Application of output feedback based dynamic sliding mode control to speed control of an automotive engine

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Abstract. The three stage mean value engine model (MVEM) of an IC engine has been first represented in Local Generalized Controller Canonical Form (LGCCF) and then dynamic sliding mode control has been employed for speed and manifold pressure trajectory tracking. The approach leads to dynamical discontinuous feedback resulting chatter free response. The particular choice of sliding surface is input dependent. Air-fuel-ratio is stabilized to its ideal stoichiometric value of 14.67 by using error as sliding surface.

1 Introduction

Output feedback stabilization of SISO and MIMO nonlinear systems in differential input-output (I-O) form via dynamic sliding mode has been studied from the theoretical perspective by Lu and Spurgeon [9, 10, 11], Sira-Ramirez [13]. Asymptotic stabilization using such techniques has been suggested to be appropriate for control of a rather general class of nonlinear systems appearing in differential I-O form, which may not be dynamic feedback linearizable. The nonlinear system does not need to be expressed in a regular form as is the case in many other sliding mode approaches, nor does it need to have full relative order which implies linearizability. This paper investigates the application of the method to automotive engine control.

Different sliding mode control strategies [7] have been applied to the engine control problem but complete control using MIMO techniques has not been presented. Choi and Hedrick [2] proposed an adaptive sliding mode control algorithm and demonstrated simulation results. They used a two-state model with throttle demand as the control variable and the resulting algorithm needs engine speed, manifold pressure and temperature, throttle body airflow and throttle position to be measured. Bhatti et al. [1]] designed a sliding mode controller for idle speed operation using a linear model for the design and taking spark advance and air bypass valve as inputs. An observer was designed to reconstruct the system state for use by the controller. Kjergaard et al. [8], Vesterholm and Hendricks [15] used a SISO mean value engine model with throttle angle as the control variable and speed as output. Recently, Weihua et al. [16] presented feedback linearization of MIMO engine model. Sliding mode control having proven robustness property is the natural choice for robust control. However, standard sliding mode control, when applied to engine, produces undesirable chattering.

In this paper, complete control of MIMO engine model using mixed sliding mode techniques is presented. A three stage MVEM model [5] of an IC engine is first represented in Local Generalized Controller Canonical Form (LGCCF) for the design purpose and then for A/F-ratio channel, traditional sliding mode control with error in A/F-ratio as the sliding variable is applied while dynamic sliding mode control is employed for the rest two channels namely speed and manifold pressure channel. In dynamic sliding mode control the particular choice of sliding surface is control dependent which always results in dynamic controller. Such dynamic policies are desirable in sliding mode as they effectively filter out the chattering of control signal.

2 Background

Consider a locally observable system in state space form

$$\dot{x} = f(x,u,t); \quad y = h(x,u,t) \tag{1}$$

$x \in |Re^n$, $u \in \Re^m$ and $f(x,u,t)$ and $h(x,u,t)$ are smooth vector functions, the following locally equivalent differential I-O form exits:

$$y_i^{(n_i)} = \phi_i(\hat{y},\hat{u},t), \quad i = 1,\cdots,p. \tag{2}$$

where $\hat{u} = (u,\cdots,u_j^{(\beta_j)})$; $j = 1,\cdots,m$ and $\hat{y} = (y,\cdots,y_i^{(n_i-1)})$ with $\sum_i^p n_i = n$. This representation is same as the Local Generalized Controller Canonical Form (LGCCF) of Fliess [4]. A differential I-O system is called proper if

(a) p=m,
(b) all $\phi_i,\ i = 1,\cdots,m$ are C^1 functions,
(c) the following regularity condition is satisfied

$$\det\left[\frac{\partial(\phi_1,\cdots,\phi_m)}{\partial(u_1^{(\beta_1)},\cdots,u_m^{(\beta_m)})}\right] \neq 0.$$

The corresponding zero dynamics of the system model (2) is defined as: $\phi_i(0,\hat{u},t) = 0$ The system (2) is called *minimum phase* if the zero dynamics is uniformly asymptotically stable.

