SPREADING CONTROL FOR DISTRIBUTED PARAMETER SYSTEMS *

KHALID KASSARA†

Abstract. This note contains a description of the concepts of spreadability and feedback spreading control (FSC) for distributed parameter systems (DPS), the approaches used until now along with applications and a list of references to works including peer-reviewed papers, communications to international meetings and theses.

Distributed biophysical processes usually involve spreads in space, e.g. biological control and invasion theory, pollution in air quality, desertification in vegetation dynamics, cancer cells in biomedicine, etc. In order to model and control such expansion phenomena, the concepts of spreadability and spreading control have been introduced by El Jai and Kassara [14] in 1994 for DPS and tested in [15] for transport systems. They consist of what follows : Let, $\Omega \subset \mathbb{R}^n$ be an open and bounded domain with sufficiently smooth boundary $\partial\Omega$. Let $-A$ stand for an unbounded densely defined linear operator which generates a C_0 analytic semigroup $(S(t))_t$ on $Z = L^2(\Omega)$ and consider the following semilinear control system,

$$\frac{\partial z}{\partial t} + Az = \varphi(z, v) \quad \text{on } \Omega \times (0, \infty[,$$

with initial data, $z(0) = z_0$, where $z_0 \in \mathrm{dom}(A)$ and φ denotes a nonlinear operator which maps $\mathcal{S} \times V$ into Z, with V another Hilbert space and $\mathcal{S}$ a closed subset of Z. For $t_1 > 0$ and a measurable function $\bar{v} : [0, t1[\to V$, we denote by $z(\cdot, \bar{v})$ a solution, when it exists, on the interval $(0, t_1[$.

Let ω be the map to be spread, defined as follows,

$$\omega : \mathcal{S} \to 2^{\Omega}.$$

A measurable function $\bar{v} : (0, t_1[\to V$ is called a spreading control with respect to the map ω if :

$$z(t, \bar{v}) \in \mathcal{S}, \tag{0.1a}$$

and,

$$\omega(z(t, \bar{v})) \text{ nondecreasing on} (0, t_1[\tag{0.1b}$$

The system is then said spreadable with respect to ω. The following approaches have been considered by several authors,

(i) Restating spreadability as a monotonicity problem has led [29, 30] to characterize feedback spreading control laws (in short FSC laws) as selections of set-valued maps involving tangential sets and enables to investigate minimal FSC laws. Out of the technical need to design spreads taking into consideration both the speed and the time of spreading, the papers [1, 31] examine in the same context, FSC laws which generate a spread either slower or quicker than a desired given speed. The above approach has been applied to cancer research [16, 32] by deriving protocol laws that destroy cancer in a PDE immunotherapy model. Even in the same framework target control in a semilinear control system has been investigated in [38].

(ii) A direct approach using optimal control techniques :

1. for linear DPS with $\omega(z) \doteq \{x \mid z(x) = z_d(x)\}$ where z_d belongs to Z. See [2, 14, 17, 22].
2. for a nonlinear DPS governing the evolution of a population of rabits among foxes, using an initial state control method and a fixed point theorem, cf. Evans and Pritchard [26].
3. for investigating spreadabililty with respect to measure [4–6].
4. spreading control is connected to cellular automata [13, 24, 45].

REFERENCES

* Posted January 2008.

†Department of Mathematics, University Hassan II of Casablanca 1, P. O. Box 5366, Maarif, 20100, Casablanca, Morocco, ($kassarak@member.ams.org$)

[1] J.P. AUBIN, K. KASSARA, Spreading control with speed constraints, Decision and Control 2003, Proceedings 42nd IEEE CSS Conference, 2(2004), 1482-1486.

[2] L. AUTRIQUE, J-P. LEYRIS, B. NOUMARE, Unconstrained Minimization Approaches to Control Determination of a Spreadable System, Engineering Optimization, Vol. 34, N. 1, 2001.

[3] C. BARBU, Modélisation de l'effet d'un insecticide et application au contrôle d'insectes vecteurs de la maladie de Chagas dans le Yucatan, Rapport de Master, Université de Perpignan, 2006.

[4] A. BERNOUSSI, Etalabilité au sens des aires, HDR, Université de Perpignan.

[5] A. BERNOUSSI, A. EL JAI, New approach of spreadability. Mathematical and Computer Modelling, 31(2000), 93-109.

[6] A. BERNOUSSI, A. EL JAI, A. J. PRITCHARD, Spreadability and evolving interfaces, Internat. J. Systems Sci. 32(2001), no. 10, 1217–1232.

[7] A. BERNOUSSI, Spreadability and vulnerability of distributed parameter systems. Internat. J. Systems Sci. 38(2007), no. 4, 305–317.

[8] A. BERNOUSSI, A, AMHARREF, M., Etalabilité, vulnrabilité, An. Univ. Craiova Ser. Mat. Inform. 30(2003), no. 1, 53–62.

[9] M. BERNOUSSI, Contrôle étaleur, PhD, Université de Perpignan, 1999.

[10] M. EDDIBI, K. KASSARA, Spreadability as a monotonicity problem, International Conference on Applied Mathematics and Engineering Sciences (CIMASI98), Casablanca, Morocco.

[11] M. EDDIBI, Etalabilité des syst'emes distribués : une approche par monotonicité, thèse de DES, Université Hassan II de Casablanca , 2000.

[12] M. EDDIBI, K. KASSARA, Spreadability as a monotonicity problem, JANO5, Kenitra,1998.

