Appendix A

Color Images

Color is one of the important properties of human vision. The human eye can distinguish thousands of color shades and intensities, but only a few shades of gray. Thus color image processing becomes an integral part of an image processing system. Although the perception of color by the human brain is not completely understood, the physical nature of color can be expressed on a basis supported by experimental and theoretical results [39]. Basically, colors that human beings perceive in an object are determined by the nature of light reflected from the object. This is perceived by *cones* in the human eye. The human eye has three sets of *cones* and hence, we can distinguish three basic colors - red, green and blue (RGB). Other colors, termed secondary colors, can be realized by a combination of these three basic colors. Examples of secondary colors are magenta (red and blue), cyan (blue and green) and yellow (green and red)

For machine analysis purposes, basic RGB may not suffice. Depending on the requirements, we need to have different color coordinates. An example is TV transmission: to have compatibility with the monochrome transmission, RGB has to be converted to monochrome value and two color values. Thus, we have different coordinates for color image (and video) representation.

Color coordinates (or spaces or planes) should have the following properties:

- Perceptually uniform: Equal steps in color spaces should be equally discriminable.
- Easy to navigate: User should find the space intuitive to move around in.
- Closely related to the physiology of the visual system. This is necessary for research kind of applications.
- Accurate color specification: Necessary to generate a hard copy.

- Implementable on an electronic display.

A.1 Color Coordinate Systems

Some of the popular color coordinates, along with their transformation, are given below. The list is not complete. For a good discussion on color coordinates and their transformation, see [55].

- RGB to YIQ transformation

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- RGB to XYZ transformation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.490 & 0.310 & 0.200 \\ 0.177 & 0.813 & 0.011 \\ 0.000 & 0.010 & 0.990 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- XYZ to $R_N G_N B_N$ transformation

$$\begin{bmatrix} R_N \\ G_N \\ B_N \end{bmatrix} = \begin{bmatrix} 1.910 & -0.510 & -0.288 \\ -0.985 & 2.000 & -0.028 \\ 0.058 & -0.118 & 0.896 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- XYZ to u, v, Y transformation

$$\begin{aligned} u &= 4X/(X + 15Y + 3Z) \\ v &= 6Y/(X + 15Y + 3Z) \\ Y &= Y \end{aligned}$$

- RGB to I_1, I_2, I_3 transformation

$$\begin{aligned} I_1 &= (R + G + B)/3 \\ I_2 &= (R - B)/2 \\ I_3 &= (2G - R - B)/4 \end{aligned}$$

A.2 H.263 and QCIF

The H.263 recommendation puts no constraint on the bit rate; however, its bit rate is up-to 64 Kbps.

The table below shows the recommended H.263 formats[1]

[1] For more details: http://rice.ecs.soton.ac.uk/peter/h263/

Format	Image Size	
	Y	Cb and Cr
Sub-QCIF	128×96	64×48
QCIF	176×144	88×72
CIF	352×288	176×144
4CIF	704×576	352×288
16CIF	1408×1152	704×576

Appendix B

MRF

Consider the entropy equation

$$S = -\sum_x P(x) lnP(x) \quad where, \;\; P(x) = P[X = x] \tag{B.1}$$

maximize S, that is,

$$\,^{max}_{\;k}[S] = \,^{max}_{\;k}\left[-\sum_x P(x) lnP(x)\right], \tag{B.2}$$

subject to

$$\sum_x U(x)P(x) = m \tag{B.3}$$

and

$$\sum_x P(x) = 1. \tag{B.4}$$

Maximizing Eqn. B.2 is equivalent to minimizing it with a + sign. That is,

$$\,^{min}_{\;k}\left[\sum_x P(x) lnP(x)\right] \tag{B.5}$$

Regularized cost function $\hat{S}$ is

$$\hat{S} = S + \lambda \sum_x P(x)U(x) + \mu \sum_x P(x) - 1 \tag{B.6}$$

Differentiating and seting the derivative to 0, we have,

$$\frac{\partial \hat{S}}{\partial P(x)} = \frac{\partial}{\partial P(x)}\left\{\sum_x P(x) lnP(x) + \lambda \sum_x P(x)U(x) + \mu \sum_x P(x) - 1\right\} \tag{B.7}$$

