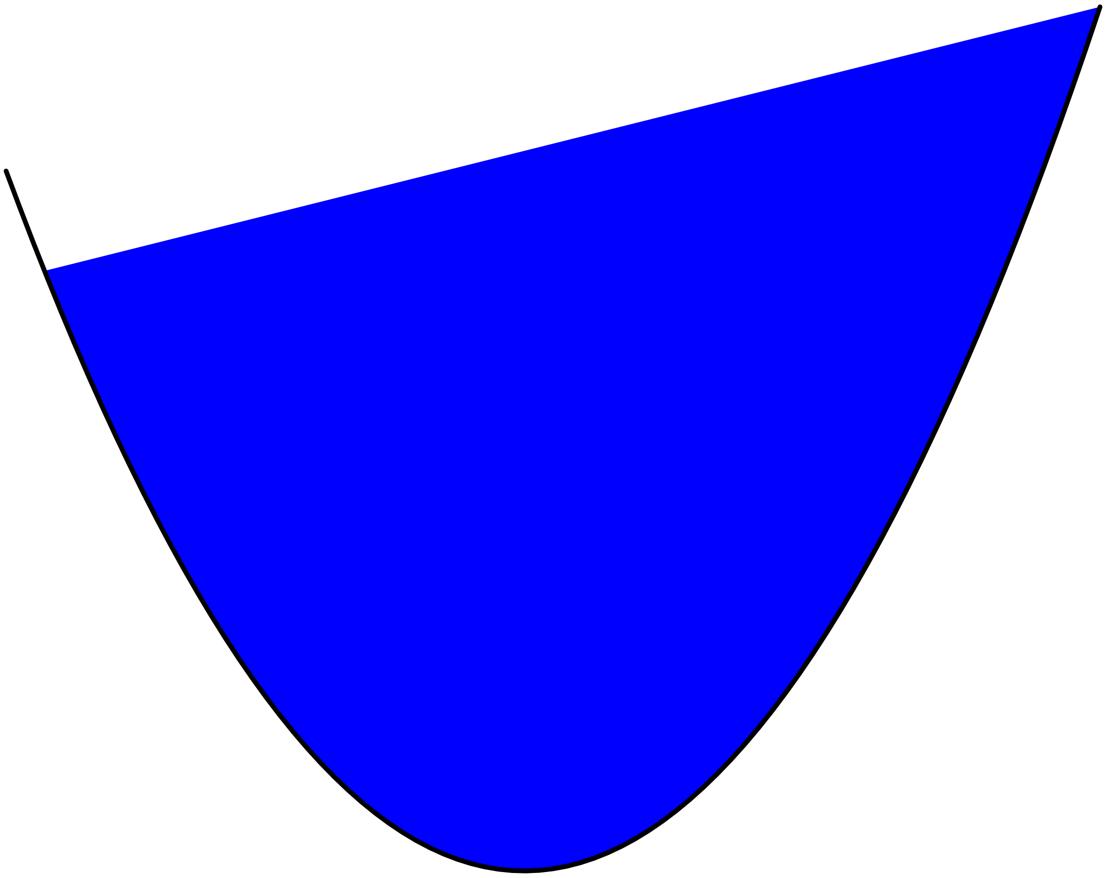
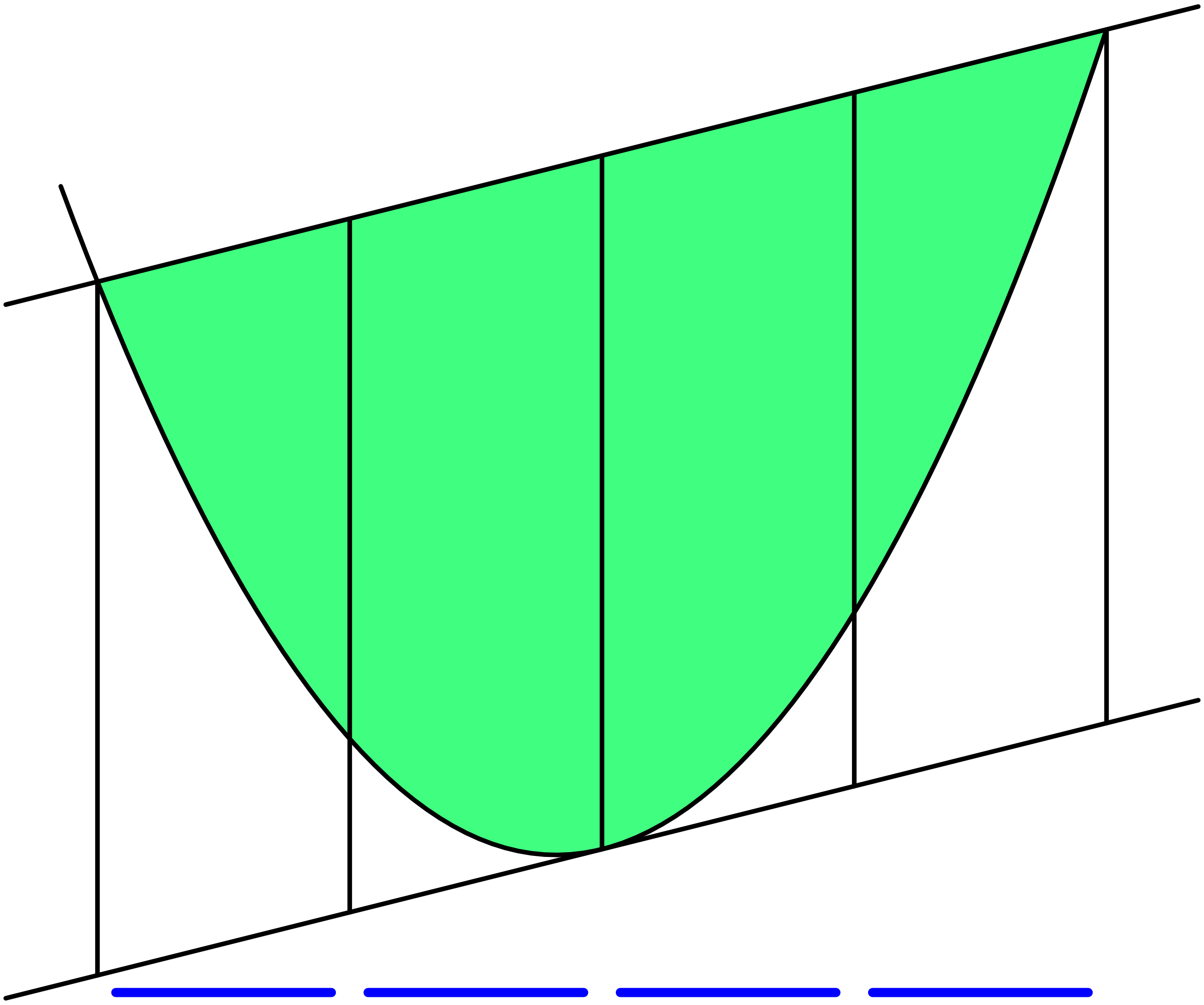
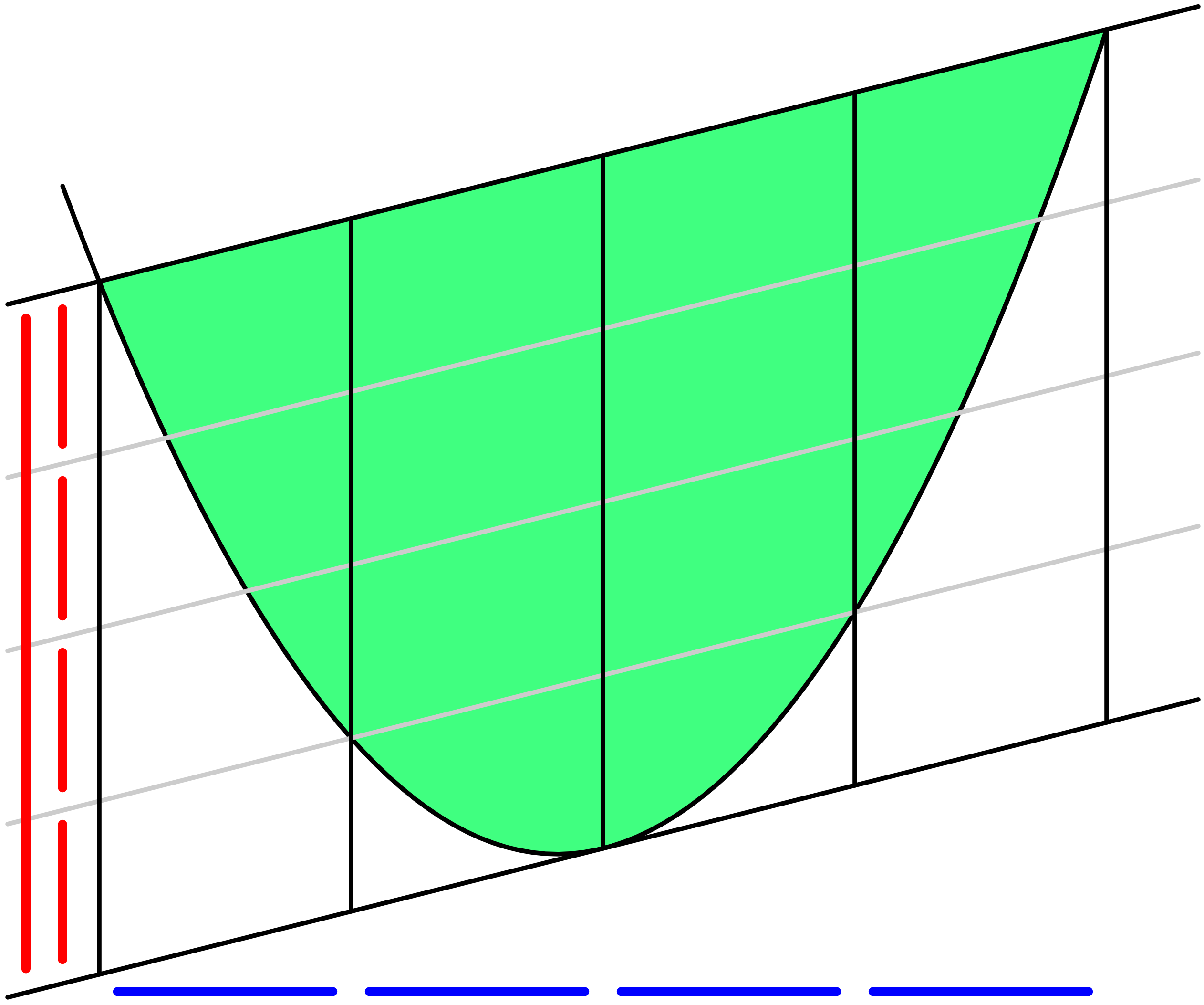


We seek to find the area above a parabola

Created by Brian Hunter
St No. 28564953
February 9, 2001



First we note the "Characteristic Property of a Parabola"
^y is proportional to ^x*^x



First we note the "Characteristic Property of a Parabola"
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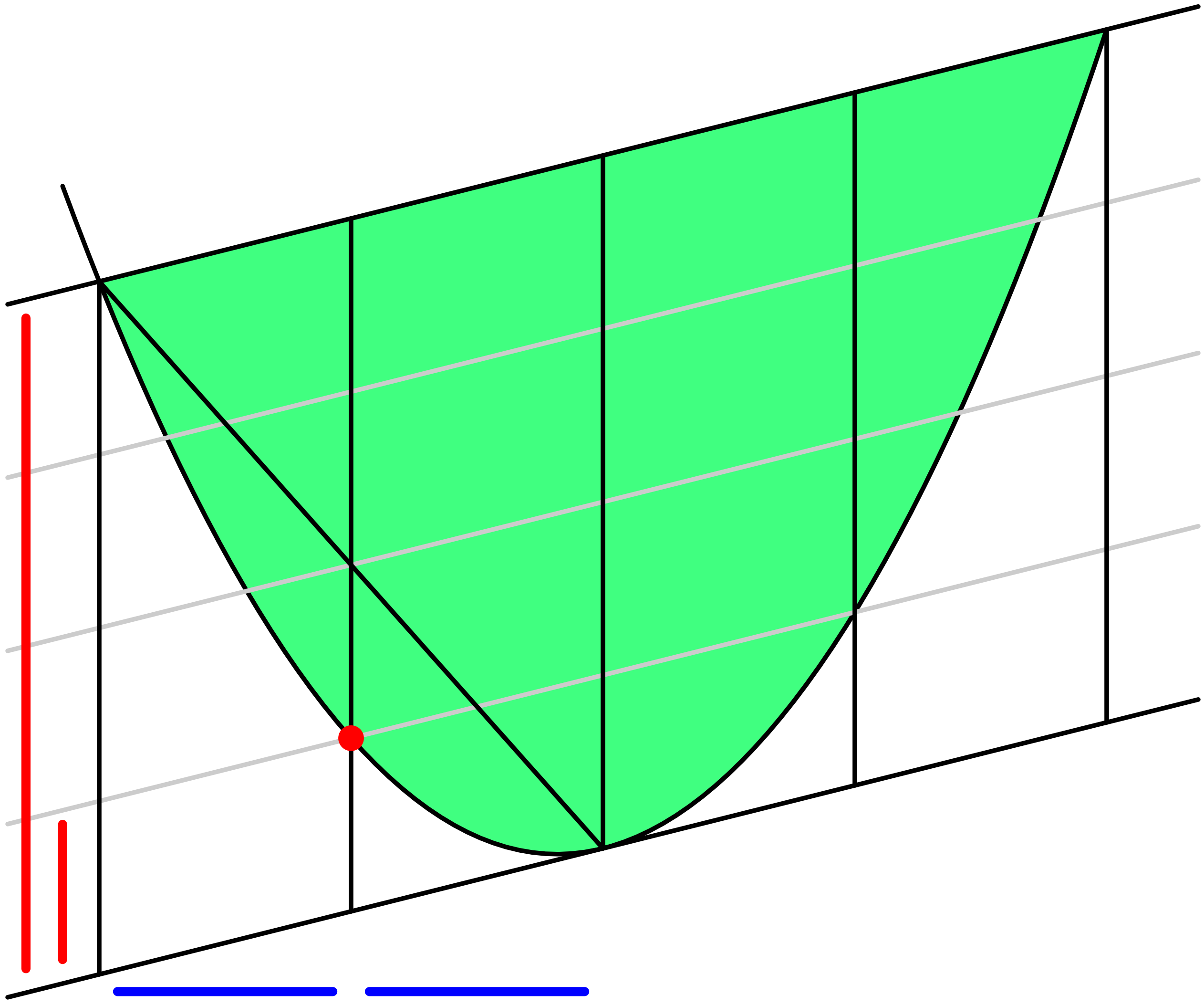
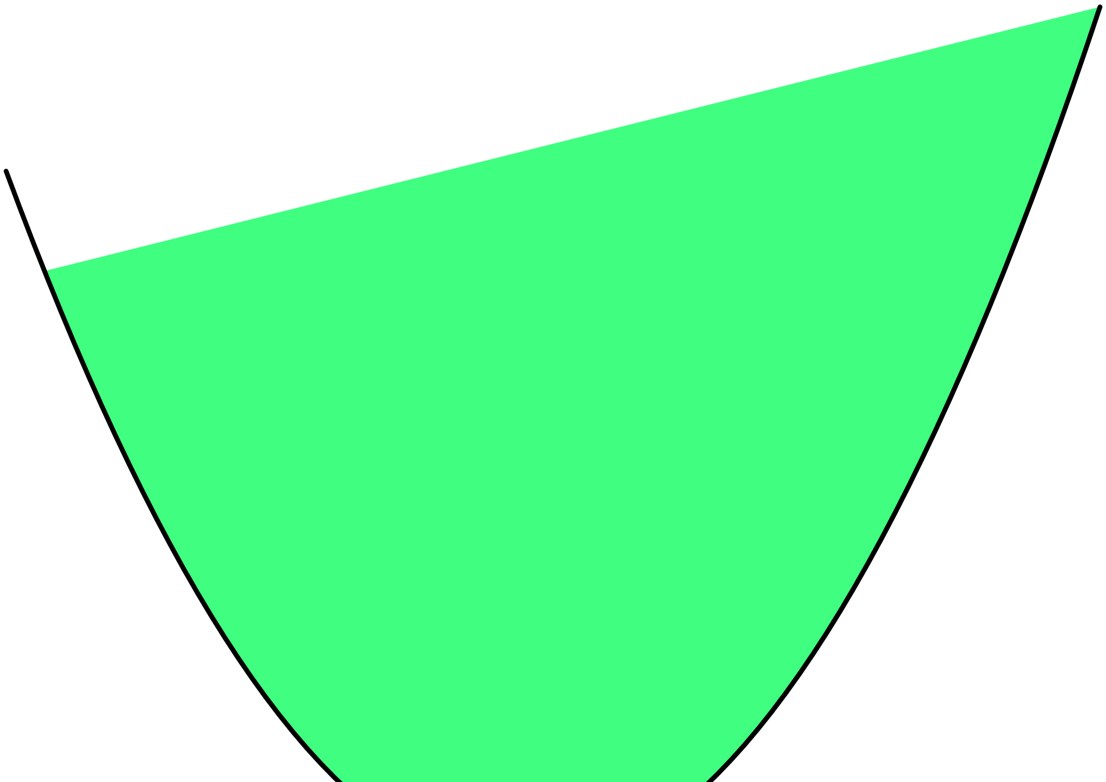
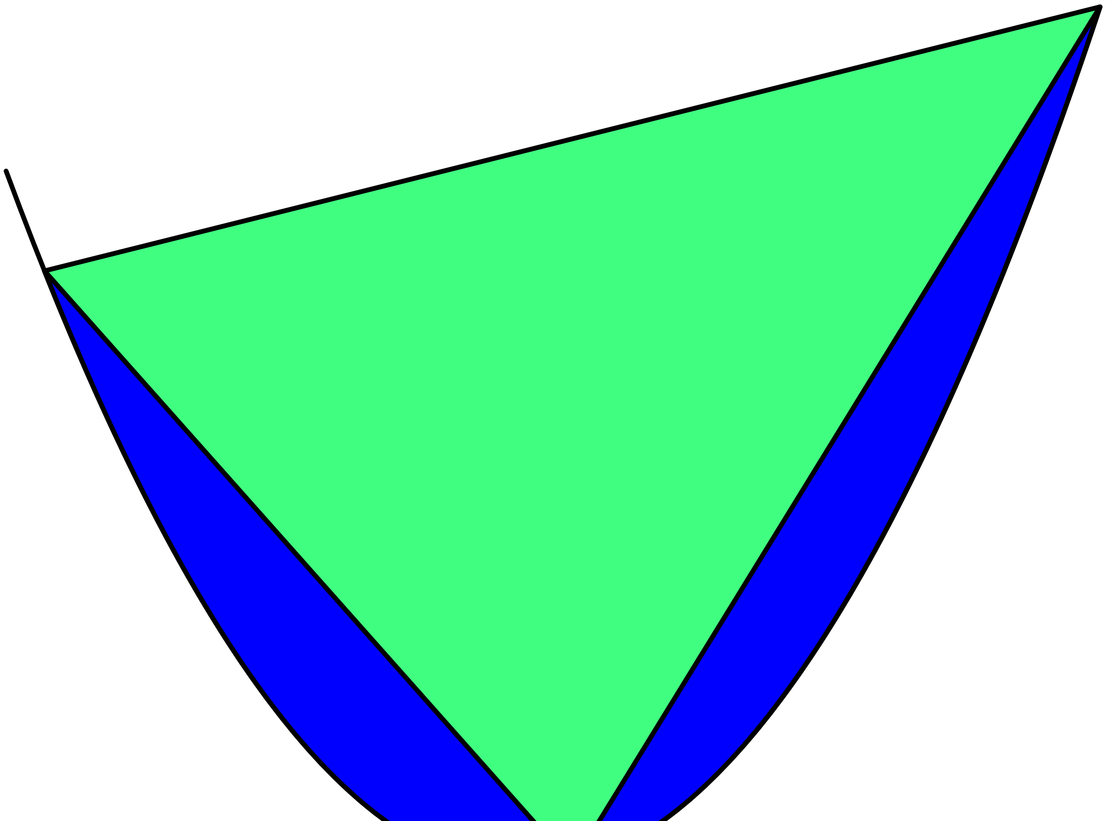


