

Production & Costs

Production Function

- The production function shows the relationship between output and input.
- It is usually expressed as $Q = f(K, L, A)$ where K =capital, L =labor, and A =Technology are the three major types of inputs.
- Note that the notation $f()$ does not specify an exact mathematical relationship, only that one exists.

Production Function

- Labor is considered a variable input as it varies directly with output.
- Capital and technology are considered fixed inputs as output can vary over a given level of input.

Production Function

Measures of Productivity (Labor)

- We can measure productivity of the factors of production (K, L, A).
- Total Product of Labor (TPL) = Q
- Average Product of Labor (APL) = Q/L
- Marginal Product of Labor (MPL) = $\text{Chg } Q / \text{Chg } L$

Production Function

- The relationship between output and input reflects productivity.

Production Function Example

L	1	2	3
TP	10	18	24
APL	10	9	8
MPL	10	8	6

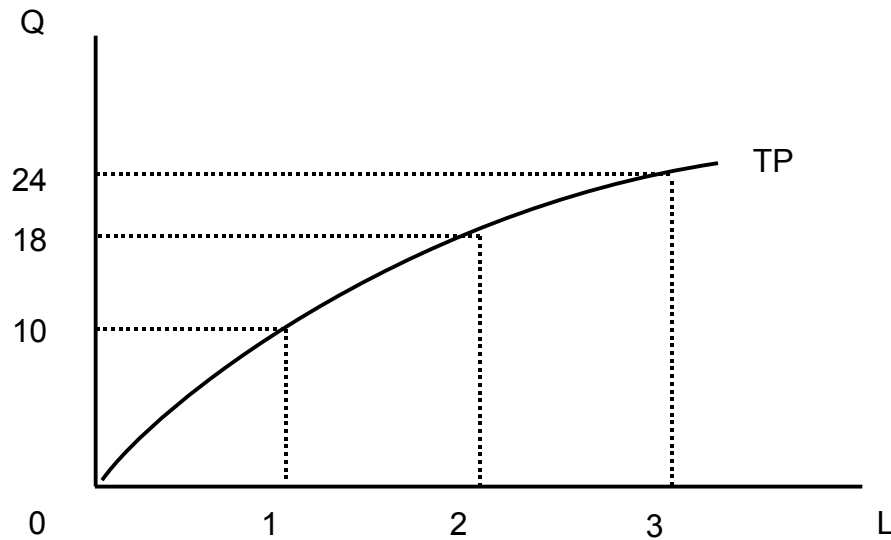
Production Function

Notes to slide

- Line 1: units of labor input
- Line 2: output based on labor input
- Line 3: Col 1 $10/1 = 10$; Col 2 $18/2 = 9$
- Line 4: Col 2 $(18-10)/(2-1) = 8$