[13] A. EL JAI, S. EL YACOUBI, Cellular automata modelling and spreadability, Mathl. Comput. Modelling, 2002(36) pp. 10591074.

[14] A. EL JAI AND K. KASSARA, *Spreadable distributed systems,* Mathl. Comput. Modelling, 20(1994) pp. 47-64.

[15] ———, *Spreadability of transport systems,* Int. J. Systems Science, 27(1996) pp. 681-688.

[16] ———, *Target control by using feedback spreading control with application to immunotherapy*, Internat. J. Control, 79(2006), 813-821.

[17] A. EL JAI, K. KASSARA AND O. CABRERA, *Spray control*, International Journal of Control, 68(1997), pp. 709–730.

[18] A. EL JAI, Nouvelle approche pour la modélisation de systmes en expansion spatiale : Dynamique de végétation. Tendances nouvelles en modlisation pour l'environnement. Elsevier. (1997) 439-445.

[19] D. UCINSKI, A. EL JAI, On weak spreadability of DPS and its achievement via LQ control techniques. IMA Journal on Control and Information Science. 14, (1997) 1-22.

[20] A. EL JAI, M. FOURNIER, K. KASSARA, B. NOUMARE, Vegetation Dynamics : a Deterministic Approach Using the Concept of Spreadability, Theoretical and Applied Climatology, 52(1995), 241-249.

[21] A. EL JAI, M. FOURNIER, O. CABRERA, Y. MAURISSEN, *A prediction correction alternative for modelling spatiotemporal systems. Application to landscape modelling*, Ecolgical modelling, 116(1999), 237-251 .

[22] S. EL YACOUBI, A. EL JAI AND J. KARRAKCHOU, *Spreadability and spray actuators*, Appl. Math. Comput. Science, 8(1998), pp. 367-379.

[23] S. EL YACOUBI, A. EL JAI, Distributed parameter systems spreadability and its achievement via cellular automata, IEEE Mediterranean Conferene on Modeling Identification and Control. Alghero, Italy, (9-11, 1998).

[24] S. EL YACOUBI, Etalabilite et automates cellulaires , Doctorat detat Thesis, University of Rabat, Morocco, 2001.

[25] S. EL YACOUBI, A. EL JA, P. JACEWICZ, J. G. PAUSAS, *LUCAS : an original tool for landscape modelling, Environmental Modelling and Software*, 18(2003), 429-437.

[26] E. D. EVANS, A. J. PRITCHARD, *A control theoretic approach to containing the spread of rabits*, Math. Medecine and Biology, 18(2003) pp. 123.

[27] K. FEDRA, A. HAURIE, *A decision support system for air quality management combining GIS and optimisation techniques*, International Journal of Environment and Pollution (IJEP), Vol. 12, No. 2/3, 1999.

[28] K. KASSARA, Etalabilité des Systèmes Distribués, Doctort d'état Thesis, University of Rabat, Morocco, 1996.

[29] ———, *Feedback spreading controls for semilinear parabolic systems*, J. of Comput. and Appl. Math., Control of Partial Differential Equations (Jacksonville, Florida, 98), 114(2000), pp. 41-54.

[30] ———, *Feedback spreading control laws for semilinear distributed parameter systems*, Systems Control Lett., 40(2000), pp. 269-276.

[31] ———, *Feedback Spreading Control under Speed Constraints*, SIAM J. Control Optim., 41(2002), 1281-1294.

[32] ———, *Feedback spreading control applied to immunotherapy*, Mathematical Population Studies, 12(2005), 211-221.

[33] ———, The concept of spreadability in distributed parameter systems, IEEE/SMC Conference , Le Touquet, France, Nov. 1993.

[34] ———, Spreadability in transport systems, First ECC workshop on Distributed Parameter Systems, Saarisalka, Finland, June 1994.

[35] ———, Spreadability of distributed systems : a concept motivated by environment, Panafrican Congres of Mathematicians, University Al Akhawayn, Ifrane, Morocco, 1995.

[36] ———, Optimal spray control problems for linear distributed parameter systems, Second ECC workshop on distributed parameter systems, Perpignan, France, 1995.

[37] ———, Spreadability of distributed systems and viability theory, International Conference on Applied Mathematics Engineering Sciences (CIMASI96), Casablanca, Morocco.

[38] ———, *Spreading control applied to target problems*, Journal of Convex Analysis, 13(2006), 363-372.

[39] ———, M. Eddibi, *Spreadability of distributed parameter systems as a monotonicity problem*, Math-Recherche Applications, 2001.

[40] ———, Feedback spreading control laws in parabolic systems, Fourth SIAM Conference on Control and Its Applications, Jacksonville, Florida .

[41] ———, Feedback spreading control for semilinear pdes as a selection problem, CIMPA UNESCO, Granada, Spain Jul. 2000.

[42] ———, Feedback Spreading Control : Theory and Algorithms, Roscoff, June 2004.

[43] ———, Feedback Spreading Control Laws under Speed Constraints, Conference on Nonlinear Phenomena : Modelling and Analysis, Errachidia, Morooco, 2005.

[44] B. NOUMARE , Traitement Numérique dun Problème de Contrôle Etaleur pour une Classe de Systèmes Distribués, Thèse de Doctorat, Université de Perpignan, 1997.

[45] R. SLIMI, S. EL YACOUBI, Spreadable Probabilistic Cellular Automata Models: An Application in Epidemiology, Vol. 4173, Lecture Notes in Computer Science, Springer Berlin / Heidelberg, 2006.