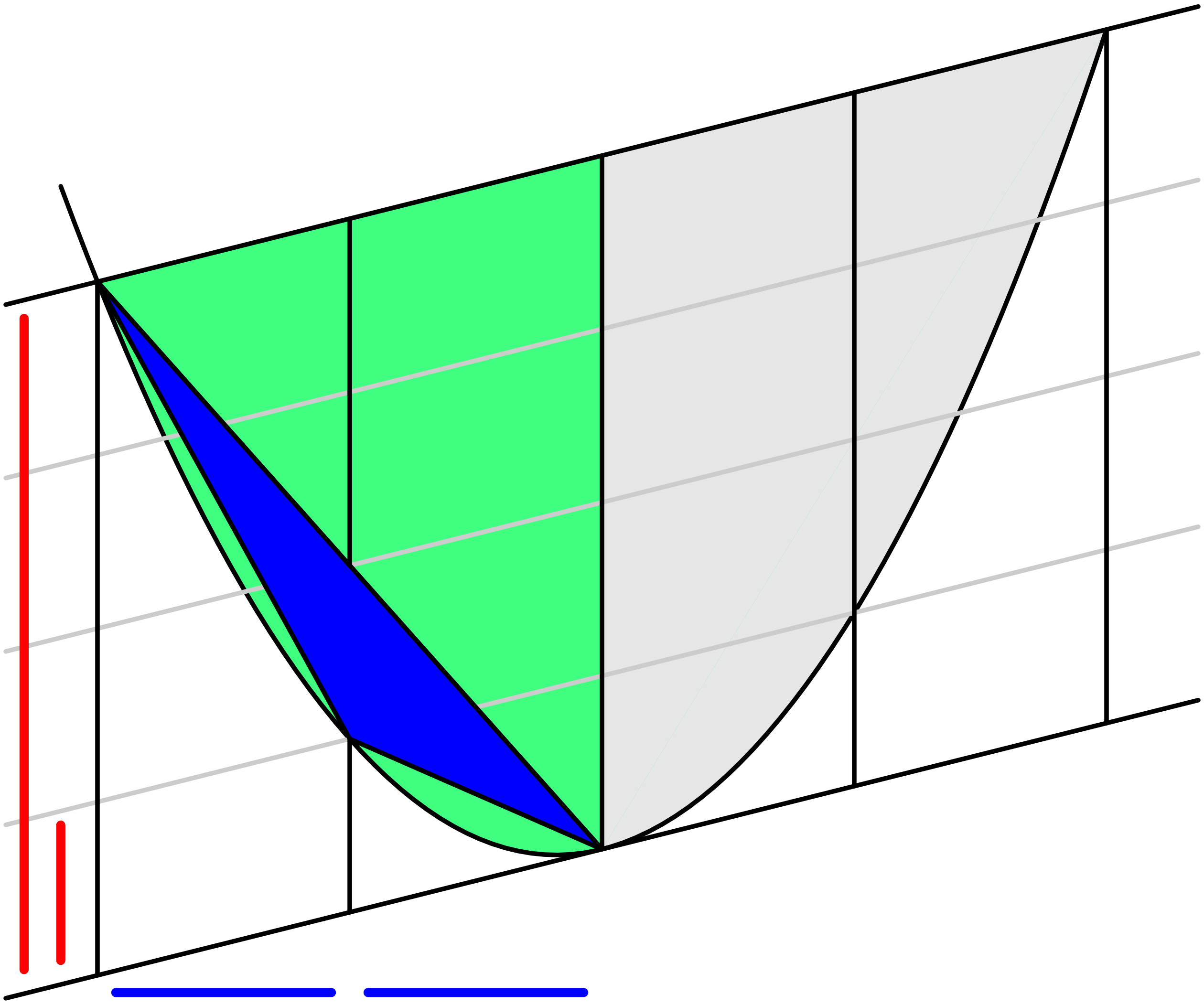
First we note the "Characteristic Property of a Parabola"
When x doubles then y quadruples