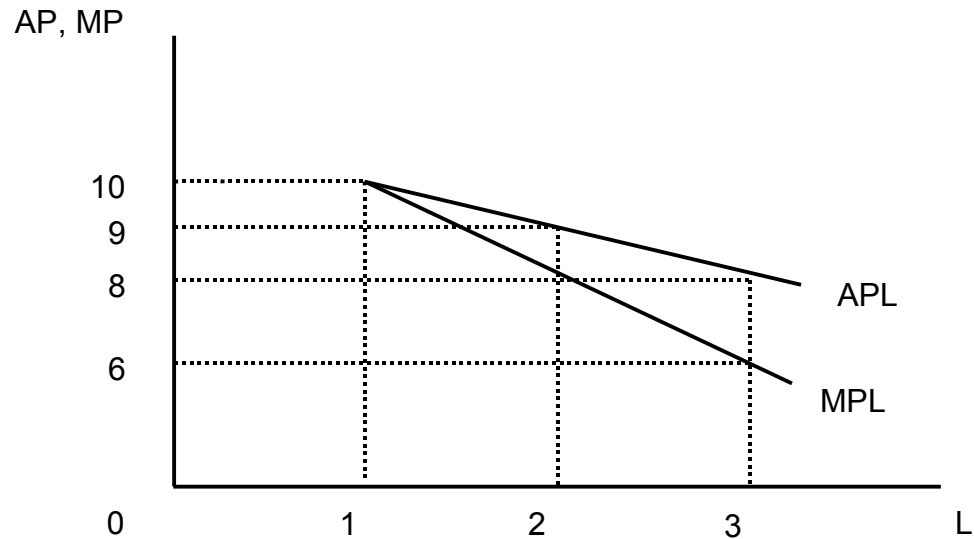
Production Function

Graphing Output



Production Function

Graphing Productivity



Production Function

Relationship between Productivity & Output

- Note that output increases at a decreasing rate.
- This occurs because the MPL is decreasing.
- The behavior of MPL determines the behavior of APL and Q as L increases.

Production Function

Effect of Capital & Technology on Labor Product Curves

- Increasing the amount of capital causes the product curves of labor to shift upward.
- Increasing the level of technology also causes the product curves of labor to shift upward.

Production Function

Relationship between the Production Function & Cost Structure

- The factors of production (capital, labor and technology) determine the cost structure and behavior.
- If capital is used extensively, then costs will tend to be fixed, that is not vary with output.
- If capital (a fixed input) is a large portion of inputs, then a majority of costs will be fixed.

Production Function

Example

- Let's compare two firms with the same output: East and West.

	FC	VC	TC
East	12	100	112
West	60	20	80

Production Function

Example

- Let's calculate the two firms costs.

	L	w	VC	K	r	FC
East	10	10	100	10	6	60
West	2	10	20	2	6	12

Production Function

Example

- Symbol explanation: Capital (K), Labor (L), Wage (w), Interest (r), Capital Cost (KC), Labor Cost (LC), and Total Cost (TC)

Production Function

Example

- Note that labor is relatively more expensive than capital in this example. A capital intensive firm has an advantage.
- In countries where labor is cheap, labor intensive firms have an advantage.

Costs

Economic Costs

- Concept of Economic Costs
 - Economic costs include explicit and implicit costs.
 - Explicit costs are costs for market supplied resources that are purchased.
 - Implicit costs are costs for the use of self owned resources. Often such costs are imputed.

Economic Costs

- The major implicit cost is that for the use of equity capital (Return On Equity, ROE).
- Economic costs include implicit costs accounting costs do not.
- When economists refer to costs, they are referring to economic costs not accounting costs.

Cost Structure

- Costs follow the structure of production process.
- Fixed inputs create fixed costs, variable inputs create variable costs.
- In traditional economics, capital is considered a fixed input.
- Labor is considered a variable input.

Cost Structure

- Total Fixed Cost (TFC)
- Total Variable Cost (TVC)
- Total Cost (TC)
- Average Fixed Cost (AFC)
- Average Variable Cost (AVC)
- Average Cost (AC)
- Marginal Cost (MC)

Cost Formulas

- $TC = TFC + TVC$
- $AFC = TFC / Q$
- $AVC = TVC / Q$
- $AC = AFC + AVC$
- $MC = \text{Chg } TC / \text{Chg } Q$
- $TVC = AVC * Q$
- $TFC = AFC * Q$

Total Fixed Cost

- Fixed costs do not vary with changes in output.
- Example
 - General & Administrative expenses
 - Marketing expenses
 - Rent/Depreciation
 - Interest Expense
 - Taxes

Java Example

- Let's say we have a small cafe, call it Java Cafe. Each month we have to pay the mortgage of \$1,500. Utilities, which do not vary with activity, cost \$400. The owner/manager takes a monthly salary of \$5,000. Other business expenses such as accounting, fees, licenses, etc. are approximately \$300 per month.

Java Fixed Costs

- Therefore, total fixed costs on a monthly basis are \$7,200 ($1,500 + 400 + 5,000 + 300$).

