{t_0}^{t_f} u^{(c)}(t) + u^{(c)}(t) \, dt \rightarrow \min
\]

subject to the dynamics

\[
\dot{z}(t) = A z(t) + B u(t)
\]

relative initial and desired terminal states \(z(t_0)\) and \(z(t_f)\) and the relative state constraint

\[
z(t)^T C^T C z(t) \geq \rho_j^2, \quad \text{for all } t \in [t_0, t_f]
\]

In this case one has

\[
a = \begin{bmatrix}
0_{3 \times 3} & I_{3 \times 3} \\
0_{3 \times 3} & 0_{3 \times 3}
\end{bmatrix}, \quad b = \begin{bmatrix}
0_{3 \times 3} \\
(1/m_j)I_{3 \times 3} & (1/m_j)I_{3 \times 3}
\end{bmatrix},
\]

\[
c = [I_{3 \times 3}, 0_{3 \times 3}]
\]

Let us point out several characteristics of the optimal dual-spacecraft reconfiguration problem as represented by Eqs. (4–6). First, we note
that the dynamics of the two spacecraft are merely coupled through
the final desired relative position and the pointwise-in-time state
constraints Eq. (6). The diameter of the collision avoidance region
$\rho_{ij}$ in Eq. (6), in general, is a reflection of the designer's assessment
of how far the actual spacecraft is away from being a point mass.
Moreover, given that we envision the dual-spacecraft reconfigura-
tion to be an integral part of a timed space mission, the final time
t has been specified in the objective functional Eq. (4). The most
distinctive feature of the optimal control problem Eqs. (4–6) is the
presence of the state inequality constraint as expressed by
Eq. (6). We note that, at the outset, it is not clear when this
transition, from constrained to unconstrained mode, and vice
versa, takes place for the optimal control forces. In the absence of
any other reasonable guess on the switching times, we assume that
the unconstrained mode corresponds to $[U] = [U_1] \cup [U_2]$, where
$[U_1] = [t_1, t_f]$ and $[U_2] = [t_2, t_f]$; and the constrained mode
corresponds to $[C] = [t_1, t_f] \cup [U_1] = [t_1, t_2]$; with this assumption intact,
we now proceed to derive the mode-dependent optimality
conditions.

First, consider the Hamiltonian $\mathcal{H} := u(t)^T A(t) + \lambda(t)^T Z(t) + Bu(t)$
and the Lagrangian $L := \mathcal{H} + \mu(t) (z(t)^T C(t) z(t) - \rho^2_{ij})$
for the optimal control problem Eqs. (4–6), where $\lambda(t)$ and $\mu(t)$
are the multipliers that adjoint the constraints expressed by
Eqs. (5) and (6), respectively. When the inequality state constraint
Eq. (6) is inactive, that is, $z(t)^T C(t) z(t) > \rho^2_{ij}$, the multiplier $\mu(t)$
is set to zero. In this case, the Lagrangian reduces to the Hamiltonian
recovering the classical, state constrained-free, optimal energy con-
trol problem. The necessary optimality conditions for the uncon-
strained mode are:

$$\dot{z}(t)^T = -\lambda(t)^T A(t) + \lambda(t)^T B(t)$$
$$z(t)^T (z(t)^T C(t) z(t) = \rho^2_{ij}$$

with the vectors $z(t_0)$ and $z(t_f)$ already specified. From Eq. (8), it
now follows that the optimal state and control trajectories during
the first unconstrained mode assume the forms

$$z(t)^{U_1} = m \left[ \begin{array}{c} \frac{(t - t_0)^2}{2} - \frac{(t - t_0)^2}{2} I_{3 \times 3} \\ \frac{(t - t_0)(t - t_1)}{2} I_{3 \times 3} \\ \frac{(t - t_0)(t - t_1)}{2} I_{3 \times 3} \end{array} \right] y^{U_1} + \left[ \begin{array}{c} z(t_0) \\ \frac{(t - t_0) z(t_0)}{2} \end{array} \right]$$