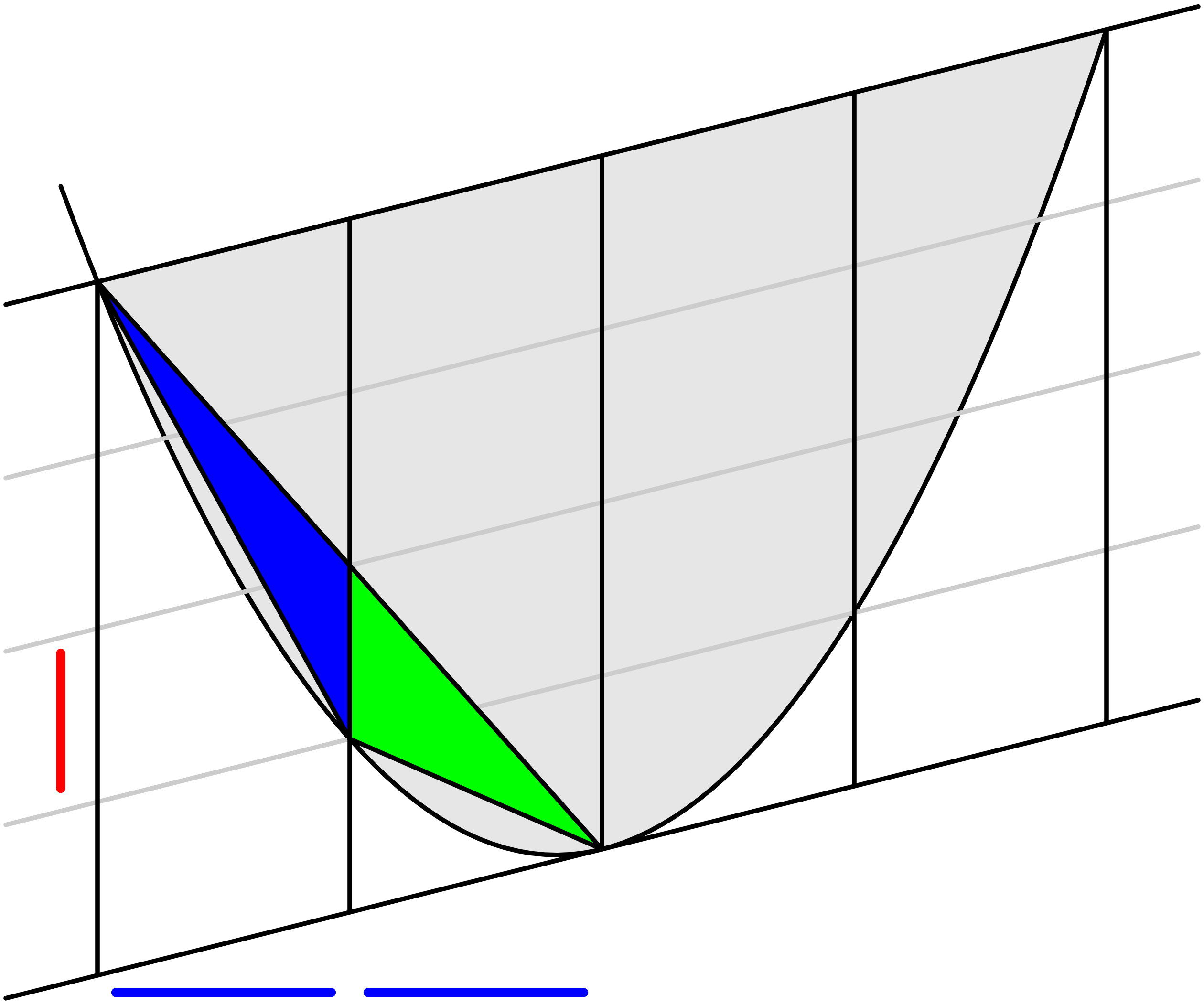


Now we will find the area above the parabola

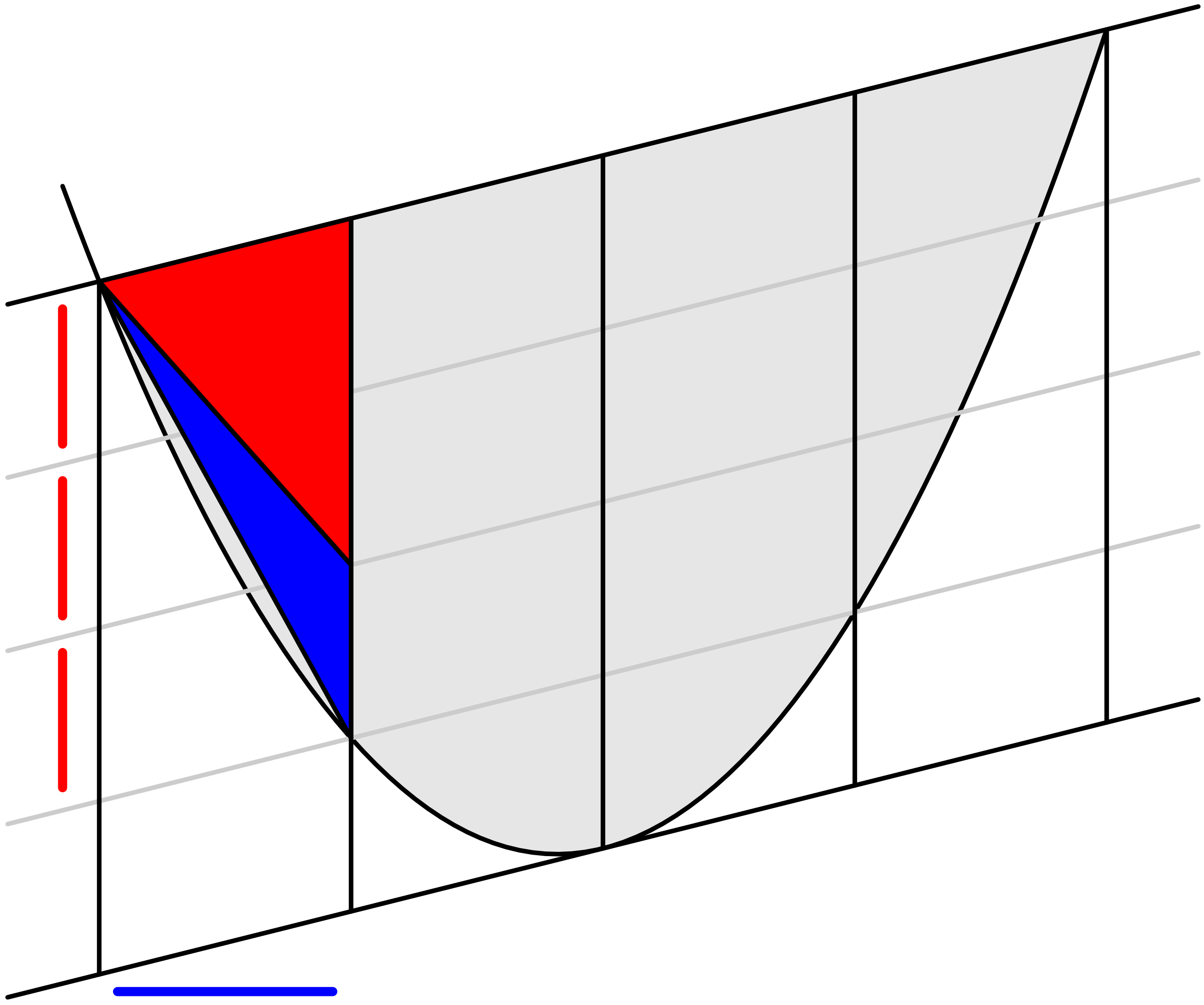


We will show that the blue is 1/3 that of the green

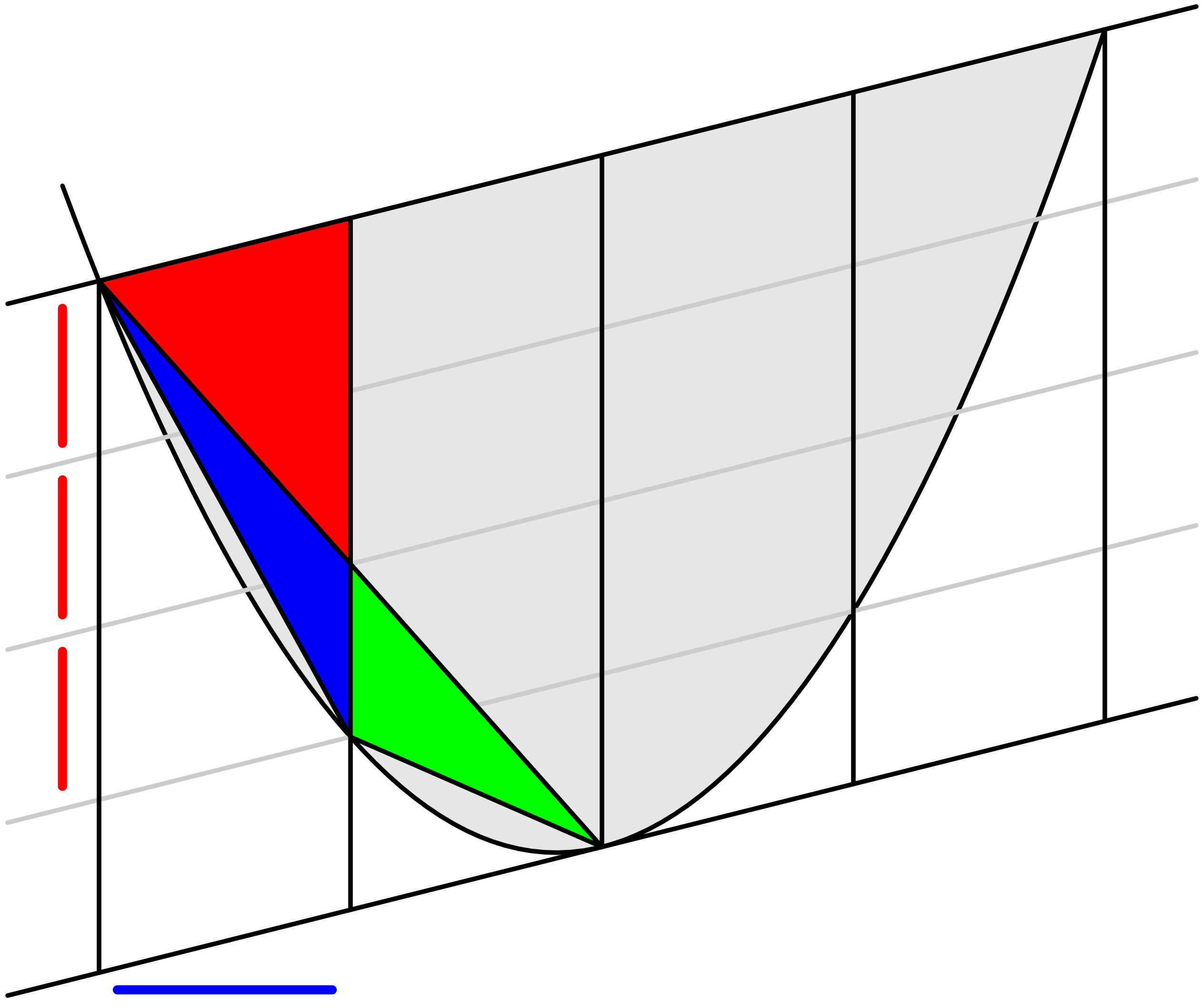




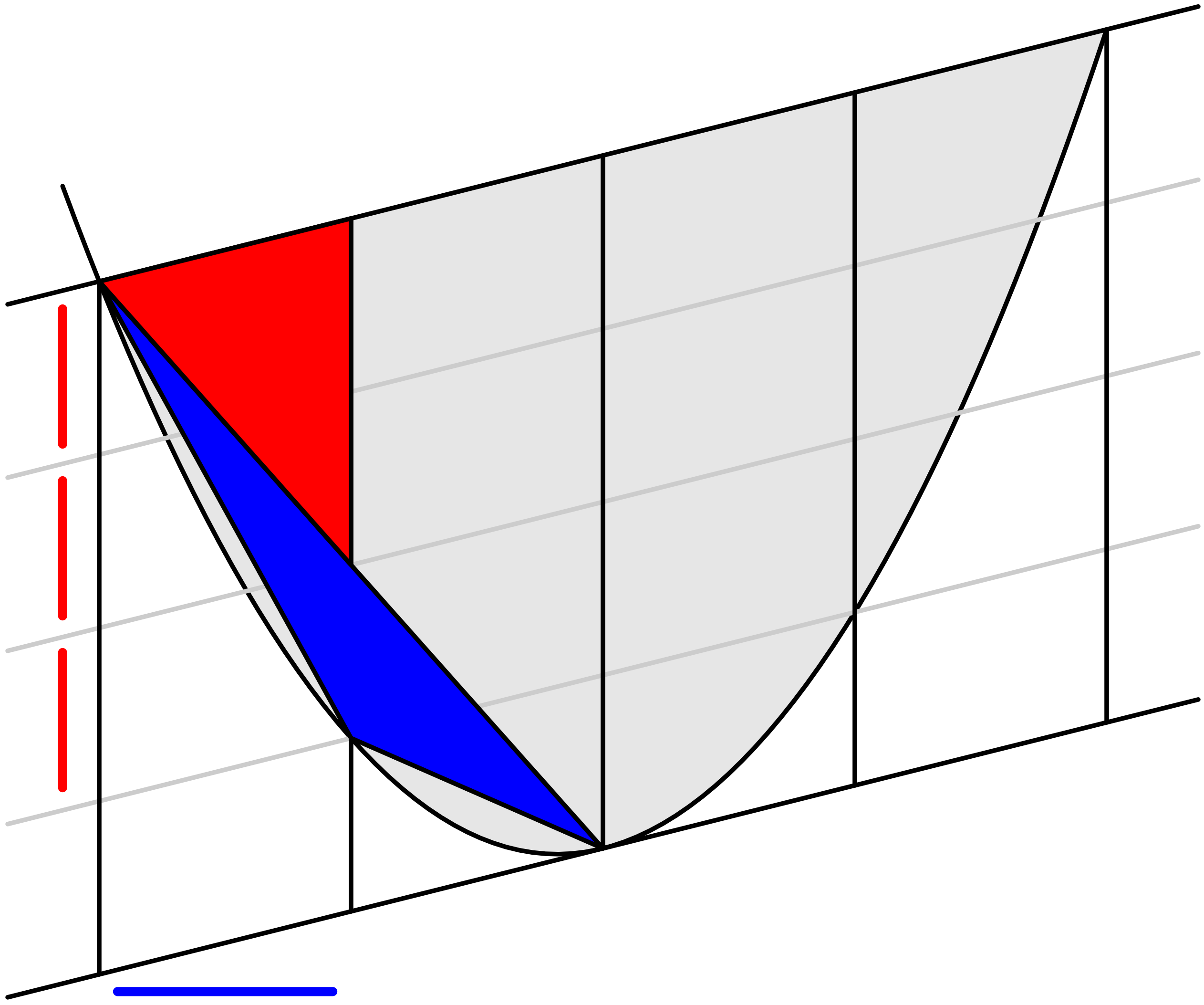
The blue equals the green
The bases and heights are equal



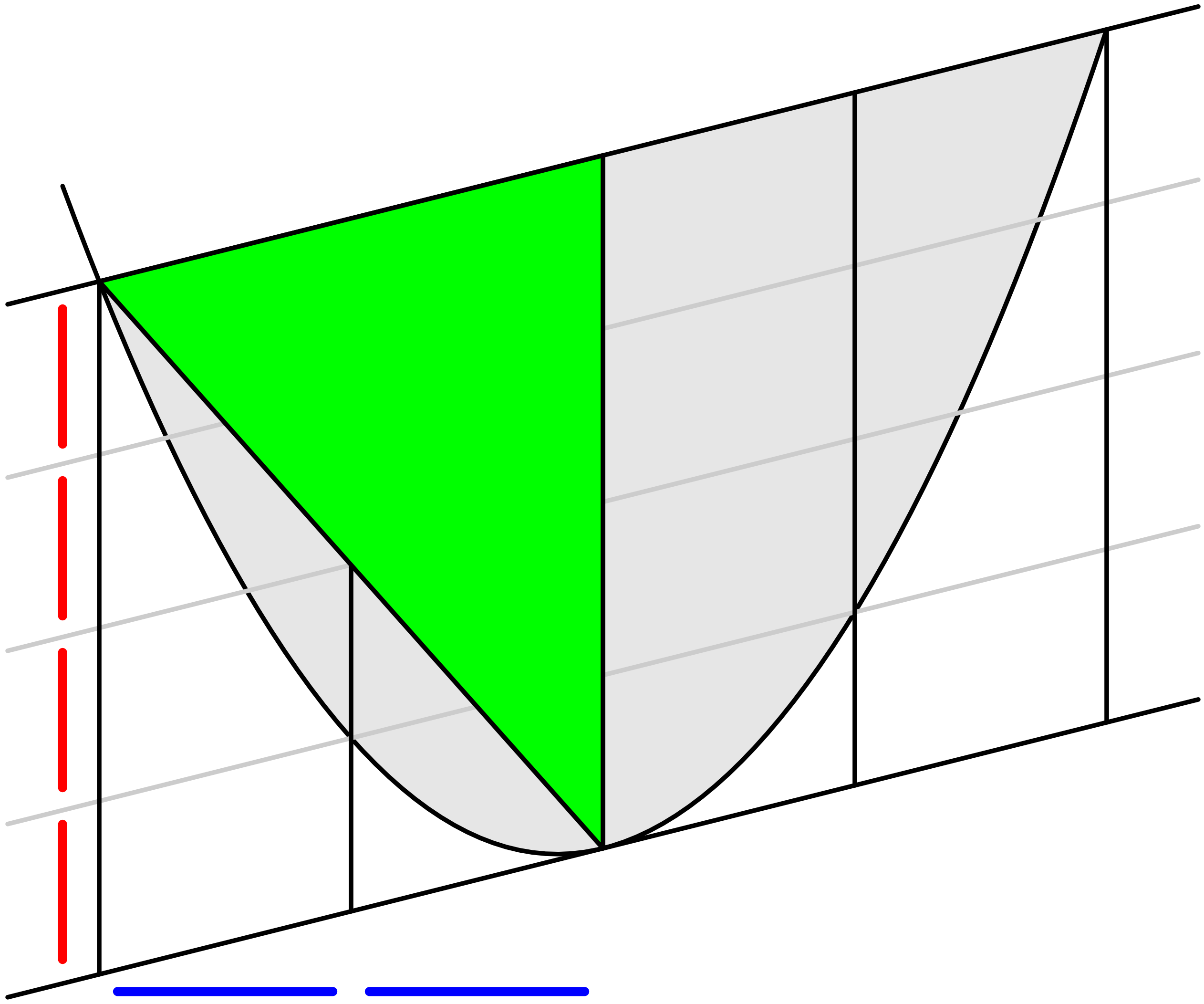
The blue equals 1/2 the red
The heights are equal <blue line>
The blue base is 1/2 that of the red base <red lines>



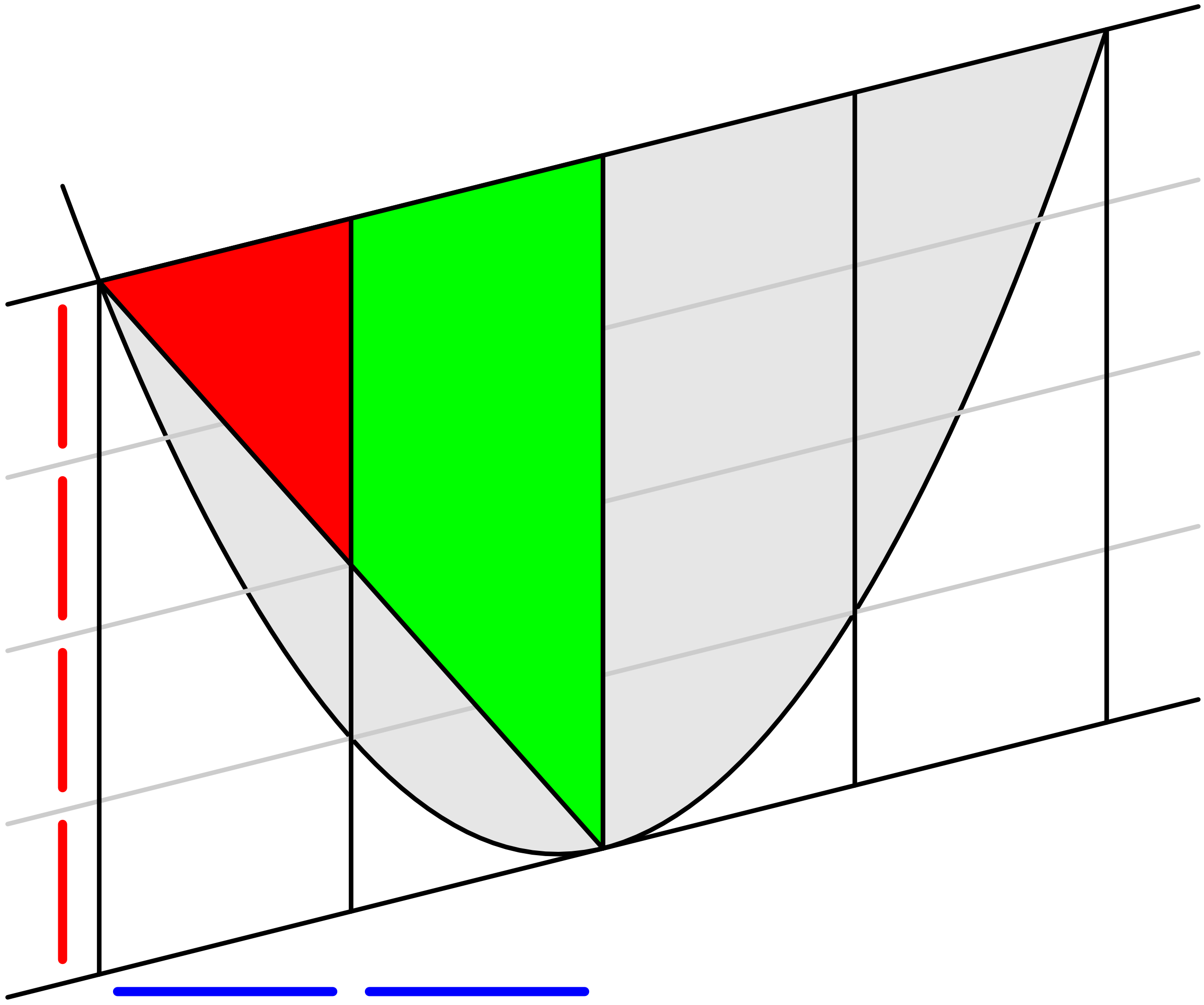
Thus, the blue and green together equal the red



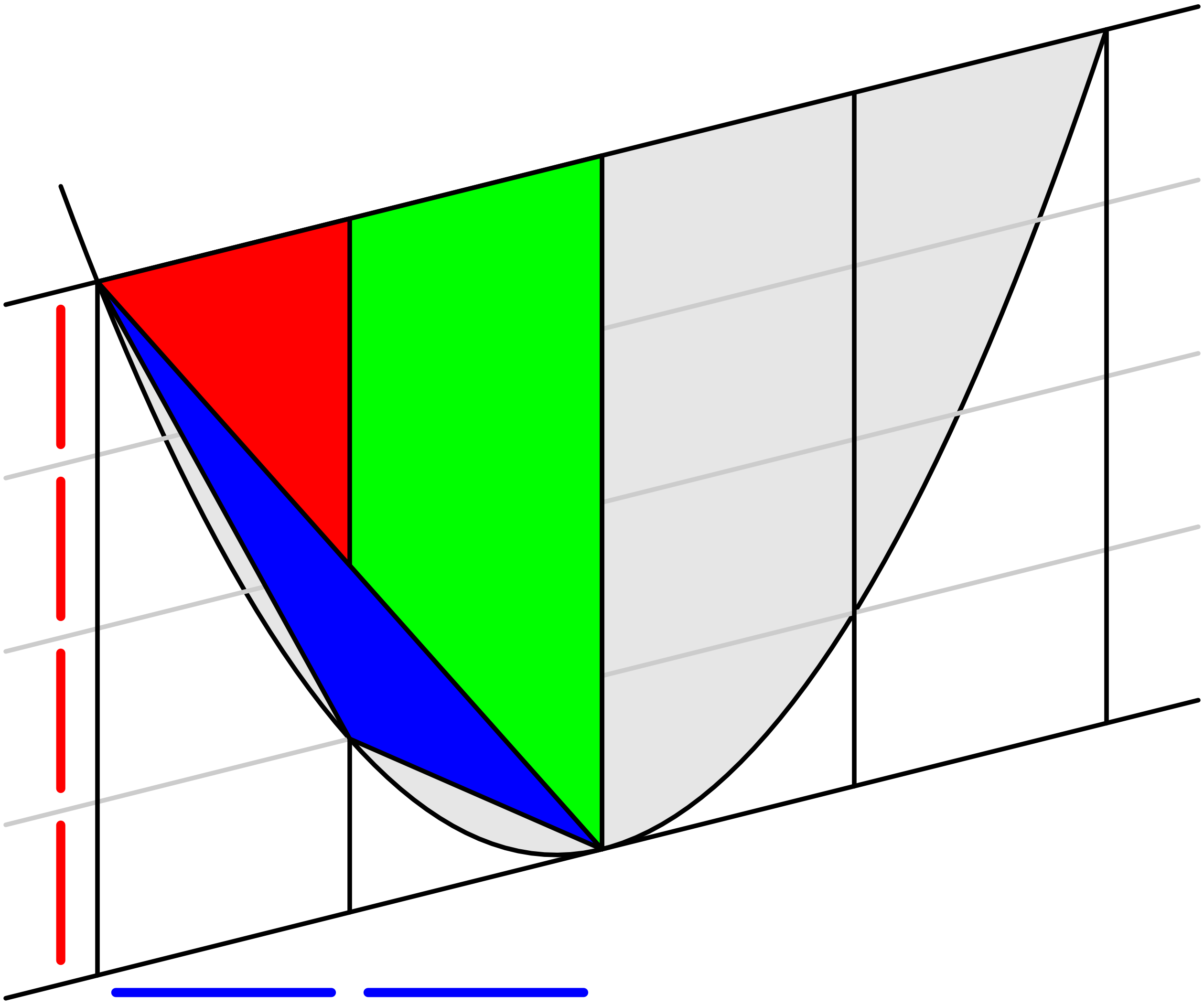
Thus, the blue equals the red



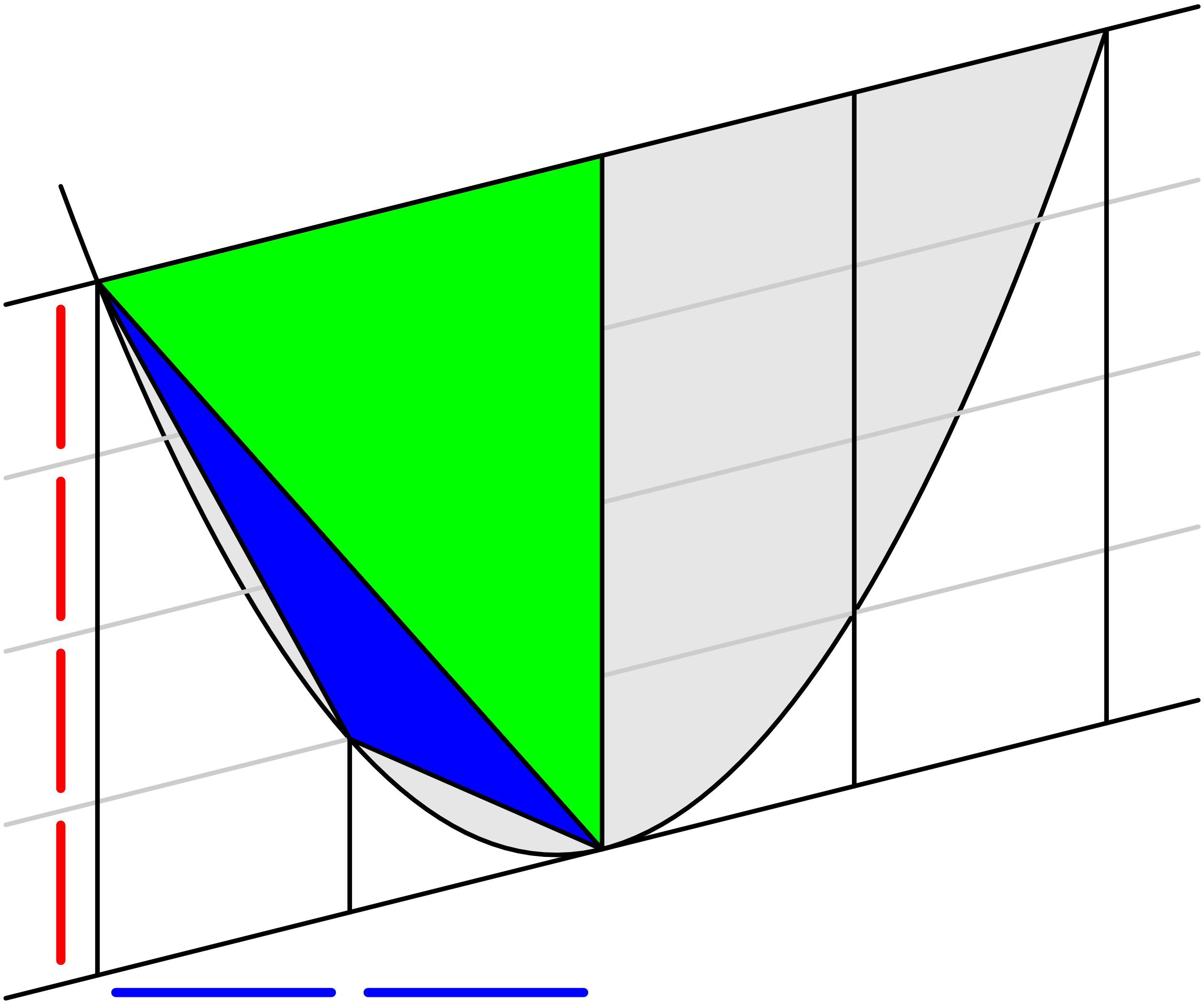
Now take the green triangle
This is a section of the original green triangle