Average Fixed Costs

- Fixed cost per unit
- Formula: $AFC = TFC / Q$

Java's Average Fixed Costs

- On any given month, Java Cafe sells 6,000 cups. Consequently, the fixed cost per unit, the average fixed cost is $\$7,200/6,000 = \1.20 .

Variable Costs

Detailed calculations

Q	L	w	TVC	AVC	APL
10	15	10	150	15	.67
20	20	10	200	10	1.00
30	35	10	350	11.7	.86
40	55	10	550	13.8	.73

Costs

Notes to calculations

- Labor hours * wage rate = Total Variable Cost
- $TVC/Q = AVC$
- Note relationship between productivity and average variable cost, highest productivity produces lowest average variable cost

Average Variable Costs

- Variable cost per unit
- Formula: $AVC = TVC / Q$

Java's Average Variable Cost

- In the Java Cafe, our baristas can make 15 cups per hour. Since the wage rate is \$15 per hour, the average labor cost per cup is \$1.00.
- The cost of the cup, coffee, and milk (only whole milk is used) is .50.
- Therefore, the average variable cost is \$1.50.

Total Variable Costs

- Total variable costs that vary directly with output.
- Formula: $TVC = AVC * Q$

Java Total Variable Cost

- Since Java Cafe sells 6,000 cups per month and its average variable cost is \$1.50, then its total variable cost is \$9,000 ($\$1.50 \times 6,000$).

Total Costs

- Aggregate cost of output.
- Formula: $TC = TFC + TVC$

Java Total Costs

- With a total fixed cost of \$7,200 and a total variable cost of \$9,000, the total cost for Java Cafe is \$16,200.

Average Cost

- Average cost of output
- Formulas: $AC = AFC + AVC$
 $AC = TC / Q$

Java's Average Cost

- Java's total cost of \$16,200 divided by its production of 6,000 cups results in an average cost of \$2.70

Marginal Costs

- Additional cost incurred when producing one additional unit.
- Formula: $MC = \Delta TC / \Delta Q$

Java's Marginal Cost

- Suppose coffee demand increases, the additional cost per cup would be \$1.50.
- Fixed costs do not change, consequently, they are ignored.
- Since average variable cost is constant, this is an important consideration, then the marginal cost = AVC.

Revenue

- Total Revenue (TR) is Price (P) * Quantity (Q)
- Average Revenue (AR) is TR / Q which is merely P, hence is not used.
- Marginal Revenue (MR) is the chg TR per a one unit change in Q.

Java's Revenue

- The average price of Java Cafe's product is \$3.00. With monthly sales of 6,000 cups, total revenue for the month is \$18,000 ($\$3.00 * 6,000$).

Profit Maximization

- Decision Criteria: Marginal Costs (MC) = Marginal Revenue (MR)
 - Marginal cost is the additional cost of producing an additional unit
 - Marginal revenue is the additional revenue gained by the sale of an additional unit (in a competitive market, MR equals price)
 - Profits (losses) are maximized (minimized) where $MC = MR$.

Cost Limitations

- Some cost data, such as marginal cost, is not tracked by accounting system.
- Problem of allocating of costs among processes, products, customers, and markets.
- Data may be outdated.

Sunk Costs

Concept of Sunk Costs

- Costs that are incurred only once.
- Include research and development, marketing campaigns, capital acquisition, etc.

Sunk Costs

- Often poor business decisions are made when costs are not recognized as sunk but as recurring.
- Accountants tend to amortize sunk costs over future time periods.
- Managers may shut down an operation because accounting costs are too high as they believe those costs will go away.

Sunk Costs

- Consequently, the firm's profitability drops as the amortization of sunk cost either remains or is written off.
- Meanwhile the profit contribution of the shutdown operation is lost forever.
- While the operation may not have generated enough profit to cover the amortization, it did reduce the loss.

Java's Sunk Costs

- As Java Cafe was starting business, it had to pay legal costs of \$5,000 to obtain and secure trademark rights as well as incurring corporate organizational fees. These costs will not be incurred again.
- Since they are non-recurring, they are not included in monthly calculations of profit.