$$u(t)^{U_1} = \frac{1}{2} \left[ \begin{array}{c} (t - t_0)^2 z(t_0) \\ -(t - t_0) z(t_0) \\ (t - t_0) z(t_0) \end{array} \right] y^{U_1}$$

where $m = (m_{1}^2 + m_{2}^2)/(2m_{1}^2 m_{2})$. Similarly, for the second uncon-
strained mode, one has

$$z(t)^{U_2} = m \left[ \begin{array}{c} \frac{(t - t_f)^2}{2} - \frac{(t - t_f)^2}{2} I_{3 \times 3} \\ \frac{(t - t_f)(t - t_f)}{2} I_{3 \times 3} \\ \frac{(t - t_f)(t - t_f)}{2} I_{3 \times 3} \end{array} \right] y^{U_2} + \left[ \begin{array}{c} z(t_f) \\ \frac{(t - t_f) z(t_f)}{2} \end{array} \right]$$

$$u(t)^{U_2} = \frac{1}{2} \left[ \begin{array}{c} (t - t_f)^2 z(t_f) \\ -(t - t_f) z(t_f) \\ (t - t_f) z(t_f) \end{array} \right] y^{U_2}$$

The yet unspecified vectors $y^{U_1}, y^{U_2} \in \mathbb{R}^{n}$ in Eqs. (9–12) will
be determined shortly via the boundary conditions. For the con-
strained mode, corresponding to the time interval $[t_1, t_2]$, we let

$$z(t)^T C(t) z(t) = \rho^2_{ij}.$$ In this case, the necessary optimality condi-
tions are:

$$\dot{z}(t)^T = -\lambda(t)^T A - 2\mu(t) z(t)^T C(t)$$
$$z(t)^T = A z(t) + B u(t),$$

$$z(t)^T C(t) z(t) = \rho^2_{ij}$$

The expressions for the state trajectories and control forces that
satisfy the necessary conditions Eq. (13) can be derived explicitly as:

$$z(t)^{C} = \begin{bmatrix} \rho_{ij} \cos(\omega t + \phi \omega) \cos \phi \omega \\ \rho_{ij} \cos(\omega t + \phi \omega) \sin \phi \omega \\ \rho_{ij} \sin(\omega t + \phi \omega) \end{bmatrix}$$

$$\frac{-1}{2m} \left[ \begin{array}{c} \rho_{ij} \cos(\omega t + \phi \omega) \cos \phi \omega \\ \rho_{ij} \cos(\omega t + \phi \omega) \sin \phi \omega \\ \rho_{ij} \cos(\omega t + \phi \omega) \end{array} \right]$$

The yet unspecified parameters $\omega, \phi, \phi \omega$, will also be determined
shortly. We note that a state feedback form of the optimal control
forces in Eqs. (10), (12), and (14) can also be obtained for the un-
constrained and constrained modes (see earlier web address).

We now consider the issues pertaining to the switching between
the unconstrained and the constrained operational modes for the
optimal dual-spacecraft reconfiguration. In this venue, we have al-
ready characterized the forms of the optimal state and control tra-
jectories in each mode, separately. Therefore, we need to fur-
ther specify 1) the time interval $[t_1, t_2]$ corresponding to the mode
switching in and out of the constrained mode and 2) the parameters
$\omega, \phi, \phi \omega$ that determine when and for how long the relative
state of the two spacecraft lives on the constrained region. We pro-
ced to address both issues using a continuity argument and a para-
meter minimization procedure. We first note that because the relative
state $z(t)$ is continuously differentiable, its continuity, in conjunc-
tion with the knowledge of the initial and terminal conditions, $z(t_0)$
and $z(t_f)$, can be employed to express the vectors $y^{U_1}$ and $y^{U_2}$
in Eqs. (9) and (11), as a function of the parameters $t_1, t_2, \omega, \phi, \phi \omega$.
This is accomplished by enforcing the boundary conditions
$z(t_1)^{U_1} = z(t_2)^{U_2}$ and $z(t_1)^{C} = z(t_2)^{C}$. By substituting these
expressions in Eqs. (9–12), we obtain a parameterization of the opti-
mal control forces and, thus, the objective function $J$ Eq. (4) in
terms of the parameters $t_1, t_2, \omega, \phi, \phi \omega$.

