At any given state of the game, let the contents of the boxes be the set $A = \{a_1,\ldots,a_n\}$, where n is the number of remaining boxes and each a_i is the contents of one box.

Assume that the banker sometimes makes an offer, of less than or equal to the arithmetic mean M(A) of the contents of the boxes.

Then the offer $B(A) \leq M(A) = (\sum_{i=1}^{n} a_i)/n$.

Let the expected win by the optimal strategy be W(A).

I will show by induction that W(A) = M(A) for all A.

Assume for now the hypothesis that W(A) = M(A) for all A for which n takes a particular value k.

Now consider the situation for an A where when n=k+1.

The banker offers $B(A) \leq M(A) = (\sum_{i=1}^{k+1} a_i)/(k+1)$.
We can treat B(A) = 0 as the case where the banker does not make an offer.

There are now two options for the player:

Deal (D):

The player takes the offer, so $W(A¦D) = B(A)$.

No Deal (¬D):

One of the boxes is eliminated at random, leaving a game state where n=k.

The optimal expected win in this case is obtained by summing the optimal expected wins after each possible elimination, and dividing by the number of possible eliminations.

So $W(A¦\neg D) = (\sum_{i=1}^{k+1} W(A\backslash\{a_i\}))/(k+1)$ which by the hypothesis = $(\sum_{i=1}^{k+1} M(A\backslash\{a_i\}))/(k+1)$. [1]

Now $M(A\backslash\{a_i\}) = ((\sum_{j=1}^{k+1} a_j) - a_i)/k = ((k+1)M(A) - a_i)/k$.

So substituting in [1], we get $W(A¦\neg D) = (\sum_{i=1}^{k+1} ((k+1)M(A) - a_i)/k)/(k+1)$
$= M(A)(k+1)/k - (\sum_{i=1}^{k+1} a_i)/(k(k+1))$
$= M(A)(k+1)/k - M(A)/k$
$= M(A)$

Now $W(A) = Max(W(A¦D), W(A¦\neg D))$
$= Max(B(A), M(A)) = M(A)$ [since $B(A) \leq M(A)$].

In other words we have shown that the hypothesis is true whenever n=k+1, if it is true whenever n=k.

Now the hypothesis is certainly true for n=1, since:
$M(A) = M(\{a_1\}) = a_1$,
and $W(A) = W(\{a_1\}) = a_1$.

Therefore by induction W(A) = M(A) for all A.