ROBUST SPEED CONTROL OF AN AUTOMOTIVE ENGINE USING SECOND ORDER SLIDING MODES

Mohammad Khalid Khan[⋆], **Sarah K. Spurgeon**[†] **and Paul F. Puleston**[‡]

[⋆†]Control Systems Research Group, Department of Engineering
University of Leicester, University Road, Leicester LE1 7RH, UK
fax: +44 116 252 2619
e-mail: {⋆mkk5,†eon}@le.ac.uk

[‡]CONICET and LEICI, Department of Electrotecnia
Universidad Nacional de La Plata (UNLP), Argentina
e-mail: puleston@venus.fisica.unlp.edu.ar

Keywords: Vehicles and Transportation Systems, Automotive Systems, Variable Structure Control, Robust Control, Flatness

Abstract

Speed control of an automotive engine based on sliding mode techniques has been presented. A 2-sliding "super twisting" algorithm has been applied since it does not require the time derivative of the sliding variable, which in this case would involve estimating engine acceleration. This and the associated flatness properties enable a sliding mode controller to be constructed which uses only measured engine speed and does not require the use of an observer. The controller tracks not only the set point speed but also shows robustness to the parameter variations and load torque disturbances and is valid across a wide operating envelope. Simulation results using a two subsystem IC engine model have also been presented.

1 Introduction

Speed control for an automotive engine is a nonlinear problem. An engine controller designed for this purpose should not only track the desired speed but also show robustness to modelling errors, load torque disturbances and be computationally efficient. Sliding mode control is known to be robust with a straightforward design formulation and thus provides a possible solution to the engine speed control problem.

Different sliding mode control strategies have been applied to the engine control problem [1, 2, 5, 15]. Choi and Hedrick [2] proposed an adaptive sliding mode control algorithm and demonstrated simulation results. They used a two-state model with throttle demand as the control variable. However, the resulting algorithm needs engine speed, manifold pressure and temperature, throttle body airflow and throttle position to be measured. Bhatti et al. [1] designed a sliding mode controller for idle speed operation using a linear model for the design and taking spark advance and air bypass valve as inputs. An observer was designed to reconstruct the system state for use by the controller. Vesterholm and Hendricks [15] used a mean value engine model with throttle angle as the control variable. They used a weighted sum of speed error and integral of speed error as the sliding variable.

In this paper, it is verified that the engine speed is an appropriate flat output and thus stabilizing this output alone will effectively stabilize the closed loop system. A two sliding algorithm is applied as presented by Levant [6–9]. The super twisting 2-sliding algorithm has been shown to be extremely robust and stable [6, 9]. Moreover it does not require the time derivative of the sliding variable, which in this case would involve acceleration. This and the associated flatness properties enables a sliding mode controller to be constructed which uses only measured engine speed and does not require the use of an observer. The engine model considered for simulation of the controller performance is adapted from that of Crossley and Cook [3] with the assumption that stoichiometric air-fuel ratio (AFR) is controlled by another independent controller.

2 Automotive Engine Model

The low frequency phenomenological representation of an automotive engine presented by Crossley and Cook [3] has been considered. The two main subsystems considered are:

1. The crankshaft dynamics
2. The manifold dynamics

The crankshaft speed state equation can be written as:

$$J\dot{n} = \tau_{eng} - \tau_l$$

where τ_{eng} is the torque produced by the engine, τ_l is the variable load torque and J is the effective engine inertia. The torque τ_{eng} is described by the following empirical function:

$$\begin{aligned}\tau_{eng} =& k_{e0} + k_{e1}m_a + k_{e2}(AFR) + k_{e3}(AFR)^2 \\ &+ k_{e4}\sigma + k_{e5}\sigma^2 + k_{e6}n + k_{e7}n^2 + k_{e8}n\sigma \\ &+ k_{e9}\sigma m_a + k_{e10}\sigma^2 m_a\end{aligned}$$

where, k_{ei}, $i = 1,\cdots,10$; are constant coefficients, AFR is the air-fuel ratio and σ is the spark advance. The variable m_a