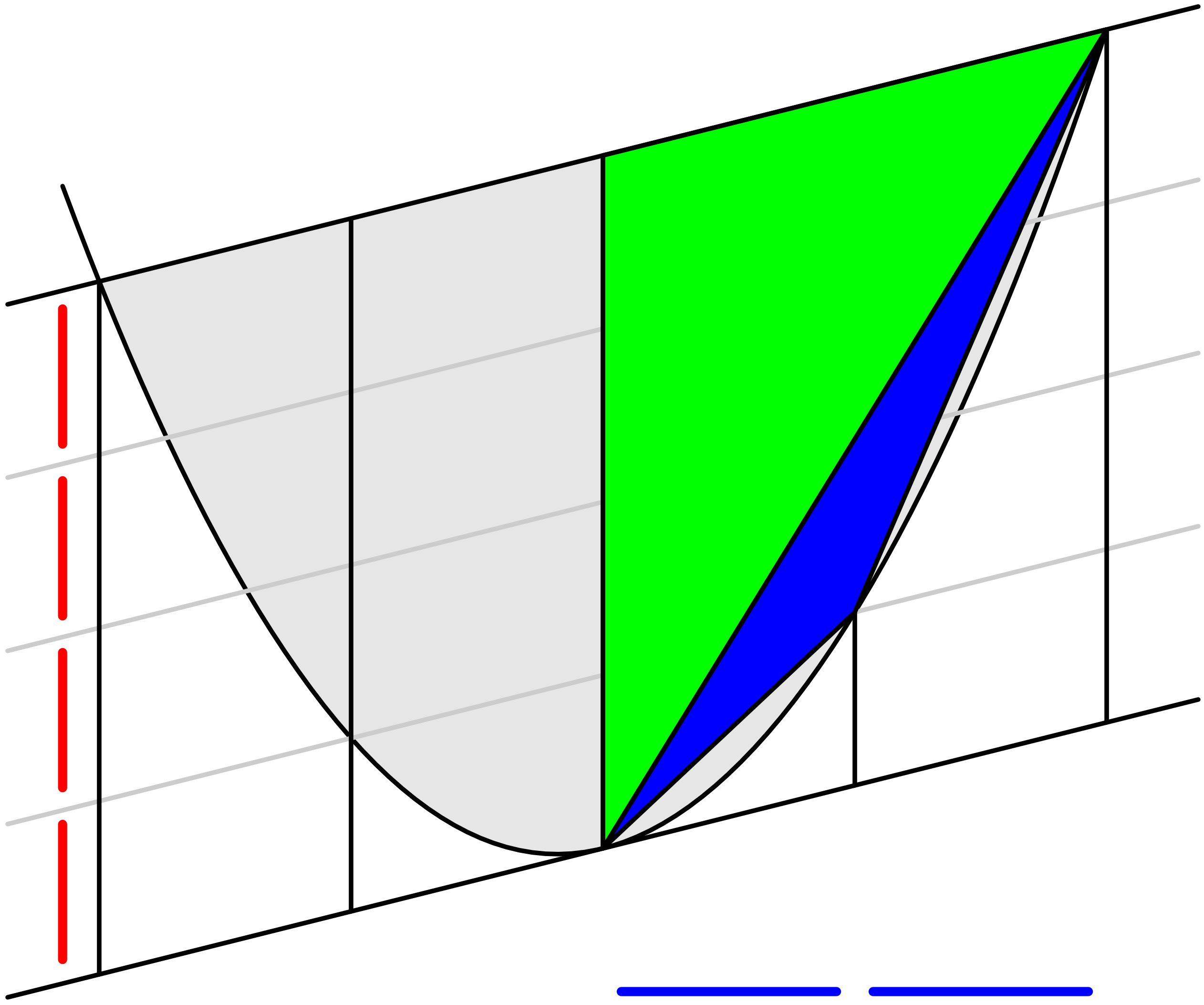
The red triangle is 1/4 of the whole triangle
Since the red base is 1/2 the green base <red lines>
and the red height is 1/2 the green height <blue lines>