2.1 Sliding Mode Control: A Review

In sliding mode control the design consists of two steps. Firstly, choice of the sliding surface (s) and secondly the selection of reachability condition. Sliding surface is chosen such that by zeroing it the control task is achieved. The reachability condition is to ensure that sliding mode is reached. The choice of sliding surface as the function of states, i.e., $s = S(x)$ is termed as static sliding surface and produces static feedback [3, 14]. This is a geometric manifold independent of control variable and the method employing this type of sliding surfaces is called direct sliding mode control method. The sliding surface which is a set of differential equations is termed as dynamic sliding surface and produces dynamic feedback [9, 12] which may be control independent or control dependent and the method is termed as indirect or dynamic sliding mode control method. Therefore, a sliding surface in general can be represented as: $s = S(t,y,\cdots,y_i^{(n_i)},u,\cdots,u_j^{(\beta_j)})$

Direct Sliding Surface In direct sliding mode control, sliding surface is linear combination of states, e.g.,

$$s_i = \sum_{j=1}^{n_i} a_{i,j} y_i^{(j-1)}; \quad i = 1, \cdots, m. \tag{3}$$

where $\sum_{j=1}^{n_i} a_{i,j}\lambda^{j-1}$ are Hurwitz polynomial with $a_{i,n_i} = 1$; $i = 1, \cdots, m.$

Indirect (Dynamic) Sliding Surface In indirect sliding mode, sliding surface have a dynamics. One of the dynamic sliding surface is

$$s_i = \sum_{j=1}^{n_i} a_{i,j}\ y_i^{(j-1)} + \phi_i(y, \dot{y}, \cdots, y_i^{(n_i-1)}, u, \dot{u}, \cdots, u^{(\beta_i)}, t); \quad i = 1, \cdots, m. \tag{4}$$

where $\sum_{j=1}^{n_i+1} a_{i,j}\lambda^{j-1}$ are Hurwitz polynomial with $a_{i,n_i+1} = 1$, $i = 1, \cdots, m.$ It is to be noted that this sliding surface is control dependent and makes system equivalent to asymptotically stable linear system in the ideal sliding mode.

In dynamic sliding mode control methods the switching occurs at the higher derivatives of the control and thus the actual control signals are continuous [9–11, 13]. This is one of the advantages of this technique.

Sliding Reachability Conditions After selecting the proper sliding surface sliding reachability condition is to be chosen which ensures that sliding mode will be attained and system trajectories will remain confined in the sliding mode once reached. One popular choice is $\dot{s}s < 0$, if $s \neq 0$, where derivative is taken along the system trajectories. There are several possible such conditions and can be broadly defined by:

Definition 1 A general sliding reachability condition is defined as

$$\dot{s} = \gamma(K, s) \tag{5}$$

where $s = [s_1, \cdots, s_m]^T$ and $K = [K_1 \cdots, K_m]^T$ are set of design parameters satisfying–

(1) $\gamma(K, 0) = 0$;
(2) $\gamma_i(K, s)$ is a C^1 function of s if $s \neq 0$, $i = 1, \cdots, m$;
(3) equation (5) is asymptotically stable.

A decoupled reachability condition is given by

$$\dot{s}_i = \gamma_i(K_i, s_i) \tag{6}$$

and each γ_i satisfy the above conditions. The decoupled reachability condition would force each $s_i \rightarrow 0$ independently. One of the decoupled reachability condition is

$$\dot{s}_i = -K_i s_i - k_{0i}\text{sign}(s_i) \quad i = 1, \cdots, m. \tag{7}$$

which has been used for this work.

Taking derivative of the sliding surface along the system trajectories and using reachability condition (7) to close the loop. The highest order derivative of control, $u_i^{(\beta_i+1)}$ which always appear linearly, can be solved, if regularity condition is satisfied, as

$$u_i^{(\beta_i+1)} = p_i(y, \dot{y}, \cdots, y_i^{(n_i)}, u, \dot{u}, \cdots, u^{(\beta_i)}, t); \quad i = 1, \cdots, m. \tag{8}$$

and then dynamic controller can be realized by chain of integrators.