is the air mass charged in the cylinder during the intake stroke, which takes place in the first π radians crankshaft rotation of the four-stroke cycle. Thus, in the model, m_a was obtained by integrating the air mass flow from the manifold and resetting the integrator at the end of each ingestion stroke. This results in a variable reset period, $t_{reset} = \pi/n$, which depends on the rotational speed. Finally, it is known that in the actual engine, a delay exists between the ingestion of the air-fuel and the related torque production. Therefore, an induction to power lag of π radians was assumed and, consequently, a variable delay (π/n) was included in the model. However, for the purpose of controller design, the output of the integrator block with variable reset can be closely approximated by:

$$m_a = \frac{\dot{m}_{ao}\pi}{n}$$

The nominal load torque, comprising of rolling friction, engine friction and aerodynamic drag torques, can be expressed as the function of the speed, n as follows:

$$\tau_l = \tau_r + \tau_f + \tau_a$$

with $\tau_r = k_{l1}$, $\tau_f = k_{l2} + k_{l3}n$ and $\tau_a = k_{l4}n^2$ where, $k_{li}, (i = 1, \cdots, 4)$ are constant coefficients.

The intake manifold dynamics, modelled as a first order differential equation, is:

$$\dot{p}_m = \frac{RT_m}{V_m}(\dot{m}_{ai} - \dot{m}_{ao})$$

where, R is the gas constant, V_m the manifold volume and T_m the manifold temperature. RT_m/V_m is assumed to be constant. The air mass flow rate into the cylinders from the manifold $\dot{m}_{ao}$, is the function of manifold pressure p_m, and speed n, given as:

$$\dot{m}_{ao} = k_{mo0} + k_{mo1}np_m + k_{mo2}np_m^2 + k_{mo3}n^2p_m$$

The air mass flow rate into the manifold $\dot{m}_{ai}$, is the function of manifold pressure p_m and the throttle angle θ, as follows:

$$\dot{m}_{ai} = f(\theta)g(p_m)$$

where,

$$\begin{aligned} f(\theta) &= k_{th0} + k_{th1}\theta + k_{th2}\theta^2 + k_{th3}\theta^3 \qquad (1) \\ g(p_m) &= \begin{cases} 1 & p_m \leq 0.5p_{atm} \\ \frac{2}{p_{atm}}\sqrt{p_{atm}p_m - p_m^2} & p_m > 0.5p_{atm} \end{cases} \end{aligned}$$

The coefficients k_{moi} and k_{thi}, $i = 0, \cdots, 3$ are constants determined by experimental data and p_{atm} is the atmospheric pressure.

Let the speed n, and the manifold pressure p_m, determine the state vector: $x = (x_1, x_2)^T = (n, p_m)^T$ and the control variable: $u = f(\theta)$. Then the state space description of the system can be written as

$$\begin{aligned} \dot{x}_1 &= \chi(x) \qquad (2) \\ \dot{x}_2 &= \xi(x) + \gamma(x)u \qquad (3) \end{aligned}$$

where

$$\begin{aligned} \chi(x) &= \frac{1}{J}\{K_1 + K_2(k_{mo0}/x_1 + k_{mo1}x_2 + k_{mo2}x_2^2 \\ &\quad + k_{mo3}x_1x_2) + K_3x_1 + k_{e7}x_1^2 - k_{l1} - k_{l2} \\ &\quad - k_{l3}x_1 - k_{l4}x_1^2\} \\ \xi(x) &= \frac{RT_m}{V_m}(-k_{mo0} - k_{mo1}x_1x_2 - k_{mo2}x_1x_2^2 \\ &\quad - k_{mo3}x_1^2x_2) \\ \gamma(x) &= g(x_2) \end{aligned}$$

with

$$\begin{aligned} K_1 &= k_{e0} + k_{e2}(AFR) + k_{e3}(AFR)^2 + k_{e4}\sigma + k_{e5}\sigma^2 \\ K_2 &= \pi(k_{e1} + k_{e9}\sigma + k_{e10}\sigma^2) \\ K_3 &= k_{e6} + k_{e8}\sigma \end{aligned}$$