Sample Problems

Costs

Sample Problem

Q	TFC	TVC	TC	MC	AC
10	100	(A)	250	15	25
20	(B)	200	(C)	(D)	15
30	100	400	500	20	(E)
(F)	100	700	800	30	20

Costs

Sample Problem

- (A) $TC - TFC = TVC: 250 - 100 = 150$
- (B) TFC does not change, still 100
- (C) $100 + 200 = 300$
- (D) $(300 - 250) / 10 = 5$
- (E) $500 / 30 = 17$
- (F) $800 / 20 = 40$

Costs

Sample Problem

Q	L	APL	MPL
10	15	(A)	.7
20	(B)	1.0	2.0
30	35	.9	(C)
(D)	55	.7	.5

Costs

Sample Solution

A $APL = Q/L = 10/15 = .7$

B $L = Q/APL = 20/1.0 = 20$

C $MPL = \Delta Q/\Delta L = (30-20)/(35-20) = .7$

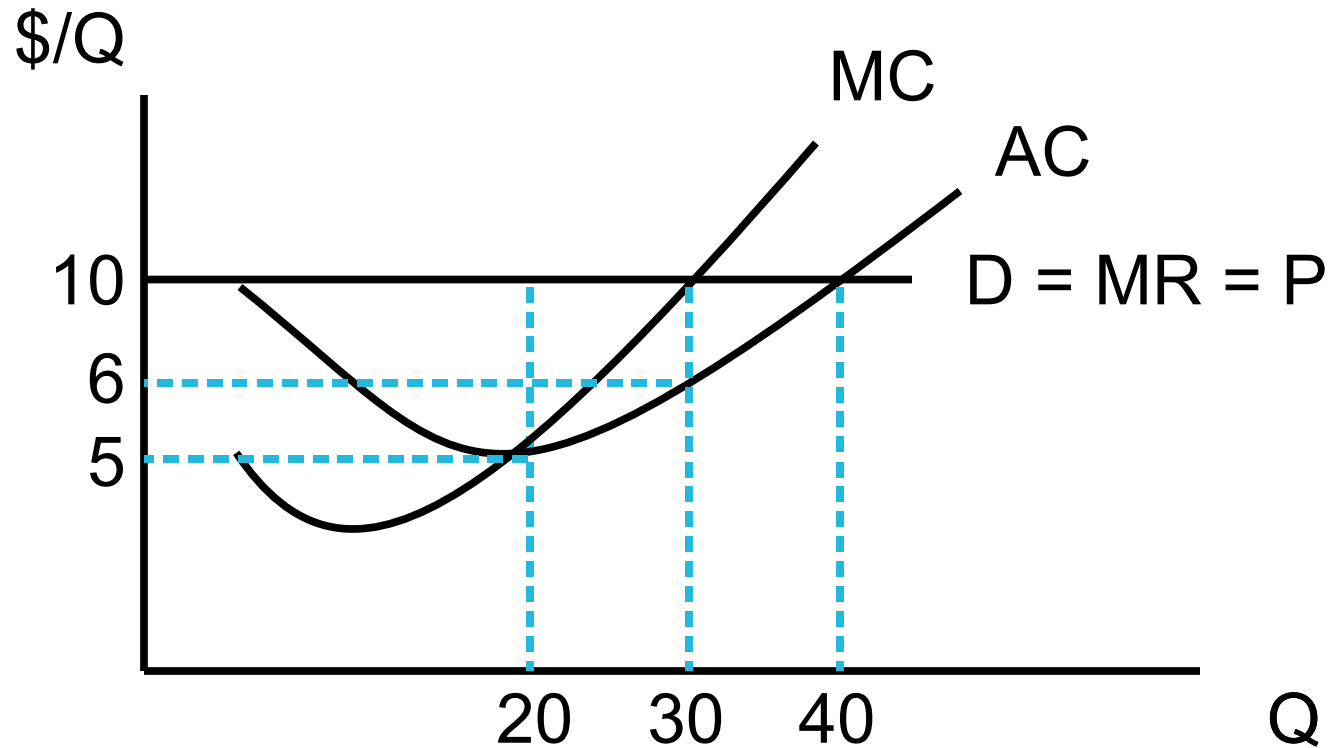
D $Q = APL * L = 55 * .7 = 40$

Costs

Sample Solution

Q	L	APL	MPL
10	15	.7	.7
20	20	1.0	2.0
30	35	.9	.7
40	55	.7	.5

Costs/Sample Problem



Costs/Sample Problem

1. What production level exists at the point of maximum profit?
2. What is the average cost at an output of 30?
3. What is the profit at an output of 20?
4. What is the marginal cost at an output of 30?
5. What is the profit margin at $Q=30$? ROS?

Costs/Sample Problem

1. What production level exists at the point of maximum profit? $Q = 30$
2. What is the average cost at an output of 30? $AC = 6$
3. What is the profit at an output of 20? $(10-5)*20 = 100$
4. What is the marginal cost at an output of 30? $MC=30$
5. What is the profit margin at $Q=30$? $(10-6)=4$
ROS? $4/10=40\%$

Operating Decisions

Operating Decisions

- Revenue and costs determine profitability.
- Operating decisions are those short term changes such as in pricing, output, and input that affect profitability.
- Decisions that focus on long term issues such as type of product, capacity, etc. are strategic decisions.

Profit

- Profit = Revenue – Costs
- Profit = TR – TC
- Net Profit Margin% (NPM) = Profit / TR

Short-Run Operating Decisions

- If $P < AVC$, then firm cannot pay for inputs that create its product. Consequently, it may have to immediately shutdown.

Java's Profit

- Java Cafe has total revenue of \$18,000 minus total costs of \$16,200, earning a profit of \$1,800. The net profit margin = 10% ($\$1,800/\$18,000$).