Furthermore, when $t_2 = \alpha t_1$ is set, for $\alpha \geq 1$, this parameterization assumes the form of
$J(t_1, \omega, \phi, \phi \omega, \alpha, \omega, \phi, \phi \omega)$. The exact functional form of this expression for
$J$ turns out to be:

$$J(t_1, \omega, \phi, \phi \omega, \alpha) = (\alpha - 1) t_1 \rho_{ij}^2 \omega^2 / 2$$
$$+ A / (t_1 - t_0)^3 + B / (\alpha t_1 - t_f)^3$$

where

$$A = 6 \rho_{ij}^2 + 2(t_1 - t_0)^2 \rho_{ij}^2 \omega^2 + 6 ||z(t_0)||^2 + 6(t_1 - t_0) z_{21}(t_0) z_{21}(t_0)$$
$$+ 2(t_1 - t_0)^2 z_{21}^2(t_0) + 2(t_1 - t_0)^2 z_{22}^2(t_0)$$
$$+ 6(t_1 - t_0) z_{21}(t_0) z_{21}(t_0) + 2(t_1 - t_0) \rho_{ij} \omega ||z(t_0)||^2$$
$$+ (t_1 - t_0) \rho_{ij} \omega ||z(t_0)||^2$$
$$+ (t_1 - t_0) ||z(t_0)||^2$$
$$+ (t_1 - t_0) ||z(t_0)||^2$$
$$+ (t_1 - t_0) ||z(t_0)||^2$$
$$+ (t_1 - t_0) ||z(t_0)||^2$$

The yet unspecified vectors $y^{U_1}, y^{U_2} \in \mathbb{R}^{n}$ in Eqs. (9–12) will
be determined shortly via the boundary conditions. For the con-
strained mode, corresponding to the time interval $[t_1, t_2]$, we let