Differentiating the first state equation we have

$$\ddot{x}_1 = \tilde{\phi}(x) + \eta(x)u \qquad (4)$$

where,

$$\begin{aligned} \tilde{\phi}(x) &= \frac{1}{J}\{(\frac{-K_2k_{mo0}}{x_1^2} + K_2k_{mo3}x_2 + K_3 + 2k_{e7}x_1 \\ &\quad - k_{l3} - 2k_{l4}x_1)\chi(x) + K_2(k_{mo1} + 2k_{mo2}x_2 \\ &\quad + k_{mo3}x_1)\xi(x)\} \end{aligned}$$

and

$$\eta(x) = \frac{K_2}{J}(k_{mo1} + 2k_{mo2}x_2 + k_{mo3}x_1)\gamma(x)$$

Remark 1: The engine model considered is differentially flat with speed as the flat output. Consider equation (2). It is possible to write x_2 as a function of x_1 and $\dot{x}_1$. Eliminating x_2 from equation (4) and using equation (3), u can be written as a function of $x_1, \dot{x}_1$ and $\ddot{x}_1$ [4, 5].

Remark 2: Due to the flatness of the system demonstrated above, it is sufficient to stabilize only the flat output which in this case is engine speed. The other variables will be automatically stabilized provided the reference trajectory does not pass through any singularities like zero speed in this case [4, 12].

3 Higher Order Sliding Modes

Higher order sliding modes (HOSM) have been developed in the literature and are currently finding useful applications [6–10, 13, 14]. The main idea behind higher order sliding is to act on the higher order derivatives of the sliding variable (s) rather than the first derivative as in standard sliding modes. Keeping the main advantage of the standard sliding modes, it has the additional advantage that it removes the chattering effect. The

r^{th} order sliding mode is determined by the equalities $s = \dot{s} = \ddot{s} = \cdots = s^{(r-1)} = 0$, which form an r-dimensional condition on the state of the dynamic system.

The sliding order is a measure for the degree of smoothness of the sliding variable in the vicinity of the sliding mode. In general, any r-sliding controller that keeps $s = 0$ needs $s, \dot{s}, \ddot{s}, \cdots, s^{(r-1)}$ to be made available [6, 7]. In the case of engine speed control, this implies that acceleration should either be measured or else an observer constructed to estimate it. However, because the "super-twisting" 2-sliding algorithm is used, no knowledge of the engine acceleration is required and hence speed control of an automotive engine based on speed measurement alone, without an observer, is possible.

The super twisting algorithm defines the control law, $u(t)$, as the combination of two terms. The first is defined in terms of a discontinuous time derivative while the second is the continuous function of the sliding variable. The trajectories of the super twisting algorithm are characterized by twisting around the origin on the phase portrait of sliding variable, see Fig. 1. Formally, consider a system of the form

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \phi(t,x) + \eta(t,x)u \end{aligned}$$

where $y_1 = s$, $y_2 = \dot{s}$ and $\phi(t,x)$, $\gamma(t,x)$ are smooth uncertain functions with $|\phi| \leq \Phi > 0$, $0 < \Gamma_m \leq \eta \leq \Gamma_M$. The so called super twisting algorithm converges to the 2-sliding set ($s = \dot{s} = 0$) in finite time and is defined by the following control law [6].

$$\begin{aligned} u(t) &= u_1(t) + u_2(t) \\ \dot{u}_1 &= \begin{cases} -u, & |u| > 1 \\ -W sign(s), & |u| \leq 1 \end{cases} \\ u_2 &= \begin{cases} -\lambda |s_0|^{\rho} sign(s), & |s| > s_0 \\ -\lambda |s|^{\rho} sign(s), & |s| \leq s_0 \end{cases} \end{aligned}$$

Where, $W > 0$, $0 < \rho \leq 0.5$ and $0 < s_0 > |s(t,x)|$ and bound on u as 1 is normalized. The simplified algorithm for systems linear in control and $s_0 = \infty$ will be employed in this study.