Profit

Concept of a Normal Operating Profit

- Net income often is referred to as operating profit
- Firms need to earn a sufficient operating profit to compensate its owners who provide the capital. The level of profits necessary to attract capital will be considered a normal operating profit.

Java Profitability

- Java Cafe competes in an industry that earns an average normal operating profit of 10%. Unless Java Cafe earns 10%, the owners would do better to sell the firm and invest the proceeds in something that earns 10%.

Short-Run Operating Decisions

- Some fixed costs are of a non-cash nature such as depreciation.
- Payment on other fixed costs can be delayed for a short-time such as bank loans through extensions.

SR Operating Decisions

- Because demand varies, profitability varies.
- Consequently, losses can occur.
- Must determine point at which shutdown of firm is necessary.
- If $P > AVC$, then a portion of revenue is covering fixed costs. Can continue operations for a period of time.

SR Operating Decisions

- The key consideration is how long demand will be depressed.
- Past history may be useful.
- Amount of cash reserves is crucial.

Java SR Operating Decision

- Every year during the 3 months of summer, demand drops to 4,000 cups per month. Later during the 3 months of winter, demand increases to 8,000 cups per month. Spring and fall maintain the average of 6,000 cups/month.

Java SR Operating Decision

- While price does not change, the contribution of revenue to cover fixed costs does change.
- $4,000 * (3 - 1.5) = \$6,000$ which means Java Cafe incurs a \$1,200 loss each month during the summer.
- The owner, wisely, does not shutdown. Instead, the salary to the owner/manager is reduced.

Costs & Pricing

Relationship between Costs & Pricing

- Ultimately, pricing has to recover all costs plus a normal operating profit.
- As costs rise, eventually prices must rise.
- Usually costs rise for all firms, not just one.