$z(t)^T C(t) z(t) = \rho^2_{ij}$.
Here $\mathbf{z}_1(t) = [z_{11}(t), z_{12}(t), \ldots, z_{1N}(t)]^T$ and $\mathbf{z}_2(t) = [z_{21}(t), z_{22}(t), \ldots, z_{2N}(t)]^T$. When this parameterization is used, it can be shown that the performance index $J$ is a nondecreasing function of $\alpha$. Thus, a judicious strategy is to choose the parameter $\alpha$ as close to one as possible. In this direction, suppose that we initiate the procedure for finding the optimal control forces and the optimal state trajectories by fixing $\alpha$. The objective functional thus assumes the form $J_{\alpha}(\mathbf{t}_1, \mathbf{t}_2, \mathbf{z}_1, \mathbf{z}_2, \mathbf{u}_1, \mathbf{u}_2, \alpha)$. Consider then finding the minimizers of $J_{\alpha}$, denoted by $t_1^*(\alpha)$, $\mathbf{t}_2^*(\alpha)$, $\mathbf{z}_1^*(\alpha)$, and $\mathbf{z}_2^*(\alpha)$. Having found these minimizing parameters, one can then proceed to construct the corresponding candidate optimal relative state trajectory $\mathbf{z}_{1\alpha}(t)$ and the optimal control forces $\mathbf{u}_{1\alpha}(t)$. The quintuple $[\alpha, t_1^*(\alpha), \mathbf{t}_2^*(\alpha), \mathbf{z}_1^*(\alpha), \mathbf{z}_2^*(\alpha)]$, however, may or may not lead to a feasible state trajectory, depending on our choice of the parameter $\alpha$. (Note that we are carefully avoiding having to solve a constrained nonconvex optimization problem.) Our proposed procedure thereby involves a search subroutine for the value of $\alpha$ that leads to a feasible as well as an optimal state trajectory. The complete procedure is summarized as follows: 1) Given the initial and desired terminal relative configurations, parameterize the objective functional $J$ Eq. (4) via the parameters $t_1, \mathbf{t}_2, \mathbf{z}_1, \mathbf{z}_2, \mathbf{u}_1, \mathbf{u}_2$, and $\alpha$ in Eq. (15). This is accomplished by using the continuity conditions at $t_1$ and $t_2$ to eliminate the vectors $\mathbf{v}^T(t) = 1, 2$, in Eqs. (9)-(11), and finally by letting $t_2 = \alpha t_1$. 2) Let us assume that, without loss of generality, $t_0 > 0$; then because $at_1 = t_2 \leq t_1$ and $t_2 \geq t_0$, we conclude that $\alpha \leq t_1/t_0$. Now for a fixed value of $1 \leq \alpha \leq t_1/t_0$, solve the system of nonlinear equations $\mathbf{V}_{\alpha}(t_1, \mathbf{v}, \mathbf{z}_1, \mathbf{z}_2) = 0$ for the minimizing parameters $t_1^*$, $\mathbf{t}_2^*$, $\mathbf{z}_1^*$, and $\mathbf{z}_2^*$, leading to the candidate optimal state trajectory and optimal control forces. 3) If this constructed relative state trajectory is feasible, the control forces obtained are in fact optimal, and we terminate the procedure. Otherwise, a line search procedure is utilized, keeping in mind that the constrained region should not be traversed more than halfway, until a feasible relative state trajectory is found.

We conclude this section by commenting on an intuitive result pertaining to the relationship between the optimal performance Eq. (4) and the diameter of the constraint region $\rho_0$, Eq. (6). In fact, it can be shown that during the constrained mode, one has $\frac{\delta J}{\delta \rho_0} = -\mu(t) = 2\omega^2/m$, where $m = (m^2 + m^2)/(2m^2)$. Because the multiplier $\mu(t)$ is set to zero during the unconstrained mode, the rather intuitive relation $\frac{\delta J}{\delta \rho_0} \geq 0$ now follows directly: The optimal performance is a nondecreasing function of the diameter of the collision avoidance region.

### III. Simulation Results

We illustrate the viability of the proposed algorithm through two representative dual-spacecraft reconfiguration scenarios: case A, where $x_1(0) = [0 0 0 0 0 0]^T$, $x_2(0) = [10 10 10 0 0 0]^T$, and $x_1(10) - x_2(10) = [10 10 10 0 0 0]^T$; and case B, where $x_1(0) = [4.5 0 0 0 0 0]^T$, $x_2(0) = [8 0 0 0 0 0]^T$, and $x_1(10) - x_2(10) = [3.5 0 0 0 0 0]^T$. For both cases, the terminal time is set to 10 time units $\rho_0$, the required minimum relative distance is set to three length units, and the mass of each spacecraft is set to one mass unit. For case A, the initial and desired final states are such that the two spacecraft are required to interelate their positions; we note that they are not a priori required to attain particular inertial positions. When $\alpha = 1$, the optimal control forces are obtained by solving the four equations $\frac{\partial J_{\alpha}}{\partial t_1} = 0$, $\frac{\partial J_{\alpha}}{\partial \mathbf{t}_2} = 0$, $\frac{\partial J_{\alpha}}{\partial \mathbf{z}_1} = 0$, and $\frac{\partial J_{\alpha}}{\partial \mathbf{z}_2} = 0$, for the minimizing parameters $t_1^*$, $\mathbf{t}_2^*$, $\mathbf{z}_1^*$, and $\mathbf{z}_2^*$, and subsequently substituting these values into Eqs. (10), (12), and (14). These values are found to be $t_1^* = 5.0000$, $\alpha_1^* = -1.7304$, $\phi_2 = -1.7281$, and $\phi_2 = \pi/4 = 0.7854$. Figure 1 shows the corresponding optimal state trajectory. We note that each spacecraft does in fact attain the initial inertial position of the other. Moreover, for case A, it turns out that the value $\alpha = 1$, that is, $t_1 = t_2$, leads to a feasible relative state trajectory. Figure 2 confirms the feasibility of the relative state trajectory obtained by setting $\alpha = 1$ and that the relative state trajectory contacts the constraint region at $t = 5$.