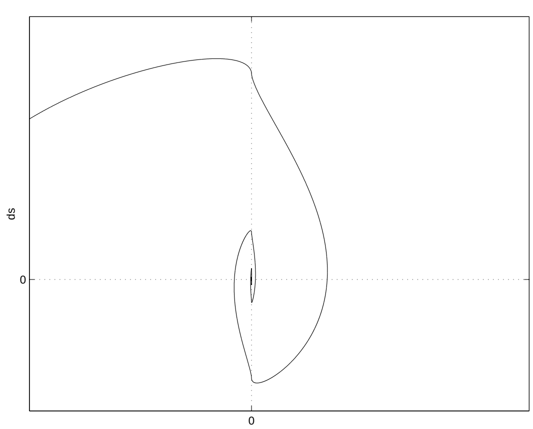

Figure 1: Phase portrait of Super Twisting algorithm

4 Engine Control Problem

The engine control is a real-life control problem. The existence of unknown torque disturbances and parameter variations, which in turn affect the speed, is an important issue to be considered. Increase in load torque results in a dip in the speed and vice-versa. Thus, the problem is to stabilize the engine speed, at the desired speed level, x_{1d}. The engine model has been discussed in Section 2, equations (2), (3) and (4). The sliding surface, $s = x_1 - x_{1d}$, i.e. the speed error, satisfies a second order differential equation of the form:

$$\ddot{s} = \phi(x) + \eta(x)u \tag{5}$$

where $\phi(x) = \tilde{\phi}(x) - \ddot{x}_{1d}$.

In the range of operation

$$|\phi| \leq \Phi > 0;$$
$$0 < \Gamma_m \leq \eta \leq \Gamma_M$$

The following control law defines the "super-twisting" 2-sliding algorithm [7, 8].

$$u(t) = -\lambda |s|^{\rho}\, sign(s) + u_1, \tag{6}$$
$$\dot{u}_1 = -W\, sign(s) \tag{7}$$

Corresponding sufficient conditions for finite time convergence to the sliding manifold are [6]

$$W > \frac{\Phi}{\Gamma_m} > 0 \tag{8}$$
$$\rho(\lambda\Gamma_m)^{\frac{1}{\rho}} > (\Gamma_M W + \Phi)(2\Gamma_M)^{\frac{1}{\rho}-2} \tag{9}$$
$$0 < \rho \leq 0.5 \tag{10}$$

The control law does not need any information on the time derivatives of the sliding variable, s, and no explicit knowledge of other system parameters. This not only reduces the number of sensors used but also reduces the computational burden of the controller. Moreover, it is also easy to tune.

5 Simulation Results

This section illustrates the performance of the controller through simulation results. The model considered here is the The Mathworks benchmark model [11]. Simulations are carried out for the speed starting from 300 rad/sec to 450 rad/sec and then back to 300 rad/sec. Within the range of operation system bounds are: $\Phi = 1.17 \times 10^4$, $\Gamma_m = 219.17$ and $\Gamma_M = 1.15 \times 10^3$. The sufficient values of W, λ and ρ satisfying equations (8), (9) and (10) are 53.5, 1.75 and 0.5 respectively. For simulation the controller coefficients chosen are: $W = 6$, $\lambda = 2$ and $\rho = 0.5$. The actual input θ has been calculated online by solving the third order algebraic equation (1). Different initial conditions chosen are: $n(0) = 300$, $P_m(0) = 0.6$ and $u_1(0) = 16$.

In the simulation it can be seen that the controller shows robustness in the presence of initial condition errors, parameter variations and unknown external disturbances. Uncertainties and disturbances have been taken in to consideration in the simulation, firstly, by modifying open loop parameters (such as rotational moment of inertia J, spark advance σ and optimal stoichiometric air-fuel ratio AFR, Fig. 7) up to 20% from their nominal values, secondly, by introducing unknown load torque variations, i.e. step change of constant power load at different speeds (representing constant power electrical appliances such as compressor of the air-conditioning unit) and filtered white noise (representing additional random torque disturbances). Variations in the nominal value of the manifold temperature (Fig. 6) were also included. The step change of constant power load (2.2 kW) is at $t = 3.5, 12, 16$ and 23 sec (Fig. 8). As can be seen in Fig. 2 and Fig. 3, there is no significant jump or dip at these points.