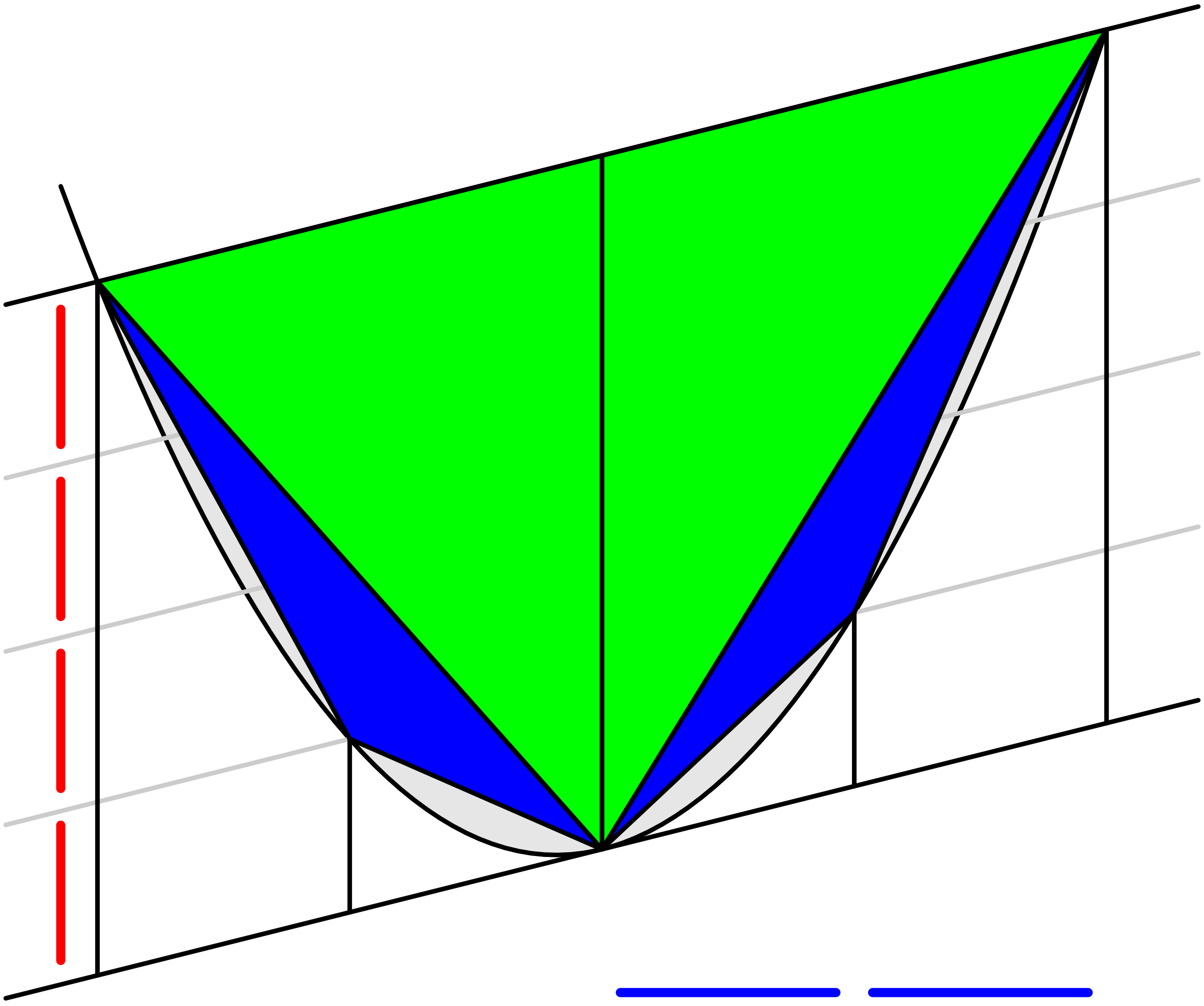
The blue equals the red



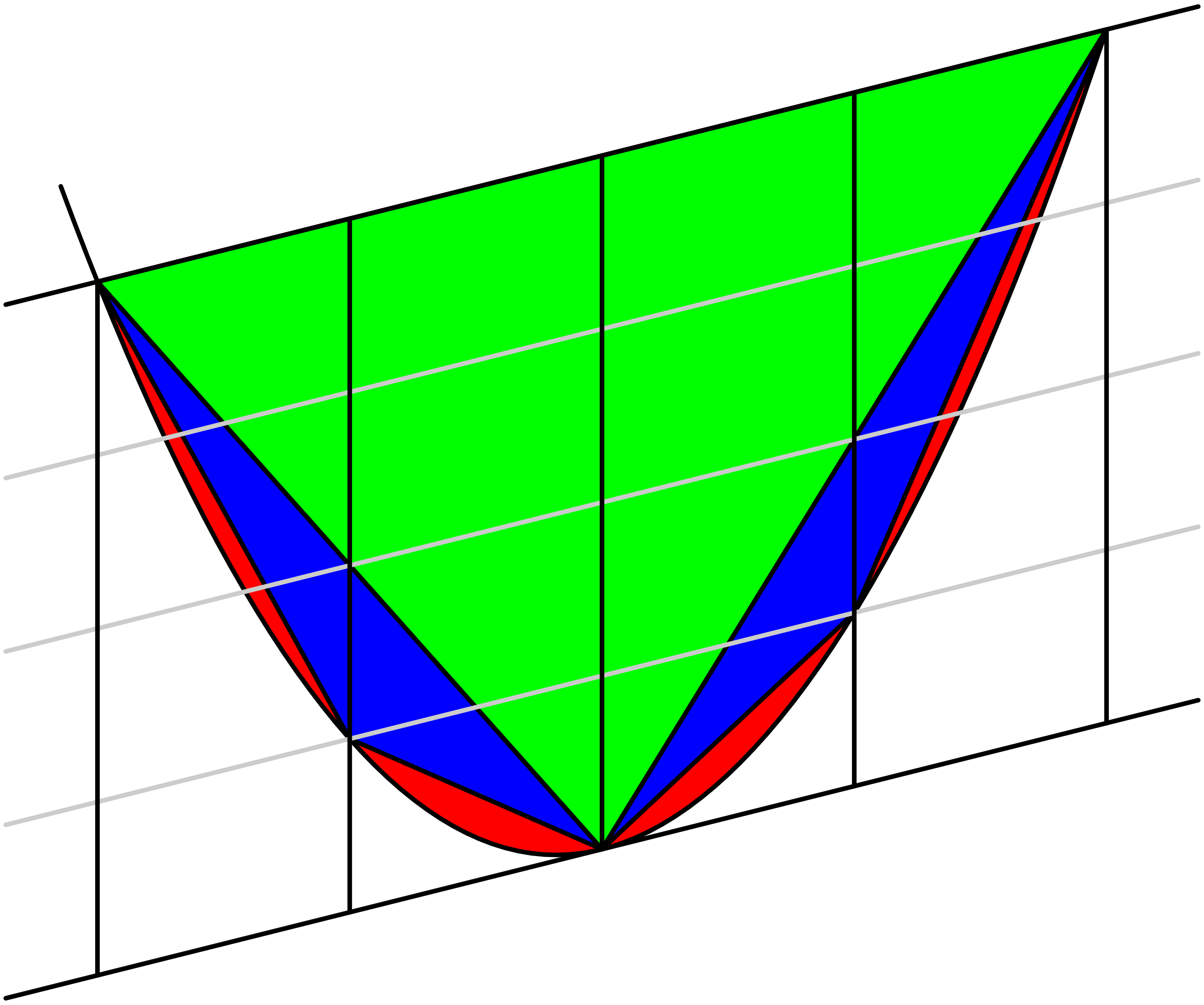
Thus the blue is 1/4 that of the green



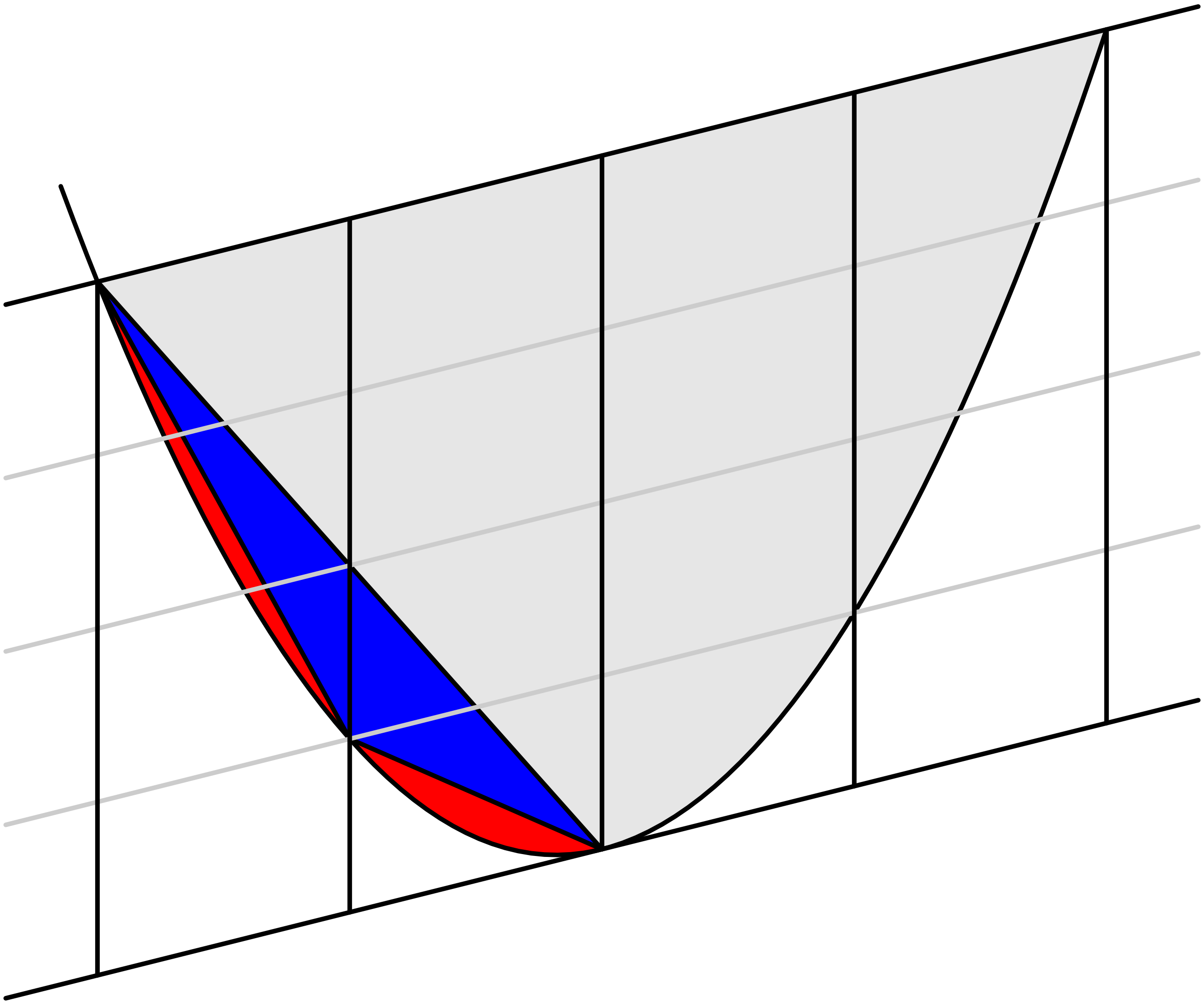
Similarly for the right side



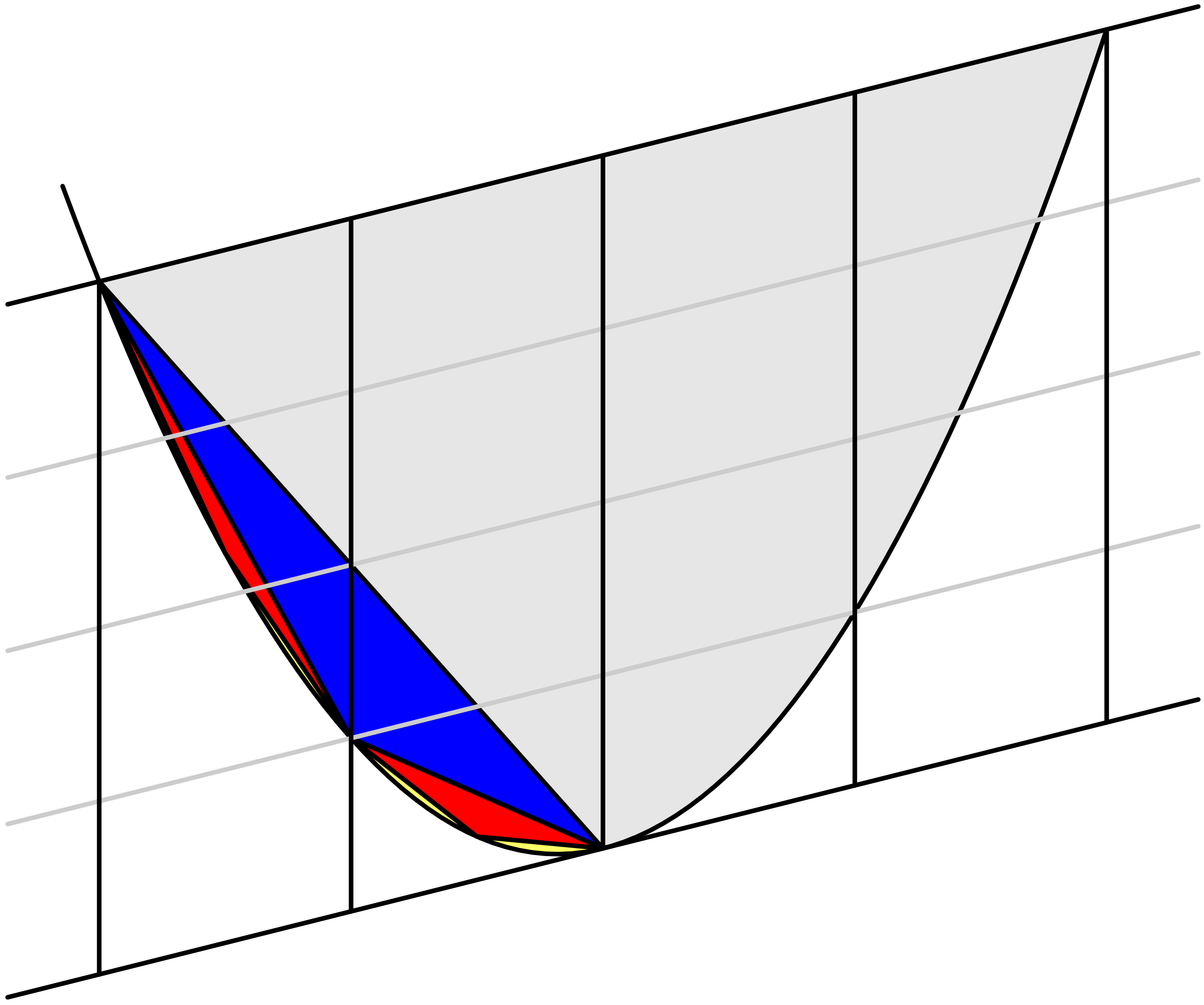
So the blue is 1/4 that of the green



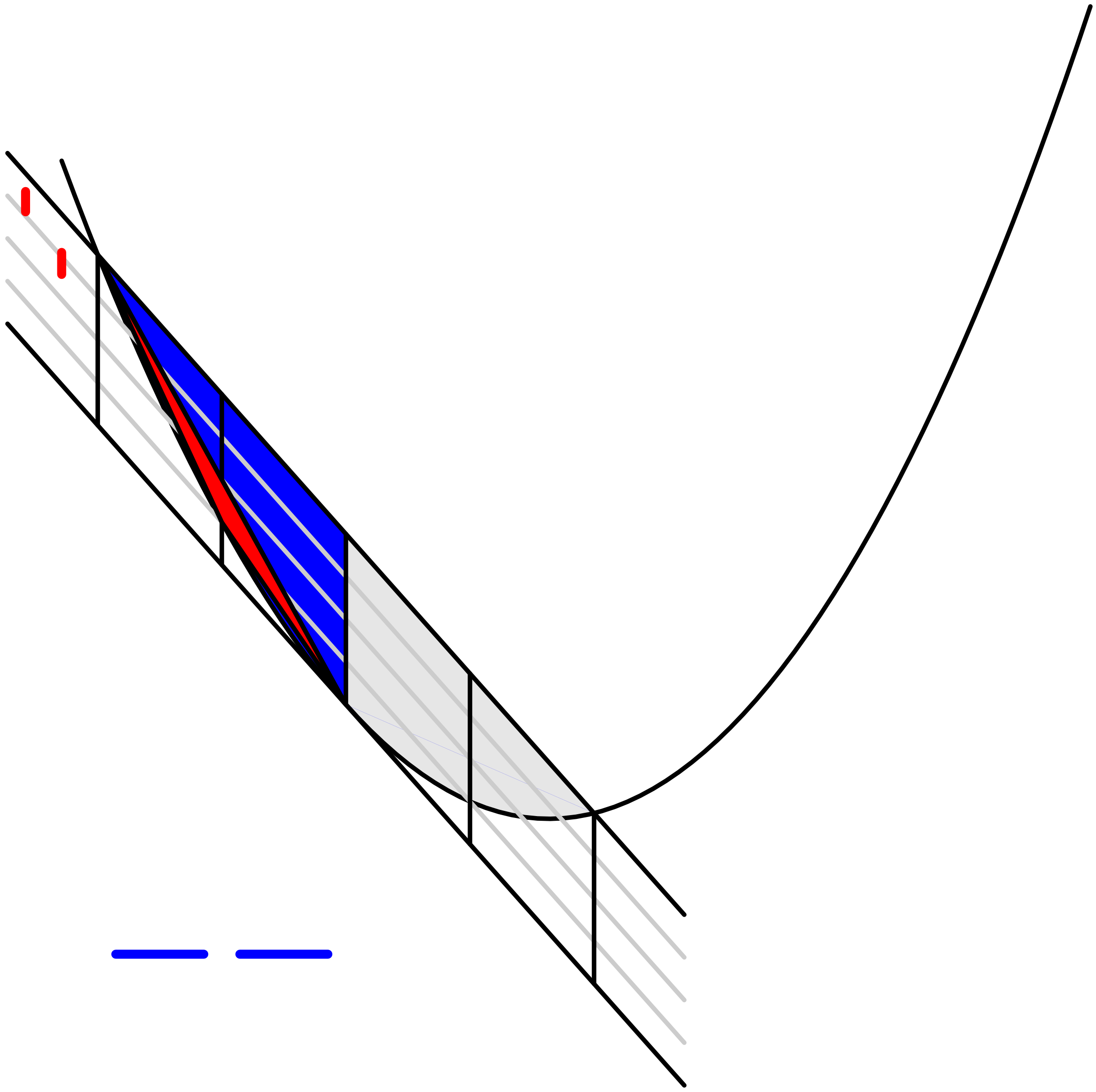
Now we will see that the red is 1/3 that of the blue
This is the same as the original problem
since we are looking at another parabola section

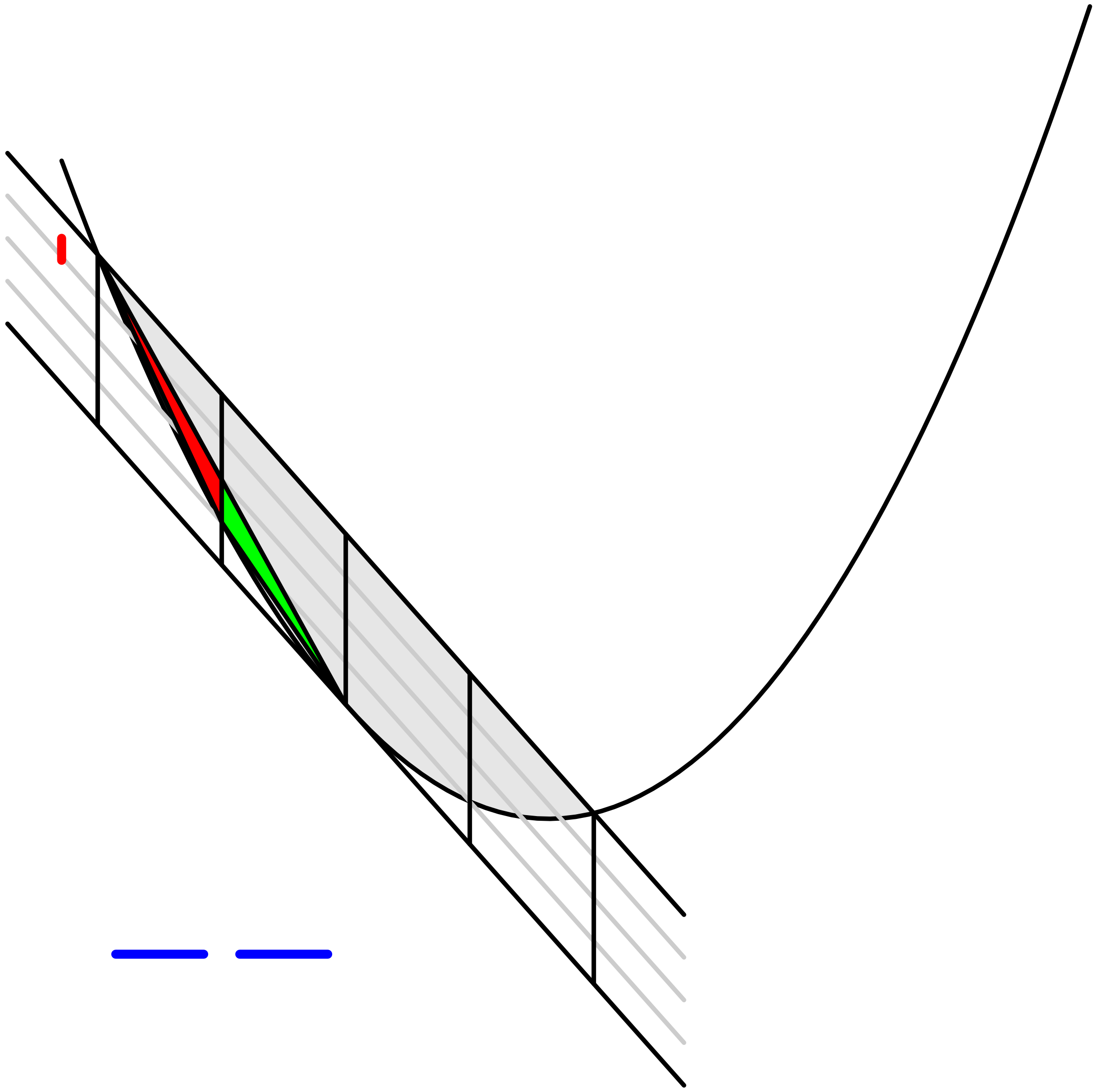


We have the original problem once again, finding the area above a parabola is the triangle (blue) plus 1/3 (the red) We thus repeat the process to find that the red is 1/3 of the blue

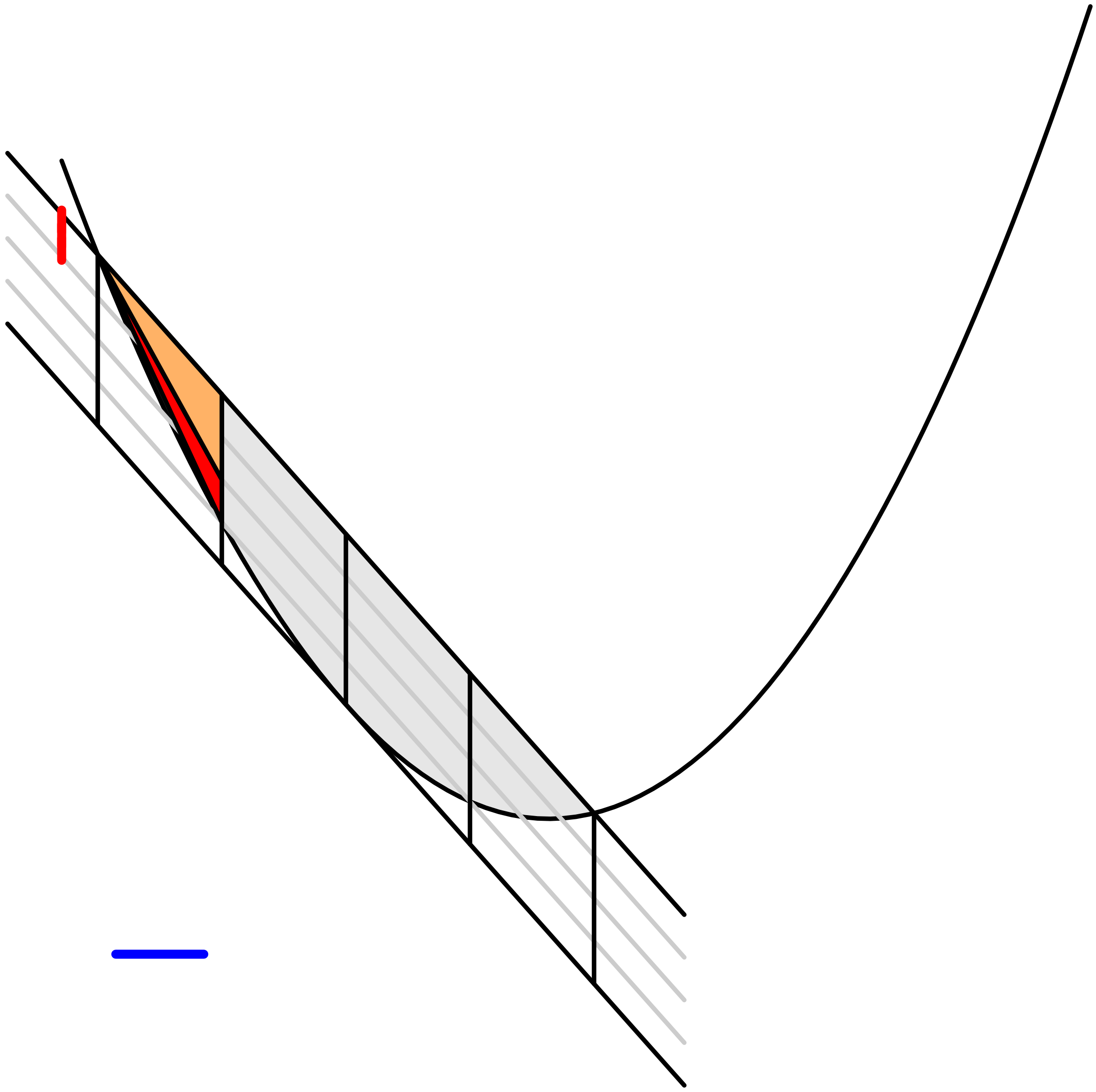


We see that the red now is 1/4 that of the blue and the yellow makes up the rest

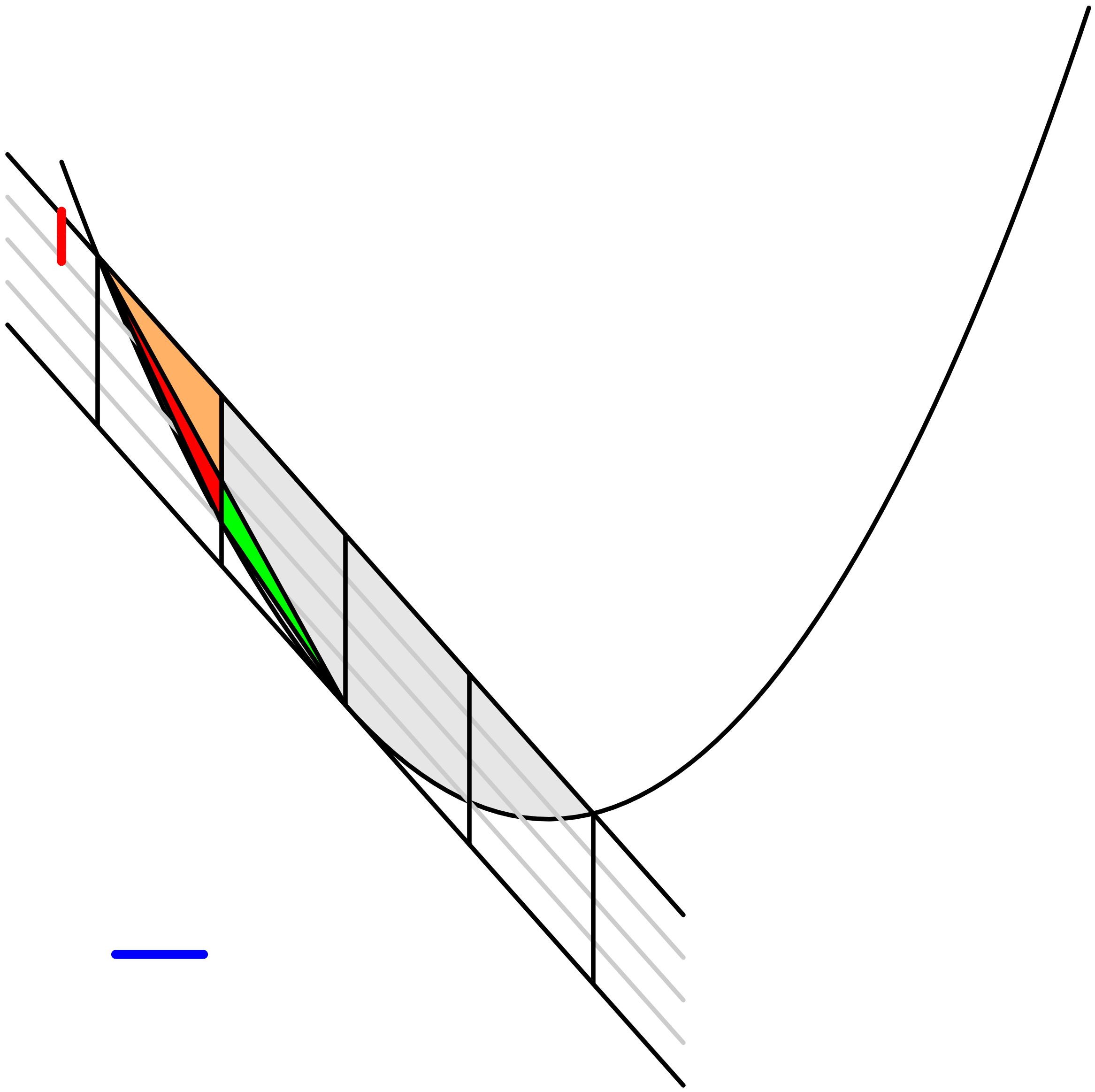




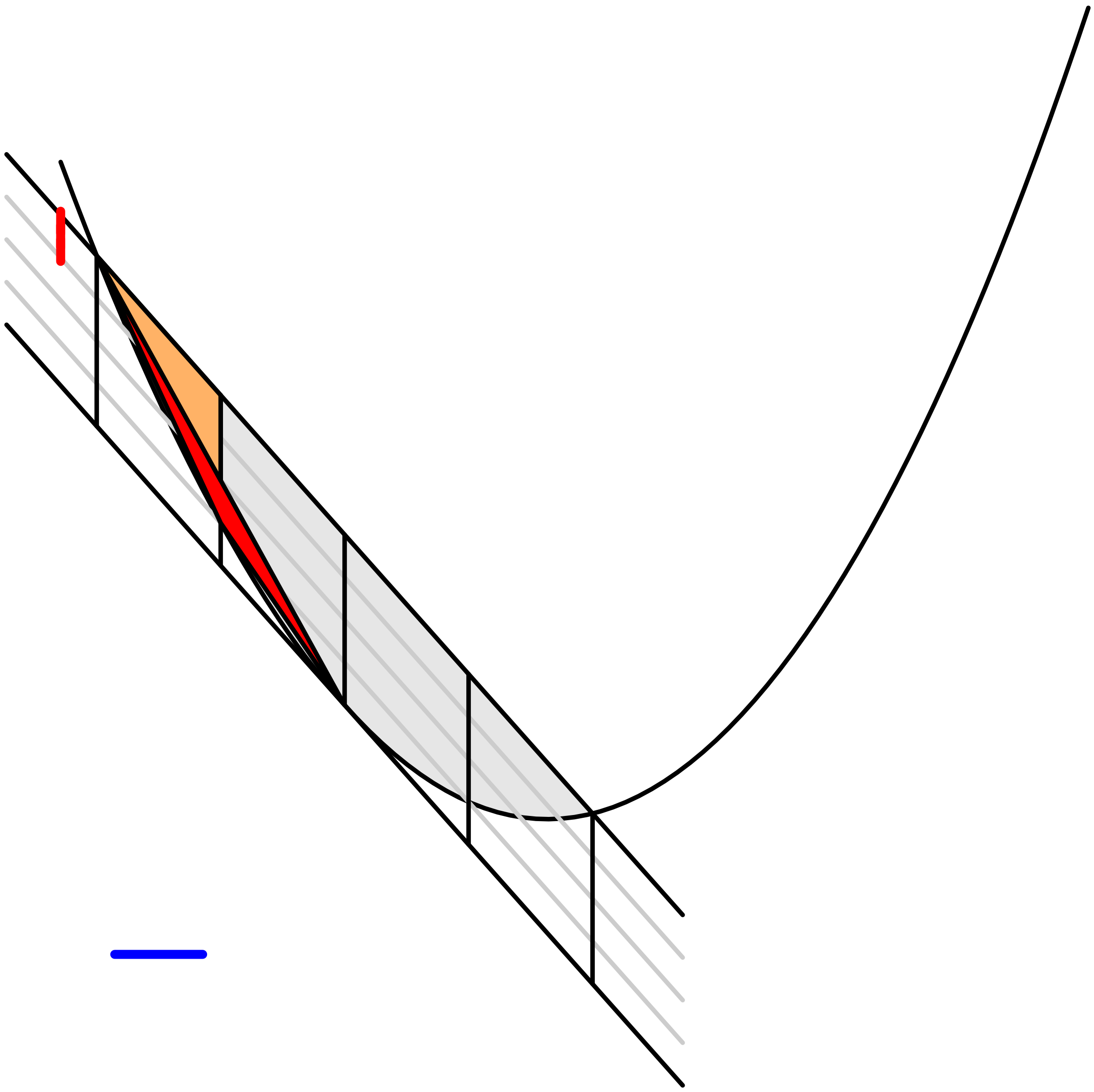
The red equals the green
The bases and heights are equal