Graphical Analysis of SR Cost Behavior

Costs

Detailed Calculations

Q	TFC	TVC	TC	MC	AC
10	100	150	250	15	25
20	100	200	300	5	15
30	100	350	450	15	15
40	100	550	650	20	16.25

Costs

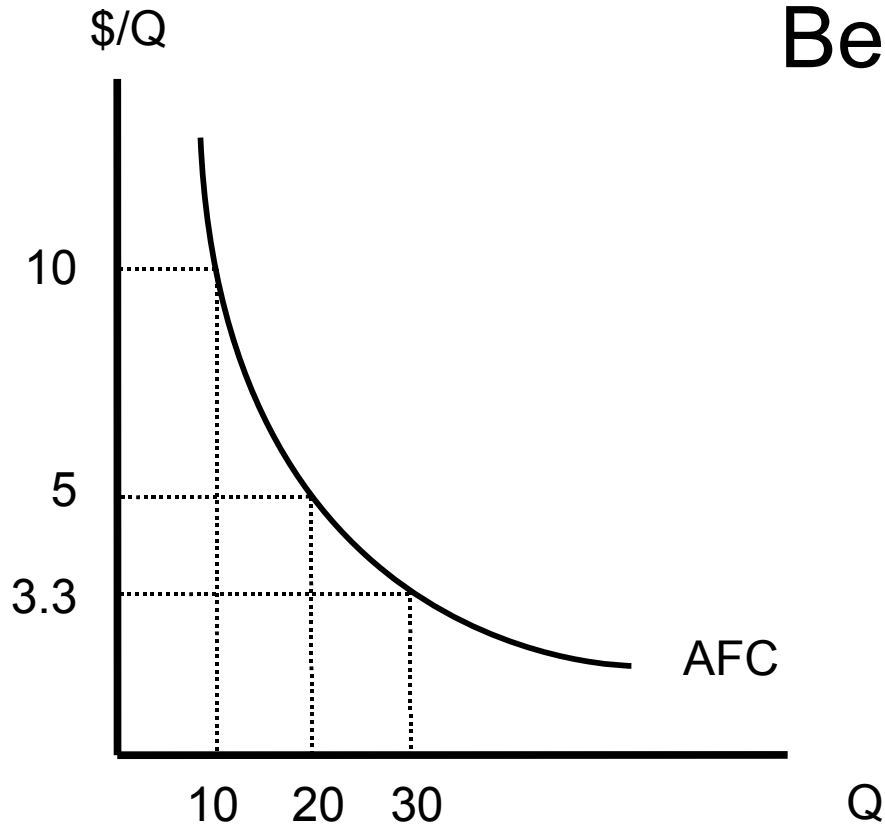
Notes to Detailed Calculations

* Total Variable Costs are calculated by multiplying $Q * AVC$, AVC could be calculated by TVC/Q .