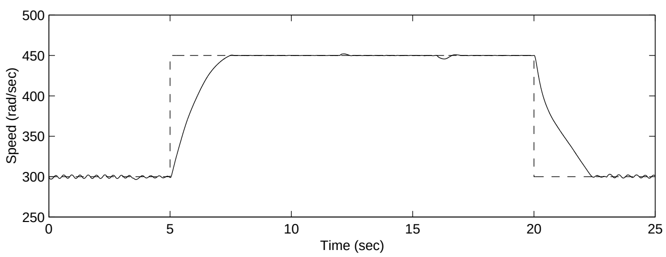

Figure 2: Rotational speed, n (rad/sec)

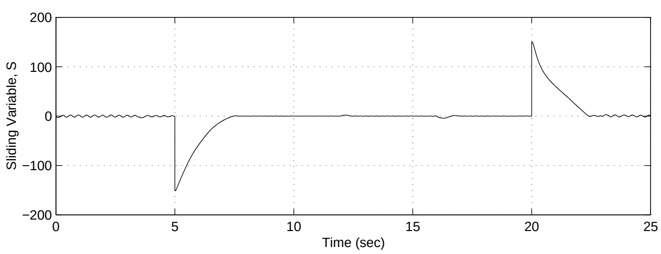

Figure 3: Sliding surface, s

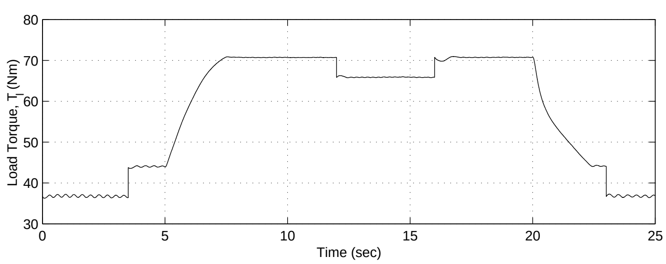

Figure 4: Load Torque, T_l (Nm)

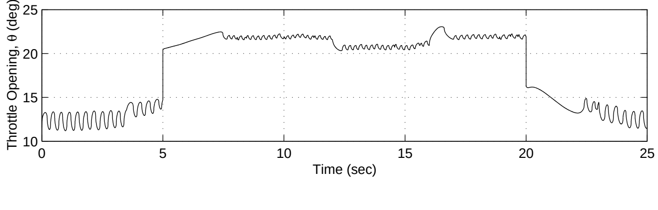

Figure 5: Throttle angle, θ (deg)

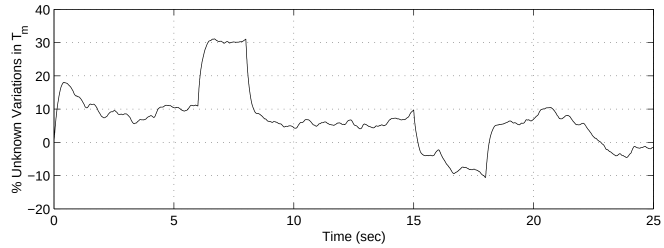

Figure 6: Unknown % variations in manifold temperature, T_m

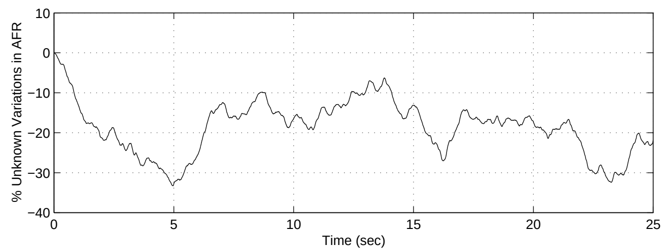

Figure 7: Unknown % variations in air-fuel-ratio, AFR

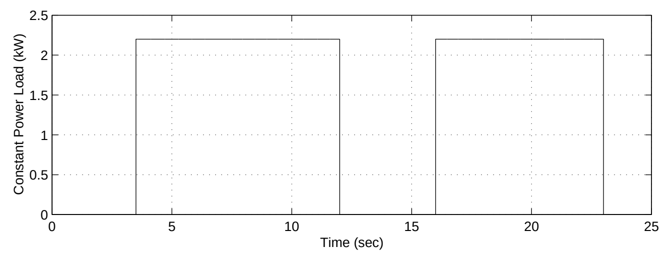

Figure 8: Unknown external disturbances due to activation and deactivation of car appliances (kW)