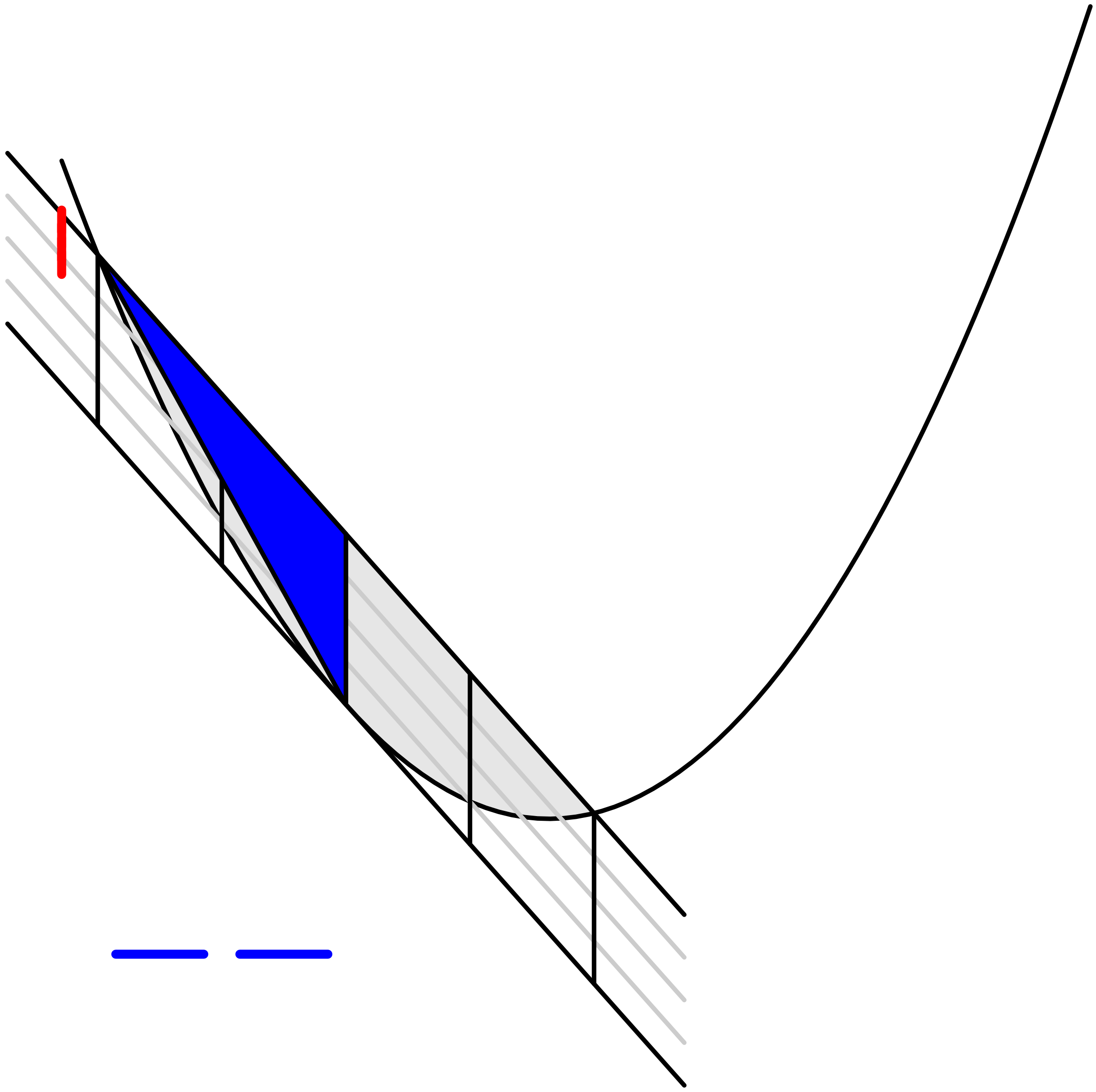
The red equals 1/2 the orange
The heights are equal <blue line>
The red base is 1/2 that of the orange base <red lines>



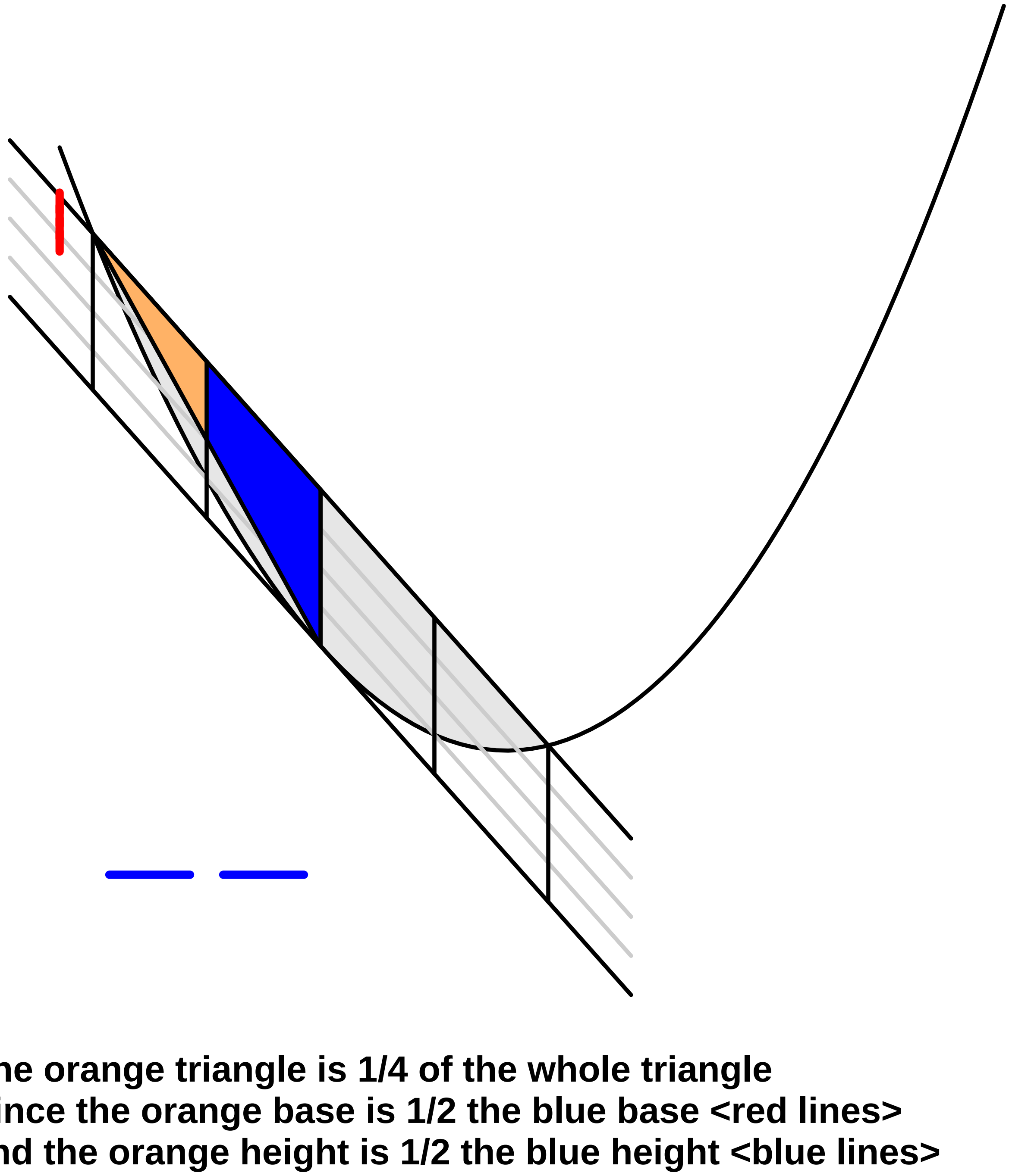
Thus, the red and green together equal the orange



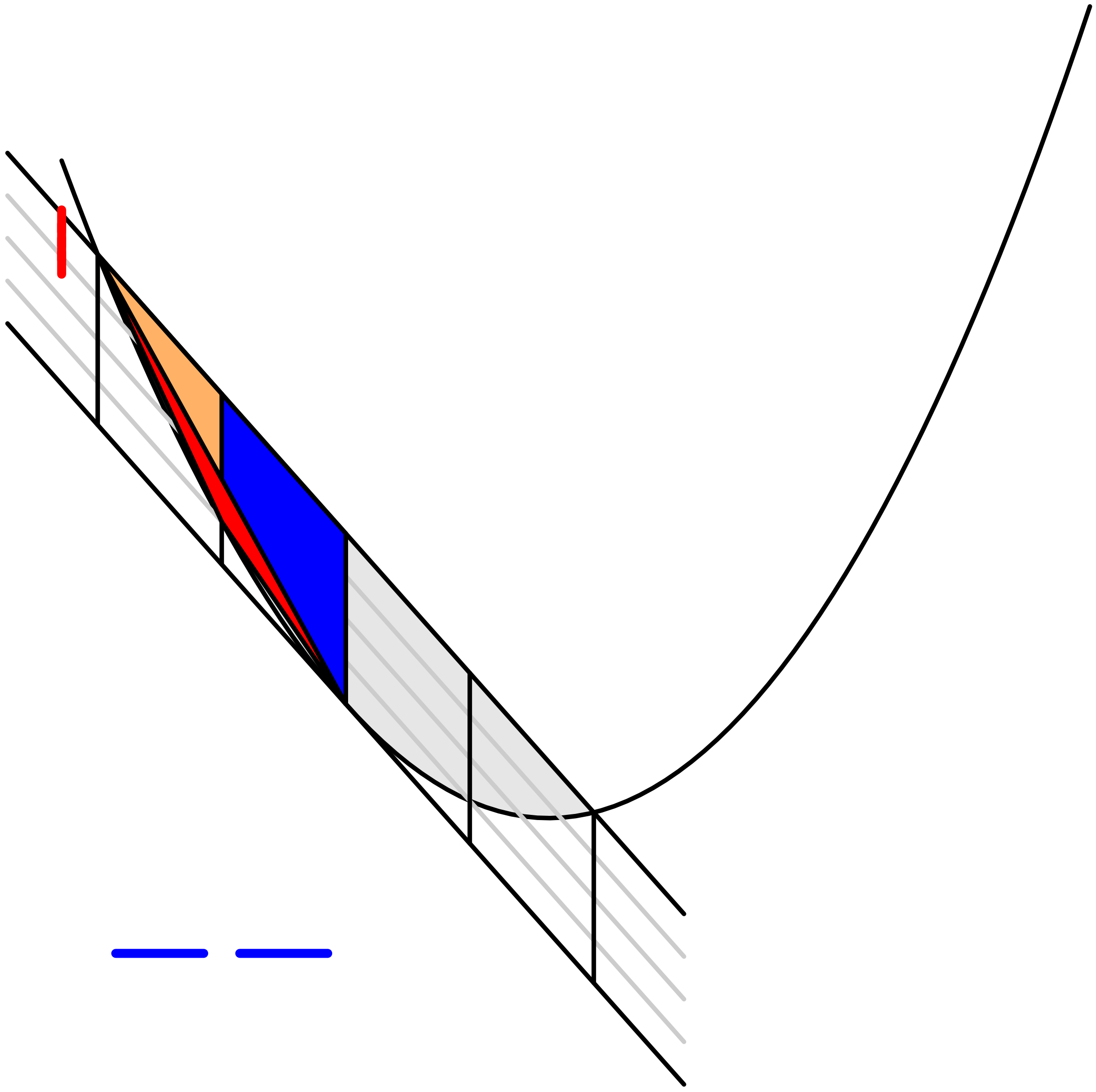
Thus, the red equals the orange



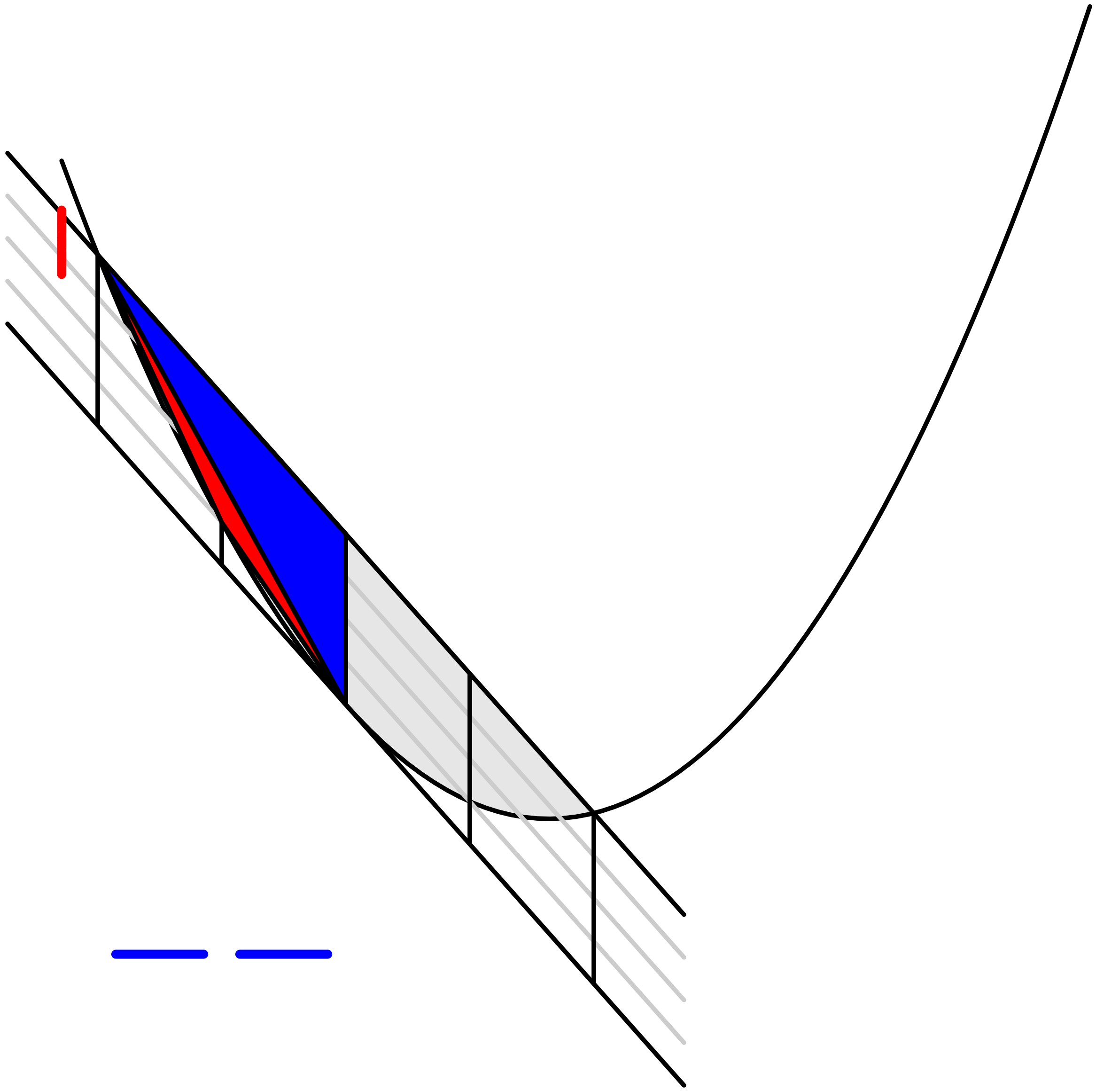
Now take the blue triangle
This is a section of the original blue
triangle of this parabola section