3 IC Engine Model

There are number of IC engine dynamic models in the literature but the Mean Value Engine Model (MVEM) developed by Hendricks et al. [5], Hendricks and Sorenson [6] is mathematically compact and can be fitted for different engines easily. Therefore, the engine model adopted in this paper is the one developed in [5]. The engine model consists of three subsystems: Fuelling system, Air Flow system and Engine Speed dynamics. These subsystems are described briefly as follows.

3.1 Fluid Film Flow Model

Fluid flow through the manifold has two components namely fuel vapor flow $\dot{m}_{fv}$ and fuel film flow $\dot{m}_{ff}$ and the total flow $\dot{m}_f$ is not measurable. The dynamical sub-model is given as:

$$\ddot{m}_{ff} = \frac{1}{\tau_f}(-\dot{m}_{ff} + X\,\dot{m}_{fi}) \tag{9a}$$

$$\dot{m}_{fv} = (1-X)\dot{m}_{fi} \tag{9b}$$

$$\dot{m}_f = \dot{m}_{ff} + \dot{m}_{fv} \tag{9c}$$

$$\lambda = \frac{\dot{m}_{ap}}{14.67\dot{m}_f} = h(x) \tag{9d}$$

where,

$\dot{m}_f$ = cylinder port fuel flow,(kg/s)
$\dot{m}_{fi}$ = injected fuel mass flow, (kg/s)
$\dot{m}_{ff}$ = fuel film mass flow,(kg/s)
$\dot{m}_{fv}$ = fuel vapor mass flow,(kg/s)
τ_f = fuel evaporation time constant
X = fraction of $\dot{m}_{fi}$ which is deposited on manifold as fuel film
λ = normalized air-fuel (A/F) ratio.

3.2 Crankshaft Speed State Equation

This sub-model is derived using energy conservation laws

$$\dot{n} = \frac{1}{In}(H_u\,\eta_i\,\dot{m}_f(t-\tau_d) - P_l - P_b), \tag{10a}$$

$$P_l(n,p_i) = n\,(k_1 + k_2\,n\,+\,k_3\,n^2)\,+\,n\,(-k_4 + k_5\,n)\,p_i \tag{10b}$$

$$P_b = k_b\,n^3 \tag{10c}$$

where

η_i = $\eta_{in}\,\eta_{ip}\,\eta_{i\lambda}\,\eta_{i\theta}$

$\eta_{in}(n)$ = $k_6(1 - k_7\,n^{-0.36})$

$\eta_{ip}(p_i)$ = $k_8\,+\,k_9\,p_i\,-\,k_{10}\,p_i^2$

$\eta_{i\lambda}$ = $-k_{11} + k_{12}\,\lambda - k_{13}\,\lambda^2$

$\eta_{i\theta}$ = $e^{-\theta^2/2\theta_{mbt}^2}$

$\theta_{mbt} = \min(\min(\theta_1,\theta_2),45)$

$\theta_1 \;= k_{14}p_i + k_{15} + n_{47}$

$\theta_2 \;= k_{16}p_i + k_{17} + n_{47}$

$n_{47} \;= \begin{cases} 4.7\,n & n < 4.8 \\ 4.7 \times 4.8 & else \end{cases}$

I = total moment of inertia (kg-m^2)
= $I_{ac}(\pi/30)^2 1000$
n = crank shaft speed (krpm)
P_l = pumping & friction power (kW)
P_b = load power (kW)
p_i = manifold air pressure (bar)

H_u = Fuel heating value (kJ/kg)
η_i = indicated efficiency
θ = spark advance (degrees)
θ_{mbt} = max. break torque spark timing
L_{th} = stoichiometric air-fuel ratio
k_i = constants

3.3 Manifold Pressure State Equation

This is dynamic equation governing the air mass flow flowing through the manifold and is derived from gas equation.

