Abstract. This note contains a description of the concepts of spreadability and feedback spreading control (FSC) for distributed parameter systems (DPS), the approaches used until now along with applications and a list of references to works including peer-reviewed papers, communications to international meetings and theses.

Distributed biophysical processes usually involve spreads in space, e.g. biological control and invasion theory, pollution in air quality, desertification in vegetation dynamics, cancer cells in biomedicine, etc. In order to model and control such expansion phenomena, the concepts of spreadability and spreading control have been introduced by El Jai and Kassara [14] in 1994 for DPS and tested in [15] for transport systems. They consist of what follows: Let, $\Omega \subset \mathbb{R}^n$ be an open and bounded domain with sufficiently smooth boundary $\partial \Omega$. Let $-A$ stand for an unbounded densely defined linear operator which generates a $C_0$ analytic semigroup $(S(t))_t$ on $Z = L^2(\Omega)$ and consider the following semilinear control system,

$$\frac{\partial z}{\partial t} + Az = \varphi(z, v) \quad \text{on } \Omega \times (0, \infty],$$

with initial data, $z(0) = z_0$, where $z_0 \in \text{dom}(A)$ and $\varphi$ denotes a nonlinear operator which maps $S \times V$ into $Z$, with $V$ another Hilbert space and $S$ a closed subset of $Z$. For $t_1 > 0$ and a measurable function $\bar{v} : [0, t_1] \rightarrow V$, we denote by $z(t, \bar{v})$ a solution, when it exists, on the interval $(0, t_1]$.

Let $\omega$ be the map to be spread, defined as follows,

$$\omega : S \rightarrow 2^{\Omega}.$$

A measurable function $\bar{v} : (0, t_1] \rightarrow V$ is called a spreading control with respect to the map $\omega$ if:

$$z(t, \bar{v}) \in S,$$  \hspace{1cm} (0.1a)

and,

$$\omega(z(t, \bar{v})) \text{ nondecreasing on } (0, t_1].$$  \hspace{1cm} (0.1b)

The system is then said spreadable with respect to $\omega$. The following approaches have been considered by several authors,

(i) Restating spreadability as a monotonicity problem has led [29, 30] to characterize feedback spreading control laws (in short FSC laws) as selections of set-valued maps involving tangential sets and enables to investigate minimal FSC laws. Out of the technical need to design spreads taking into consideration both the speed and the time of spreading, the papers [1, 31] examine in the same context, FSC laws which generate a spread either slower or quicker than a desired given speed. The above approach has been applied to cancer research [16, 32] by deriving protocol laws that destroy cancer in a PDE immunotherapy model. Even in the same framework target control in a semilinear control system has been investigated in [38].

(ii) A direct approach using optimal control techniques:
1. for linear DPS with $\omega(z) = \{ x \mid z(x) = z_d(x) \}$ where $z_d$ belongs to $Z$. See [2, 14, 17, 22].
2. for a nonlinear DPS governing the evolution of a population of rabbits among foxes, using an initial state control method and a fixed point theorem, cf. Evans and Pritchard [26].
3. for investigating spreadability with respect to measure [4–6].
4. spreading control is connected to cellular automata [13, 24, 45].

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† Department of Mathematics, University Hassan II of Casablanca 1, P. O. Box 5366, Maarif, 20100, Casablanca, Morocco, (kassarak@member.ams.org)


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