For case B, we chose the initial and the terminal desired relative positions such that the dual spacecraft is close to violating the constraint at both instances. In fact, we note that one of the spacecraft has an initial velocity that is pointing toward the constrained region. For this case the value of the parameter $\alpha$ that leads to a feasible trajectory turns out to be strictly greater than one, that is, $t_1 \neq t_2$. It is in view of the fact that after calculating $t_1^*$, $\alpha_1^*$, $\phi_2^*$, and $\phi_2$ for $\alpha = 1$, the candidate optimal relative state trajectory is not feasible. Thus, we proceeded to search for a suitable value of $\alpha$ as prescribed in Sec. II. In this venue, we determined that, when $\alpha = 3.19$, the resulting relative state trajectory traverses the boundary during the time interval $[C] = [2.1347, 6.8097]$. Figures 3 and 4 show several state trajectories of the dual-spacecraft system for various choices of the parameter $\alpha$. As the parameter $\alpha$ varies toward the value $0$.

![Fig. 1 Optimal spacecraft trajectories for case A.](image)
3.19 (Fig. 4), a feasible and optimal trajectory is realized. Figures 3 and 4 also show that, for parameter value $\alpha = 5 > 3.19$, the corresponding state trajectory remains feasible; however, it now traverses the constrained mode for a longer time period. Figure 5 shows the optimal control forces for case B when $\alpha = 3.19$. As in the earlier case, the solid and the dashed lines represent, respectively, the first and the second spacecraft optimal control forces. We note that the control forces associated with the two unconstrained modes $u^{(U_1)}$ and $u^{(U_2)}$ are linear, as they are for case A; this linear behavior, however, vanishes during the constrained mode $[C] = [2.1347, 6.8097]$. For case B, the values of the parameters $\omega^*, \Phi_1^*, \Phi_2^*, t_1^*, t_2^*$, and $J^*$ are tabulated for several values of $\alpha$ in Table 1.

### Table 1 Parameter values for several choices of $\alpha$ for case B

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<th>$\alpha$</th>
<th>$\omega^*$</th>
<th>$\Phi_1^*$</th>
<th>$\Phi_2^*$</th>
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### IV. Concluding Remarks

We solved the deep space optimal dual-spacecraft reconfiguration problem in the presence of a collision avoidance requirement. Our approach is distinguished by its reliance on the necessary optimality conditions, similar in spirit to the Euler–Lagrange equations. In this venue, we identified the collision avoidance guarantee as a state inequality constraint in the corresponding optimal control problem. This modification significantly changes the solution characteristic of the optimal control and involves the introduction of the non-smooth multiplier, which dynamically monitors the violation of the state constraints. The time evolution of this multiplier essentially determines the switching times, which in turn, lead to the characterization of the optimal relative state trajectory. At the same time, we recognize that the extension of the proposed approach to more than two spacecraft scenarios involves expressions for the control parameters that are not easy to solve. However, the proposed scheme can be employed to obtain nontrivial upper bounds for the general optimal multiple spacecraft reconfiguration by reducing it to a sequence of two-at-a-time collision-free reconfigurations.

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