$$\dot{p}_i = \frac{RT_i}{V_i}(-\dot{m}_{ap} + \dot{m}_{at}) \tag{11a}$$

$$\dot{m}_{ap} = \frac{V_d}{120\,R\,T_i}(k_{18}\,p_i + k_{19})\,n \tag{11b}$$

$$\dot{m}_{at} = m_{at1}\frac{p_a}{\sqrt{T_a}}\beta_2(p_r)\,\beta_1(\alpha) = k_{20}\beta_2(p_r)\beta_1(\alpha) \tag{11c}$$

$$\beta_1(\alpha) = 1 - \cos(\alpha - \alpha_0) \tag{11d}$$

$$\beta_2(p_r) = \begin{cases} \frac{1}{p_n}\sqrt{p_r^{p_1} - p_r^{p_2}} & \text{if}\, p_r \geq p_c, \\ 1 & \text{if}\, p_r < p_c \end{cases} \tag{11e}$$

where,

$\dot{m}_{ap}$ = air mass flow rate into cylinder (kg/sec)
$\dot{m}_{at}$ = air mass flow rate past throttle plate (kg/sec)
α = throttle opening (degrees)
α_0 = closed throttle angle (degrees)
$p_r(= p_i/p_a)$ = pressure ratio
p_a, T_a = atmospheric pressure (bar) and temperature ($^\circ$K)
R = gas constant
p_1, p_2, p_c, p_n, k_i = constants

3.4 Error Dynamics

The IC engine model is a nonlinear, MIMO, coupled system having three states $x = (\dot{m}_f, n, p_i)$ and three inputs $u = (\dot{m}_{fi}, \theta, \alpha)$. The system outputs are (λ, n, p_i). System model can be written in controller canonical form as: $\dot{x} = f + G\tilde{u}$, where,

$$f = \begin{pmatrix} -x_1/\tau_f + u_1/\tau_f \\ -\frac{1}{Ix_2}(p_l + p_b) \\ -\frac{RT_i}{V_i}\dot{m}_{ap} \end{pmatrix} \text{ and } \tilde{u} = \begin{pmatrix} \dot{u}_1 \\ e^{-\theta^2/2\theta^2_{mbt}} \\ 1 - \cos(\alpha - \alpha_0) \end{pmatrix}$$

$$G = \text{diag}([g_1, g_2, g_3]) = \begin{pmatrix} 1 - X & 0 & 0 \\ 0 & \frac{1}{Ix_2}H_u\eta_{in}\eta_{ip}\eta_{i\lambda}x_1 & 0 \\ 0 & 0 & \frac{RT_i}{V_i}k_{20}\beta_2(x_3) \end{pmatrix}$$

The error dynamics associated with the trajectory tracking problem can be written as:

$$\dot{e}_1 = \phi_1 = L_f h + \sum_{i=1}^{3} L_{g_i} h\tilde{u}_i \tag{12a}$$

$$\dot{e}_1 = \phi_2 = f_2 - x_{2d} + g_2\tilde{u}_2 \tag{12b}$$

$$\dot{e}_1 = \phi_3 = f_3 - x_{3d} + g_3\tilde{u}_3 \tag{12c}$$

where $y = (e_1, e_2, e_3) = (\lambda - 1, x_2 - x_{2d}, x_3 - x_{3d})$. x_{2d} and x_{3d} are the desired trajectories of speed, x_2 and manifold pressure, x_3. The system model is dynamic feedback linearizable [16] and in proper I-O form as:

- Number of inputs and outputs are equal;
- $\dot{y}_1, \dot{y}_2, \dot{y}_3$ are C^1 function;
- Regularity condition

  $$\det\left[\frac{\partial(\phi_1, \phi_2, \phi_3)}{\partial(\dot{u}_1, u_2, u_3)}\right] \neq 0$$

  is satisfied.