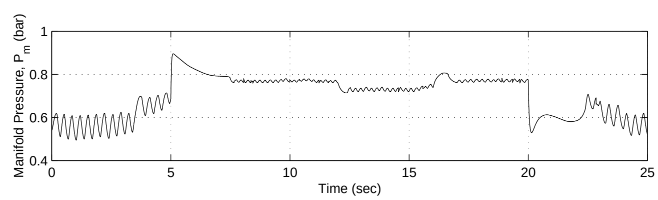

Figure 9: Manifold pressure, P_m

6 Conclusions

A sliding mode controller using a 2-sliding algorithm has been used for engine speed control. Due to the properties of the 2-sliding algorithm and a proven flatness property of the engine speed dynamics, the controller requires only measurement of the engine speed that is readily available. The performance of the closed loop system has been assessed using a comprehensive nonlinear simulation model. It has been shown that a 2-sliding controller not only tracks the set point speed but also shows robustness to parameter variations, initial speed and changes in the load torque.

References

[1] A. I. Bhatti, S. K. Spurgeon, R. Dorey, and C. Edward. Sliding mode configuration for automotive control. *Int. J. Adapt. Control Signal Process*, **13**:49–69, (1999).

[2] S.-B. Choi and J. K. Hedrick. Robust throttle control of automatic engines: theory and exprements. *J. Dynamic Systems, Measurement and Control*, **118**, (1996).

[3] P. R. Crossley and J. A. A. Cook. A nonlinear engine model for drive train system developement. In *Proc.IEE Int. Conf., Control'91*, **2**:921–925, Edinburgh, UK, (1991). Conference publication 332.

[4] M. Fliess, J. Lévine, P. Martin, and P. Rouchon. Flatness and defects of nonlinear systems. *Int. J. Control*, **61**, (1995).

[5] L. Kjergaard, S. Nielsen, T. Vesterholm, and E. Hendricks. Advance nonlinear engine idle speed control system. Technical Paper No. 940974. SAE, (1994).

[6] A. Levant. Sliding order and sliding accuracy in sliding mode control. *Int. J. Control*, **58**(6):1247–1263, (1993).

[7] A. Levant. Higher order sliding: Collection of design tools. In *Proc. ECC'97*, Brussels, (1997).

[8] A. Levant. Arbitrary-order sliding modes with finite time convergence. In *IEEE Mediterraniean Conf. on Control and Systems*, Alghero, Sardinia, Italy, (1998).

[9] A. Levant. Robust exact differentiation via sliding mode technique. *Automatica*, **34**(3):379–384, (1998).

[10] A. Levant and L. Fridman. Higher order sliding modes as a natural phenominon in control theory. In E. Garofalo and L. Glielmo, editors, *Robust control via variable structure and Lyapunov Techniques*, *Lecture Notes in Control and Optimization*, **217**, 107–133. Springer, London, (1996).

[11] Mathworks. Engine model. ftp://ftp.mathwork.com/pub /product-info/examples/autobook.zip.

[12] R. Rothfuß, J. Rudolph, and M. Zeitz. Flatness based control of a nonlinear chemical reactor model. *Automatica*,**32**:1433–1439, (1996).

[13] H. Sira-Ramirez. On the sliding mode control of nonlinear systems. *Systems and Control Letters*, **19**:303–312, (1992).

[14] H. Sira-Ramirez. On the dynamic sliding mode control of nonlinear systems. *Int. J. Control*, **57**(5):1039–1061, (1993).

[15] T. Vesterholm and E. Hendricks. Advance nonlinear engine speed control systems. In *Proc. of the American Control Conference*, pages 1579–1580, (1994).