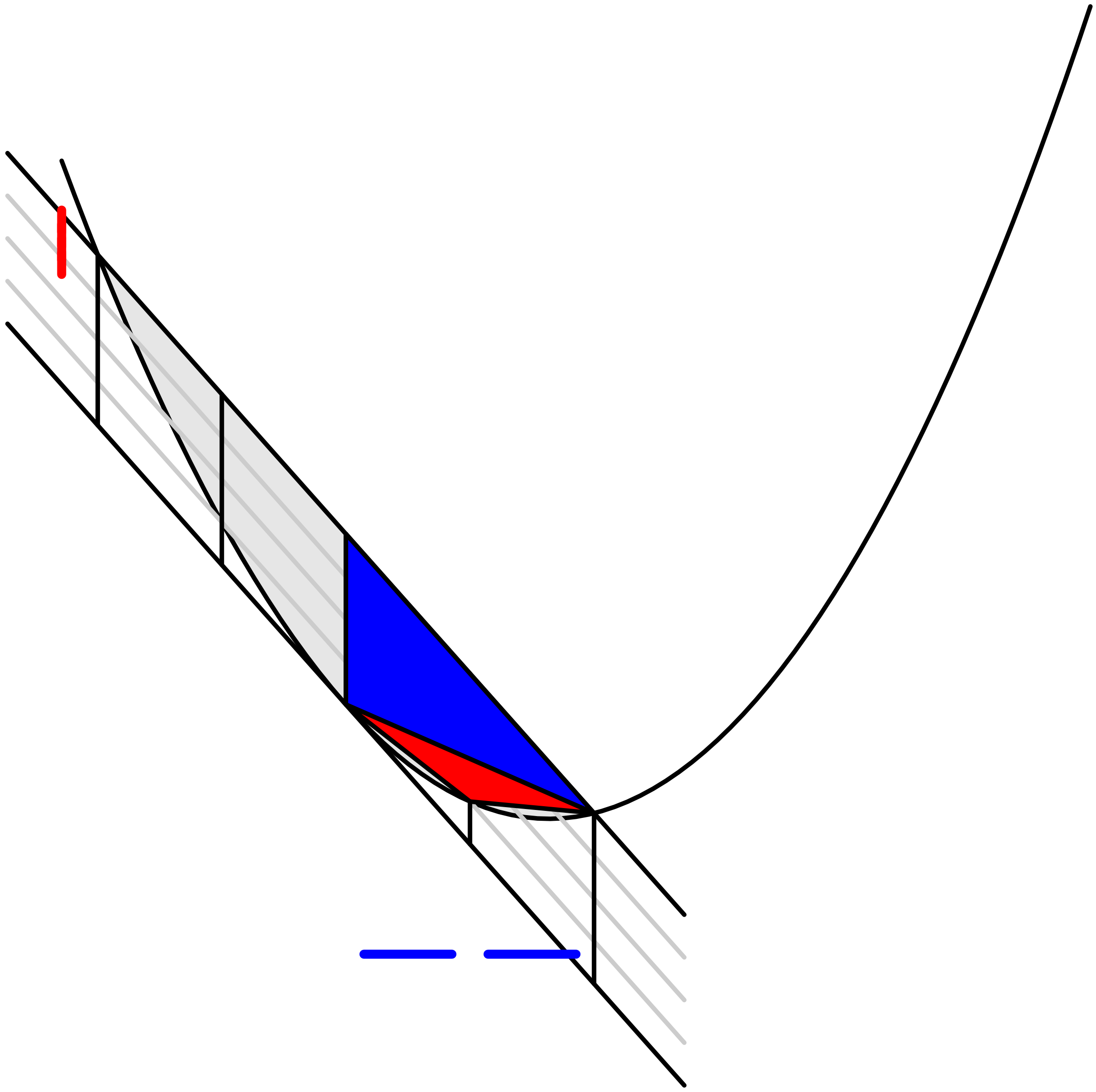
The orange triangle is 1/4 of the whole triangle
Since the orange base is 1/2 the blue base <red lines>
and the orange height is 1/2 the blue height <blue lines>



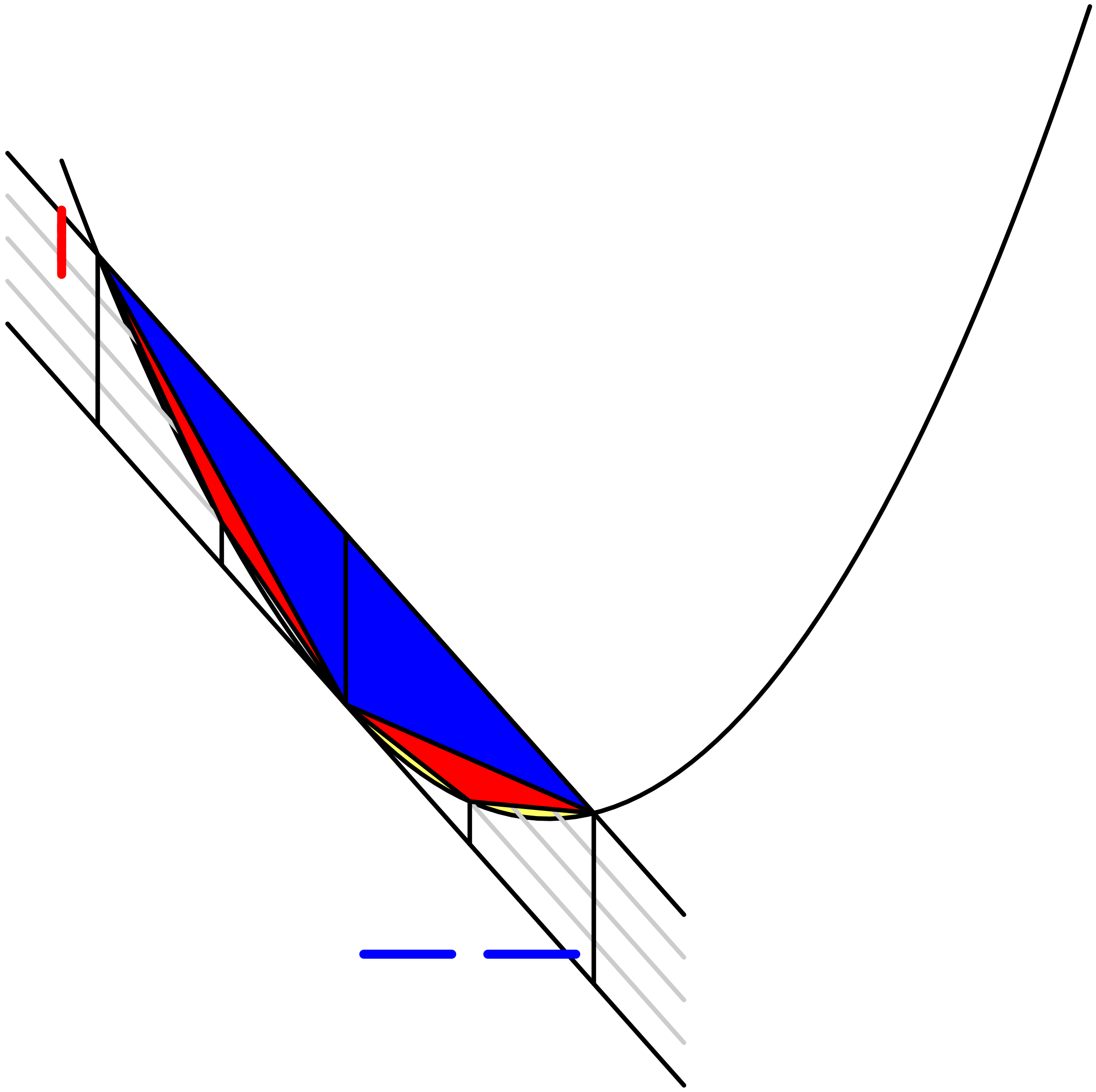
The red equals the orange



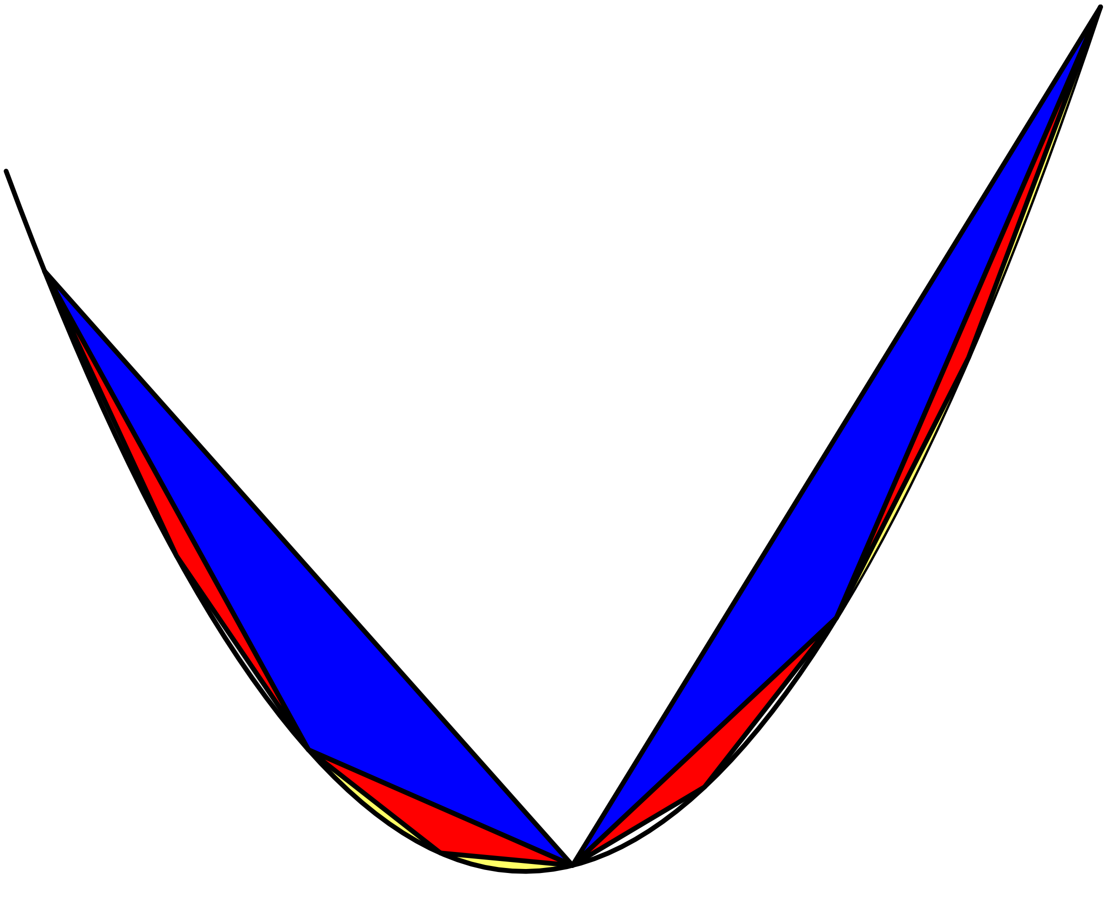
Thus the red is 1/4 that of the blue



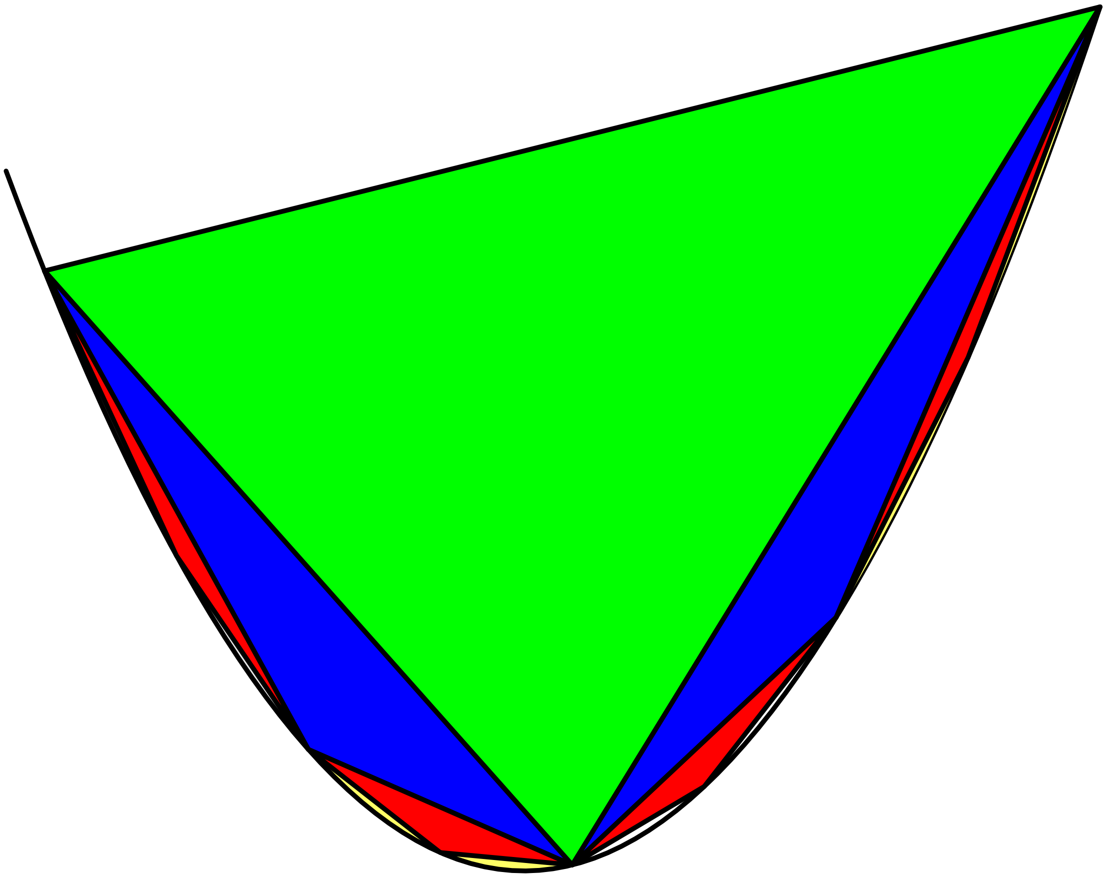
Similarly for the right side



So the red is 1/4 that of the blue



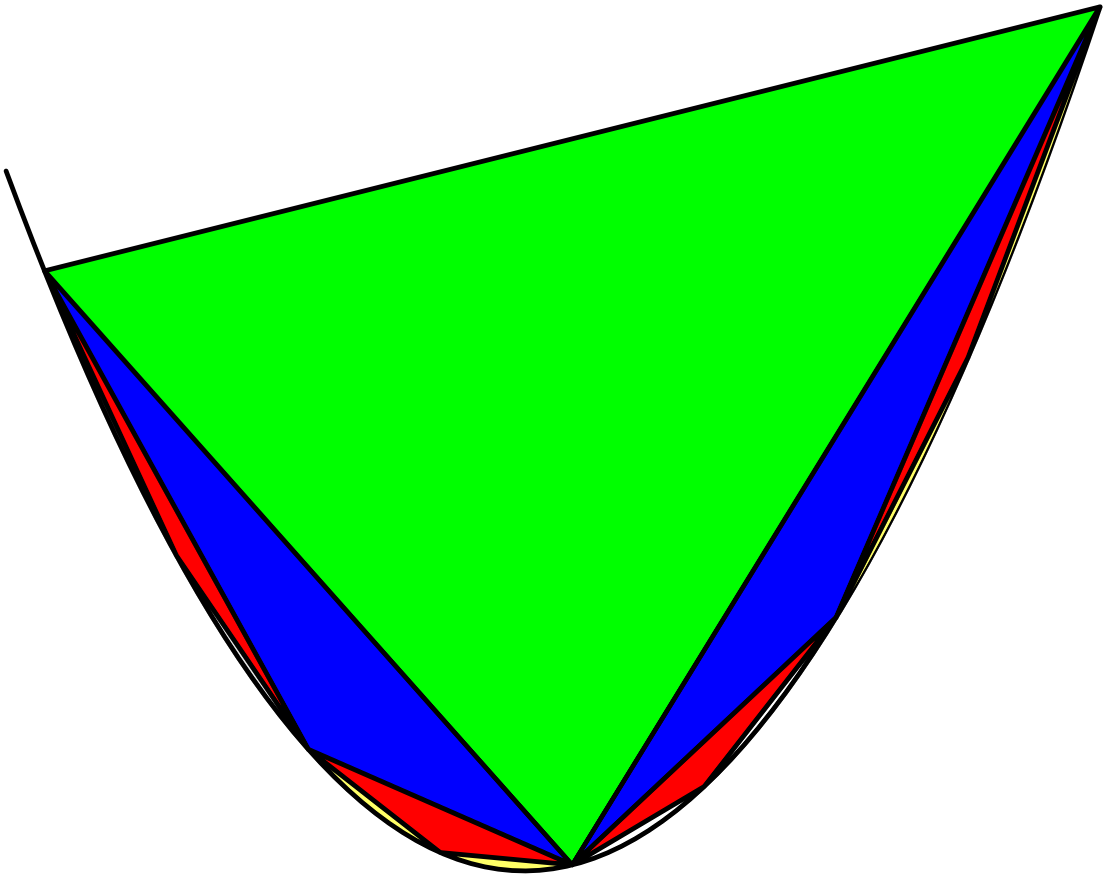
So the red is 1/4 that of the blue



So the red is 1/4 that of the blue
And the blue is 1/4 that of the green
And there is a little yellow leftover

So the area above the parabola is:
1 green + 1/4 green + 1/4 blue + yellow bit

= 1 green + 1/4 green + 1/16 green + yellow bit

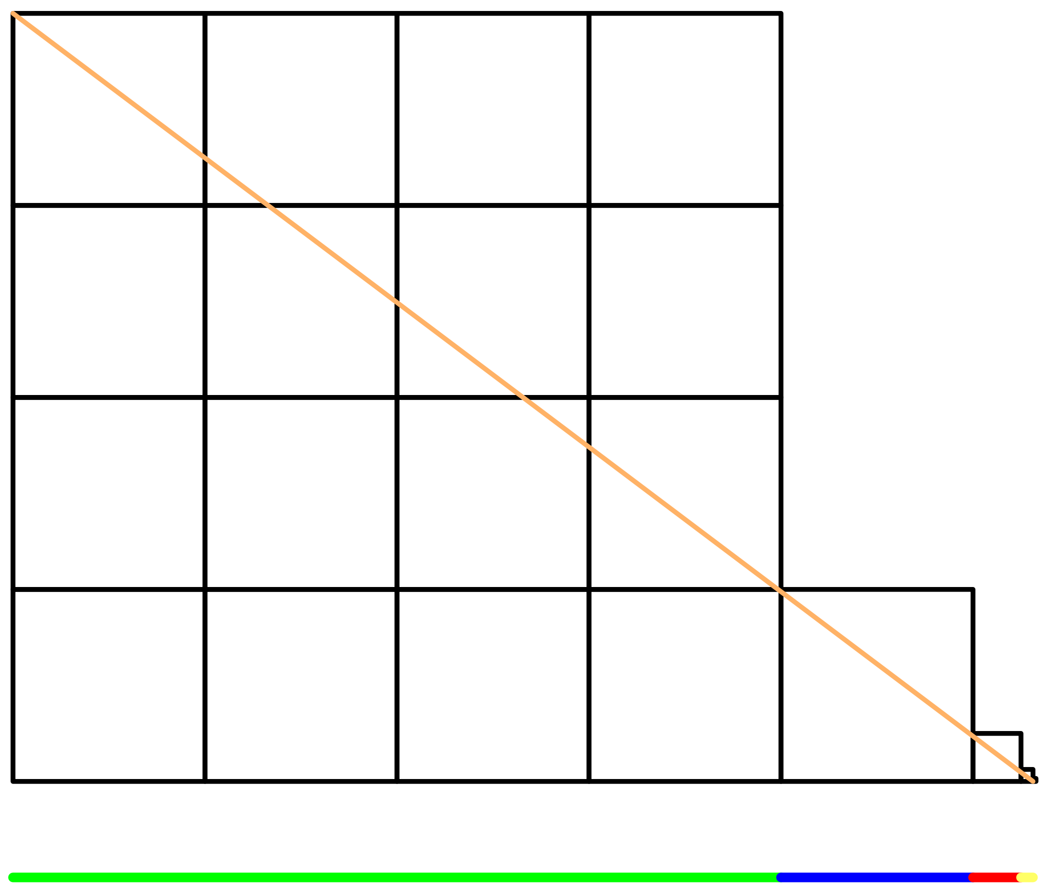


This seems to indicate that the whole area above the parabola is:

1 green + 1/4 green + 1/16 green + 1/64 green + 1/256 green + ...