That is:

$$\begin{aligned} &1 + lnP(x) + \lambda U(x) + \mu = 0 \\ &\Rightarrow lnP(x) = -(\lambda U(x) + \mu + 1) \\ &\Rightarrow P(x) = exp(-\lambda U(x) - \mu - 1) \\ &\qquad\qquad = exp(-\lambda U(x)) exp(-\mu - 1) \end{aligned} \tag{B.8}$$

Multiply both sides by $U(x)$ and summing over x,

$$\sum_x P(x)U(x) = m = exp(-\mu - 1)\sum_x exp(-\lambda U(x))$$

Take sum over $\forall x$ in Eqn. B.8,

$$\sum_x P(x) = exp(-\mu - 1)\sum_x exp(-\lambda U(x))$$

$$\Rightarrow exp(-\mu - 1) = \frac{1}{\sum_x exp(-U(x))} \tag{B.9}$$

Substitute Eqn. B.9 in Eqn. B.8 we get

$$P(x) = \frac{exp(-\lambda U(x))}{\sum_x exp(-\lambda U(x))} \tag{B.10}$$

Since B.10 is true for all values of λ, it has to be true for a special case when $\lambda = 1$. Putting $\lambda = 1$ in Eqn. B.10, we get

$$P(x) = P[X = x] = \frac{exp(-\lambda U(x))}{\sum_x exp(-\lambda U(x))} \tag{B.11}$$

The Posterior Distribution: Assume the image observation model to be $Y = \Phi(\mathbf{H}X + N)$. Consider $P[X = x|Y = y]$,
using Baye'srule, we have

$$P[X = x|Y = y] = \frac{P[Y = y|X = x].P[X = x]}{P[Y = y]} \tag{B.12}$$

Where, Y is the *observed* image; this implies $P[Y = y]$ is fixed or known. Moreover, $P[X = x]$ is Gibbs' distributed.

$$\begin{aligned} P[Y = y|X = x] &= P[(\mathbf{H}X + n) = y|X = x] \\ &= P[N = Y - \mathbf{H}X|X = x] \end{aligned} \tag{B.13}$$

Since N is statistically independent of X, $P[Y = y|X = x] = P[N = Y - \mathbf{H}X]$. Let N also be Gibbsian (special case - Gaussian), and is independent and identically distributed Gaussian with mean μ and variance σ^2. Then,

$$P[X = x] = \frac{1}{(2\pi\sigma^2)^{N^2/2}} exp(-\mu\|Y - \mathbf{H}X\|^2/2\sigma^2) \tag{B.14}$$

Substitute in Eqn. B.12

$$P[X = x|Y = y] = \frac{exp(-\frac{\mu - \|Y - \mathbf{H}X\|^2}{2\sigma^2})}{(2\pi\sigma^2)^{-N^2/2}\, P[Y = y]} P[X = x] \tag{B.15}$$

Substituting for

$$P[X = x] = \frac{exp(-U(x))}{\sum_x exp(-U(x))}$$

$$P[X = x|Y = y] = K\ exp\left\{-U(x) - \frac{\mu + \|Y - \mathbf{H}X\|^2}{2\sigma^2}\right\} \tag{B.16}$$

where,

$$K \triangleq \frac{1}{(2\pi\sigma^2)\ P[Y = y]\ \sum_x exp(-U(x))}$$

But,

$$U(x) = \sum_{c \in \mathcal{C}} V_c(x).$$

Therefore,

$$P[X = x|Y = y] = K\ exp\left\{-\sum_{c \in \mathcal{C}} V_c(x) - \frac{\mu - \|Y - \mathbf{H}X\|^2}{2\sigma^2}\right\} \tag{B.17}$$

Let

$$U_p(x) \triangleq \sum_{c \in \mathcal{C}} V_c(x) + \frac{\mu - \|Y - \mathbf{H}X\|^2}{2\sigma^2}$$

Then

$$P[X = x|Y = y] = K\ exp\{U_p(x)\} \tag{B.18}$$