Standard sliding mode control can be applied and control can be obtained but chattering will occur which is unacceptable in mechanical systems like the engine. The first equation involves input derivative while other two do not. The idea here is that standard sliding mode control will be applied to the air-fuel channel while dynamic sliding mode control strategy for the rest two channels to get smooth control.

4 Controller Design

The objectives of a typical control system for an internal combustion (IC) engine are not only to maintain the desired engine speed but also to satisfy regulations relating to exhaust emissions despite the ever-present torque disturbances. Moreover control should be robust for engines operating with different aging history of the components under vastly different environmental conditions. Sliding mode control, being a robust technique, is a natural candidate for engine control.

Throttle opening (α), spark advance(θ) and injected fuel mass flow $\dot{m}_{fi}$are the three control variables. Throttle provide large authority but is relatively slow while spark advance provide much faster actuation path but with limited authority. A good controller has to utilize both judiciously. The injected fuel mass flow rate has been used to stabilize the air-fuel ratio at the stoichiometric value where catalytic converter has high efficiency. Sliding surface chosen for the three channels are

$$s_1 = e_1, \tag{13a}$$

$$s_2 = a_2 e_2 + \phi_2, \tag{13b}$$

$$s_3 = a_3 e_3 + \phi_3 \tag{13c}$$

and the decoupled reachability condition with switching as:

$$\dot{s}_i = -K_i s_i - k_{0i}\text{sign}(s_i); \quad i = 1, 2, 3. \tag{14}$$

4.1 A/F-ratio Channel

Taking derivative of equation (13a) along the trajectories of (12a) we get,

$$\dot{s}_1|_{(12a)} = L_f h + \sum_{i=2}^{3} L_{g_i} h \tilde{u}_i + L_{g_1} h \dot{u}_1 \tag{15}$$

where L indicate Lie derivative.

The zero dynamics $\dot{u}_1 + k_{u_1} u_1 = 0$ is asymptotically stable. From equation (14) and (15) derivative of the first control is obtained as

$$\dot{u}_1 = \left(-K_1 s_1 - k_{01} s_1 - L_f h - \sum_{i=2}^{3} L_{g_i} h \tilde{u}_i \right) / L_{g_1} h \tag{16}$$

4.2 Speed and Manifold Pressure Channel

The zero dynamics of the remaining two inputs are $g_2 \tilde{u}_2 = 0$ and $g_3 \tilde{u}_3 = 0$ which is stable in the range of operation. Taking derivatives of equation (13b) and (13c) along system trajectories, we have

$$\dot{s}_i|_{(12b,12c)} = (L_{f_i} + \tilde{u}_i L_{g_i})\phi_i + f_i + g_i \tilde{u}_i + g_i \tilde{u}_i' \dot{u}_i - \dot{x}_{id} - \ddot{x}_{id}; \quad i = 2, 3. \tag{17}$$

From equation (14) and (17), the derivative of the other two control variables are solved as

$$\dot{u}_i = (-K_i s_i - k_{0i}\text{sign}(s_i) - (L_{f_i} + \tilde{u}_i L_{g_i})\phi_i - f_i - g_i \tilde{u}_i + \dot{x}_{id} + \ddot{x}_{id}) / g_i \tilde{u}_i'; \quad i = 2, 3. \tag{18}$$

where,

$$L_{f_i} = \left[\frac{\partial f_i}{\partial x_1}, \frac{\partial f_i}{\partial x_2}, \frac{\partial f_i}{\partial x_3}, \right]_{2\times 3}; \quad L_{g_i} = \left[\frac{\partial g_i}{\partial x_1}, \frac{\partial g_i}{\partial x_2}, \frac{\partial g_i}{\partial x_3}, \right]_{2\times 3}; \quad \tilde{u}_i' = \frac{\partial \tilde{u}_i}{\partial u_i}$$

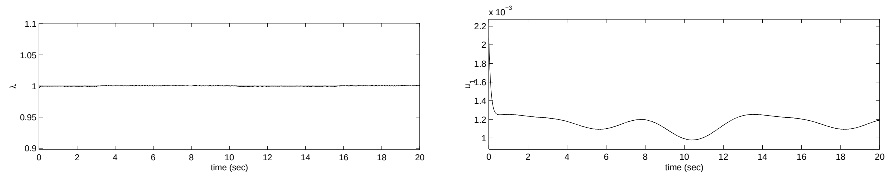
The controller is implemented by integrating the derivative of the control obtained in equations (16) and (18). Closed loop system dimension is equal to the open loop system dimension plus dimension of controller, i.e., 6 in this case.

