" PROOF ABOUT DELAY OF THE SYSTEM TO OUTPUT W.R.T. INPUT SIGNAL "

By
Kishor K V,
III E & C A Section
S J COLLEGE OF ENGINEERING,
MYSORE 570 006
kvkish83@yahoo.com

Let

X1(t)=Cos(w1*t);

then

Y1(t)=|H(jw1)|Cos(w1*t − w1*d1);
Where d1 is the slope of the phase response at w=w1;

Similarly

Y2(t)=|H(jw2)|Cos(w2*t − w2*d2);
Where d2 is the slope of the phase response at w=w2;

And if

X(t)=X1(t)+X2(t)=Cos(w1*t) + cos(w2*t);
−−−−−−−−−−−−−−−−−−−−−> (1)

then by Linearity,

Y(t)=|H(jw1)|Cos(w1*t − w1*d1) + |H(jw2)|Cos(w2*t − w2*d2);
−−−−−−−−−−−−−−−−−−−−−> (2)

Now, we assume, for a known bandwidth of frequency |H(jw)|=1,
i.e, for Wc1<w1<w2<Wc2,
Where Wc1 is the lower cutoff frequency Wc2 is the upper cutoff frequency

Bandwidth of system = Wc2 − Wc1;

then Eqn(2)=>

Y(t)=Cos(w1*t − w1*d1) + Cos(w2*t − w2*d2); −−−−−−−−−−−−−−−−−−−−−> (3)

Let ‘d‘ be the value such that

Y(t)=X(t − d);   −−−−−−−−−−−−−−−−−−−−−> (4)

Eqn(1) =>

X(t − d)=Cos(w1*t w1*d) + Cos(w2*t −w2*d);   −−−−−−−−−−−−−−−−−−−−−> (5)

Comparing Eqn(3) & Eqn(5), the condition (4) is satisfied

‘if and only if‘

" d1=d2=d "   −−−−−−−−−−−−−−−−−−−−−> (6)

But in general, if the input signal is the summation of different frequency components, then

condition (6) =>

‘ d1=d2=d3=d4=.....=d ‘   −−−−−−−−−−−−−−−−−−−−−> (7)

The condition (7) implies , at every point of frequency in the phase response of a system

with in the specified bandwidth of frequency , the slope of the phase response should be

constant which implies the phase response should be ‘ LINEAR ‘.

Finally, we can conclude that "If the system has linear phase response and unit frequency

response then ***Whatever*** **may be the input signal, the delay in the output w.r.t. input signal is always constant and is equal to slope(d) of the phase response of the system**.