We will show that this equals

1 green + 1/3 green = 4/3 green



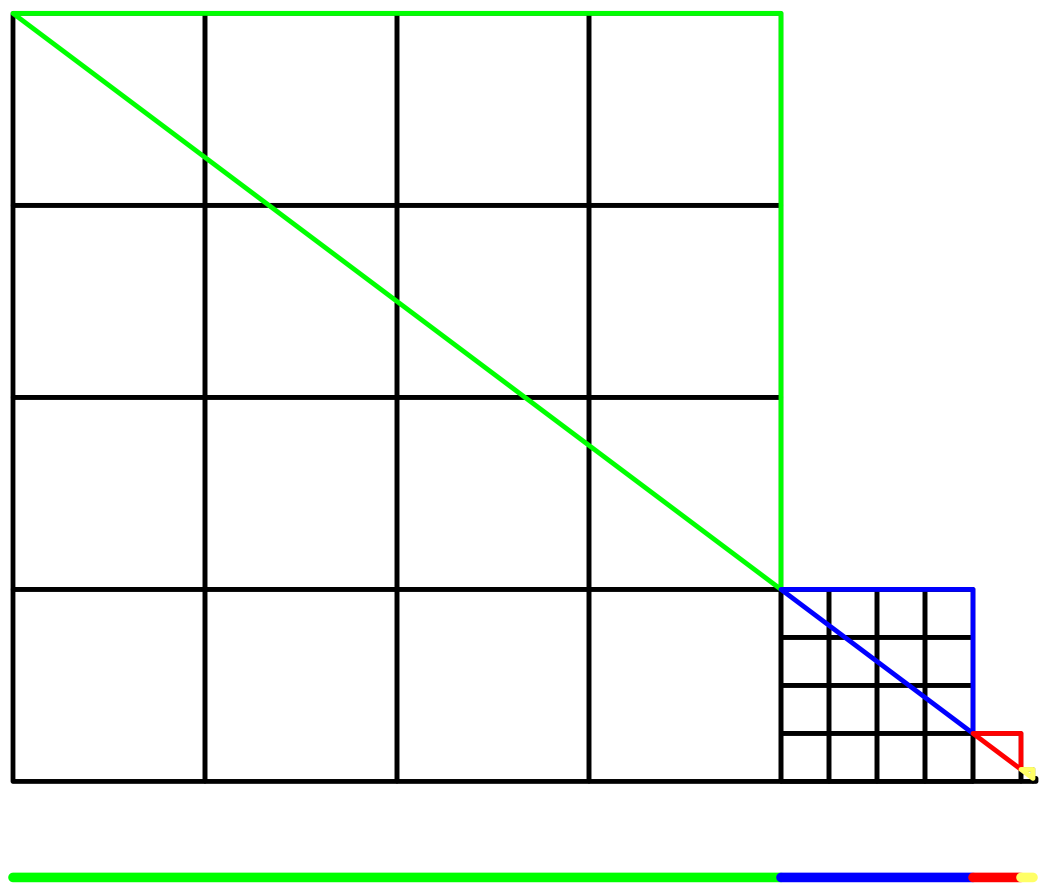
We can see the sum of this series by looking at the sum of lines
Here, the green line has length 1, the blue line 1/4 and so on

This seems to indicate that the whole length of the line is:
1 green + 1 blue + 1 red + 1/64 ...

1 green + 1/4 green + 1/16 green + 1/64 green + 1/256 green + ...

= 1 + 1/4 + 1/16 + 1/64 + ...

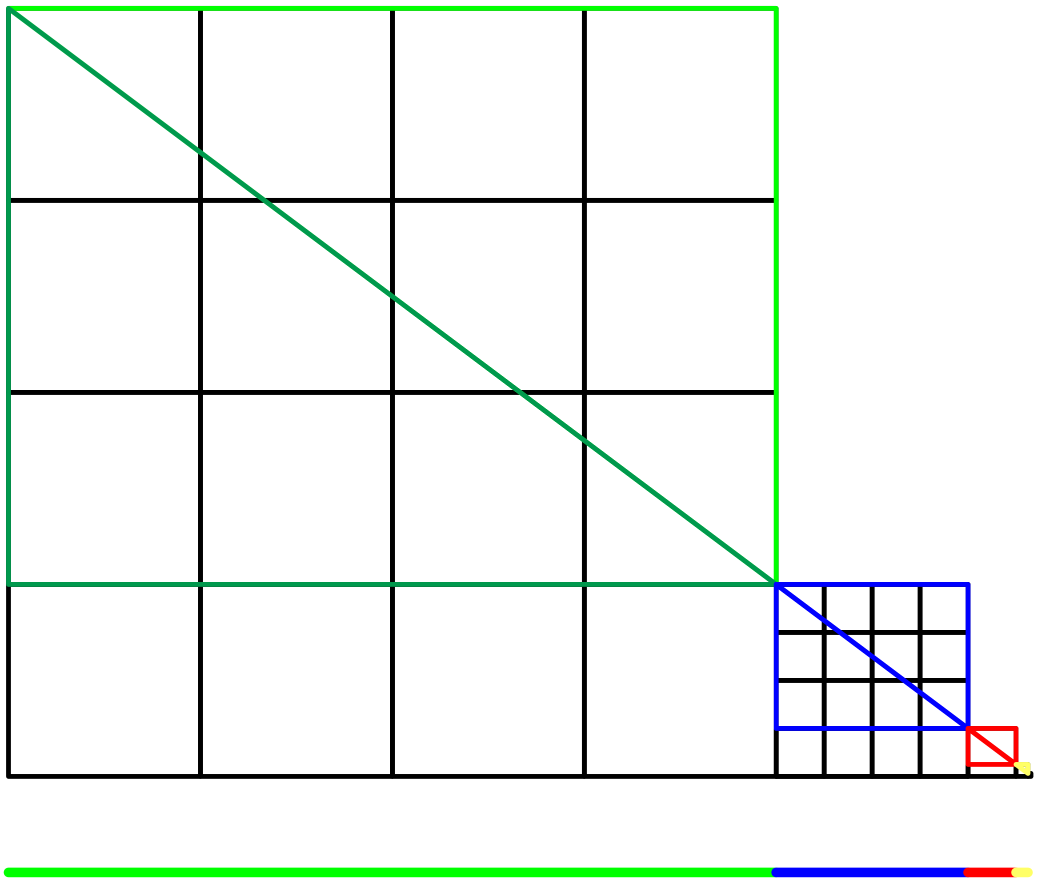
= 4/3



We can see that the green, blue and red triangles are similar

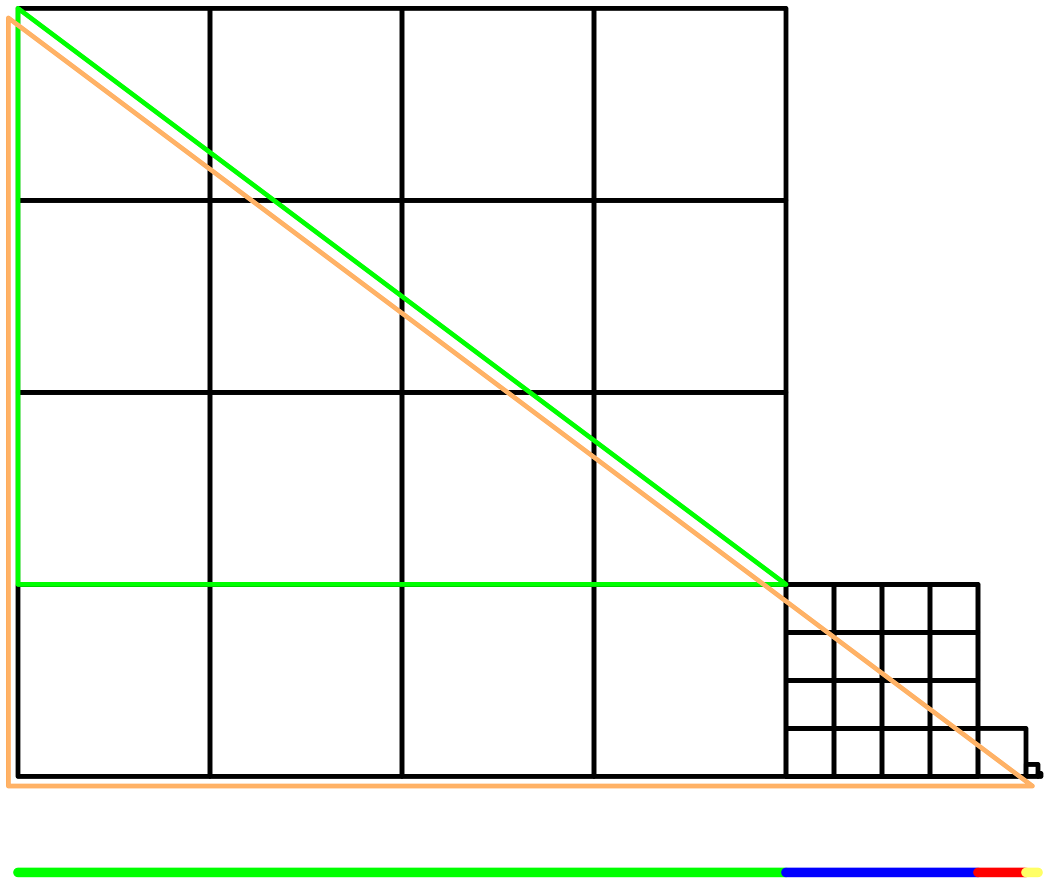
Since they are right triangles and have side ratios 4:3

So the angled bisecting coloured line is straight



Therefore, the triangles opposite are congruent (rectangle bisectors)

And they are similar to each other



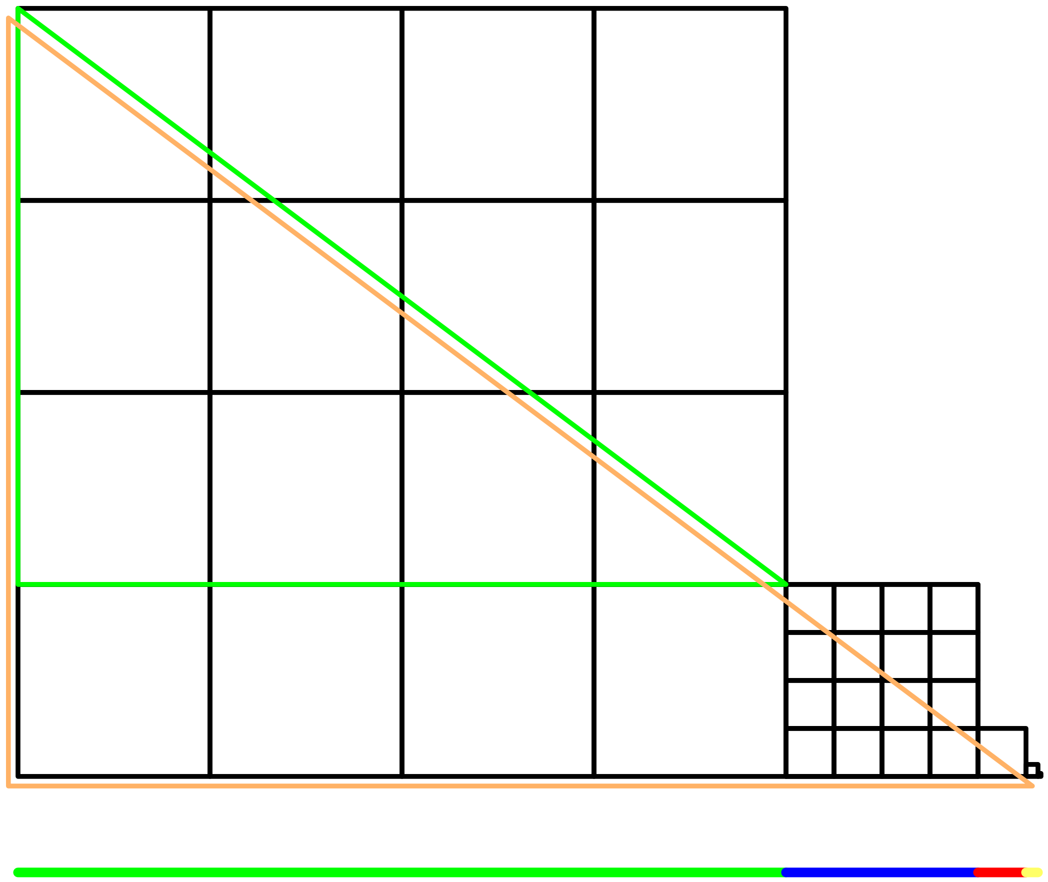
And the large orange triange is similar to the green triangle

Therefore, their sides have the same ratios 4:3

That is, the base is 1/3 longer than the height

So the length of the green, blue, red, yellow lines is:

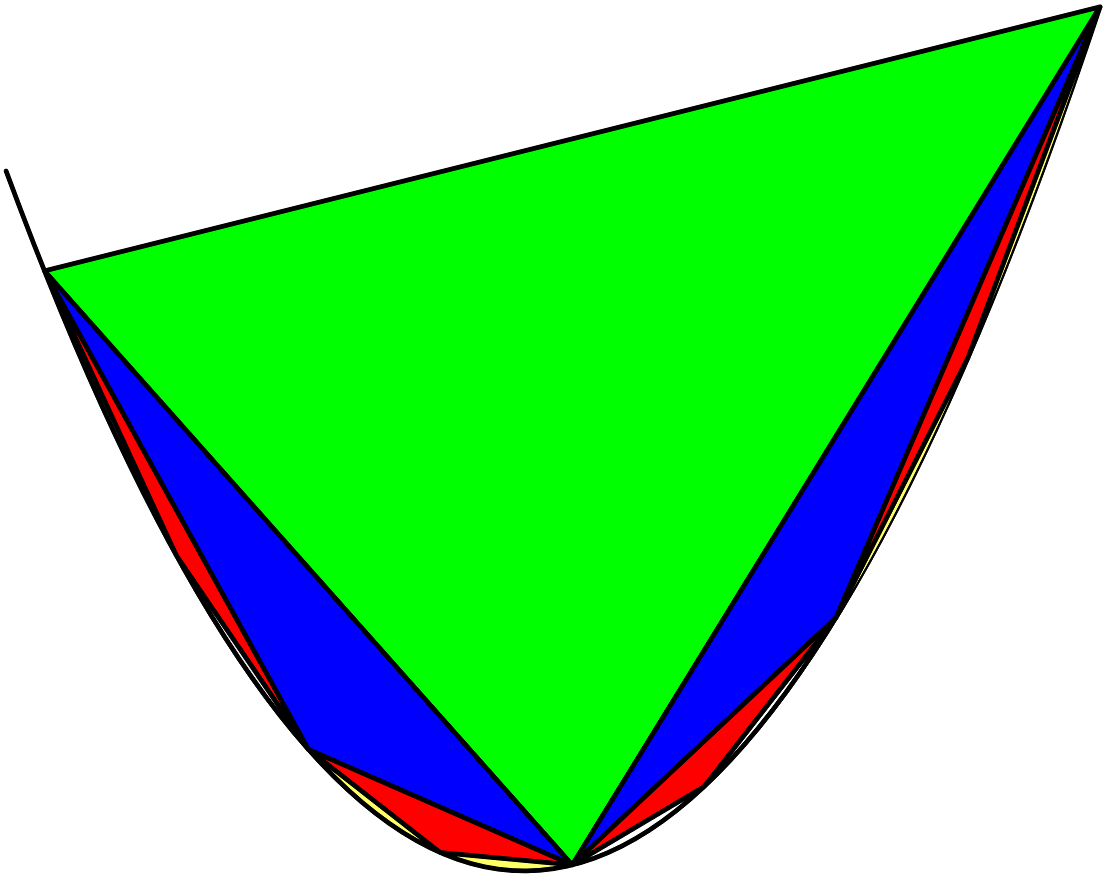
1 block + 1/3 of 1 block = 4/3 blocks



This shows that the sequence

1 block + 1/4 block + 1/16 block + ...

= 1 + 1/4 + 1/16 + ... = 4/3 [the ratio of the sides]



This proves that the area of the triangles converge to 4/3

1 green + 1/4 green + 1/16 green + 1/64 green + ... = 4/3 green

Thus the area above the parabola is

4/3 of the green triangle

Q. E. D.