5 Simulation Results

The simulation results presented here are for 1275 cc British Leyland engine for which data is published by Hendricks et al. [5]. The injection-time torque delay τ_d has not been considered for these results. Normalized air-fuel ratio has been stabilized to 1 and the desired trajectories of speed and manifold pressure are as follows:

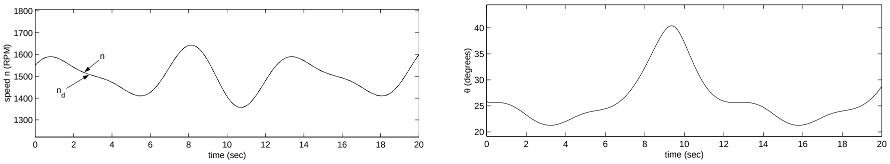
$$n_d = 1.5 + 0.1\sin(t) + 0.05\cos(1.5t)$$
$$p_{id} = 0.9 + 0.06\sin(1.5t)$$

Initial condition for system sates are [1.2e-3, 1.55, 0.9] and that of dynamic controller are [2.1e-3, 25.5, 35]. The design constants $a_2 = a_3 = 29$, $K = 25$ and $k_0 = 5$. From simulation results, it is evident that inputs are smooth and well with in the range.



(a) Tracking air-fuel ratio λ

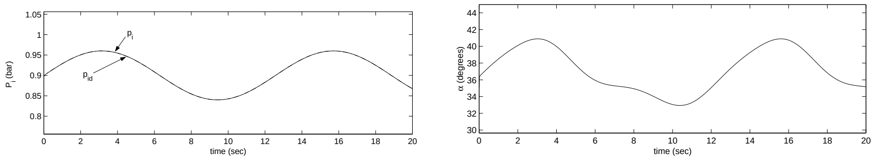
(b) Injected fuel mass flow, $u_1 = \dot{m}_{fi}$ (kg/sec)



(c) Tracking speed n

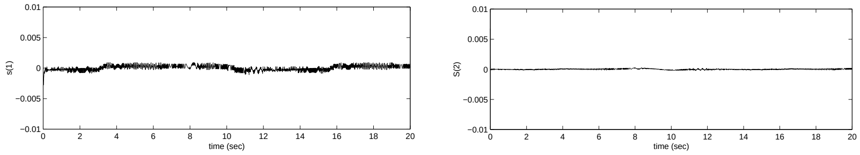
(d) Spark advance angle, θ, (degree)

Fig. 1. Simulation Results-I



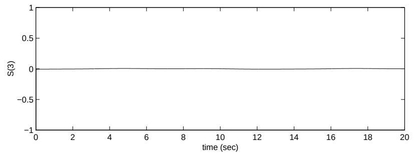
(a) Tracking manifold pressure p_m

(b) Throttle plate angle, α (degree)



(c) Sliding surface, s_1

(d) Sliding surface, s_2



(e) Sliding surface, s_3

Fig. 2. Simulation Results-II

6 Conclusions

Application of sliding mode techniques to multi-input multi-output IC engine model has been presented. Undesirable chattering effect at the actuators was avoided by dynamic sliding surface design. The particular choice of sliding surface is input dependent which always results in dynamic controller. Such dynamic policies are desirable in sliding mode as they effectively filter out the chattering of control signal. The designed controller is valid over the entire operating range. The air-fuel ratio is stabilized to its ideal stoichiometric value of 14.67 while speed and manifold pressure tracked the given trajectories.

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