*

Costs

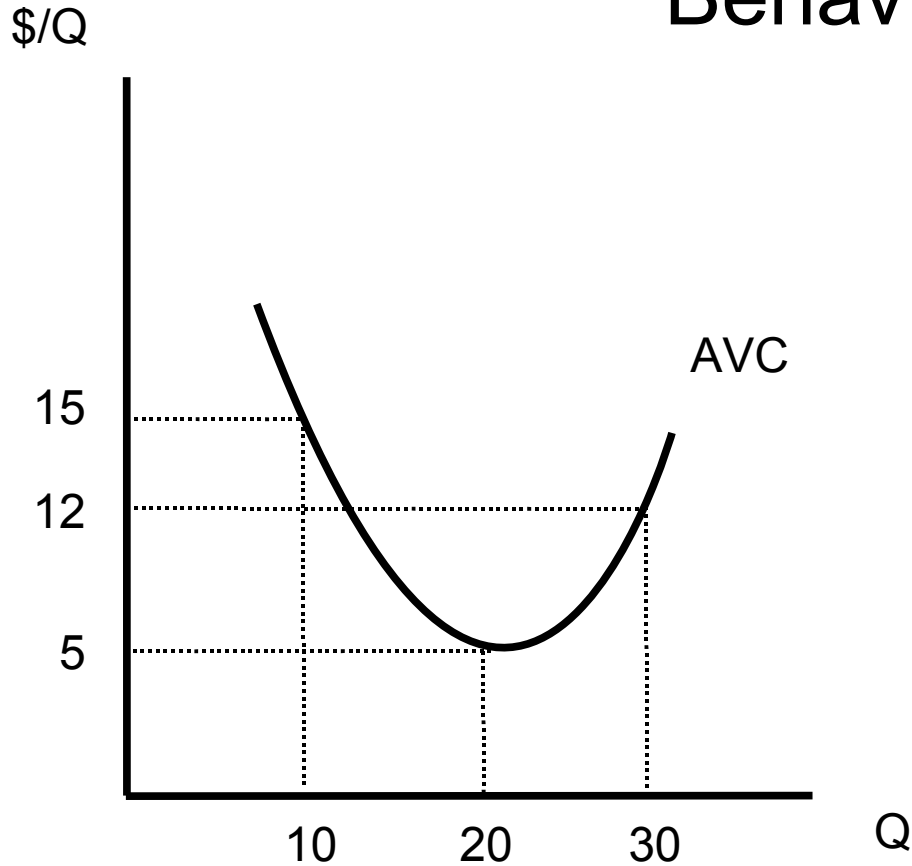
Behavior of Fixed Costs



Q	TFC	AFC
10	100	10
20	100	5
30	100	3.3

Costs

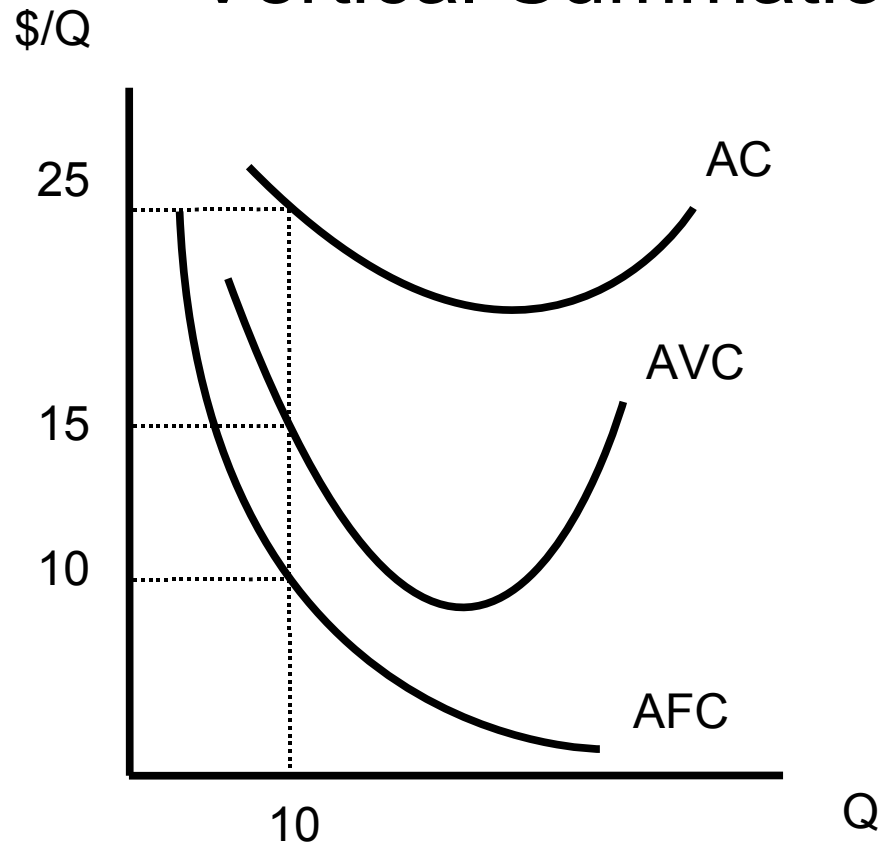
Behavior of Variable Costs



Q	TVC	AVC
10	150	15
20	200	10
30	350	11.7
40	550	13.8

Costs

Vertical Summation of Cost Curves

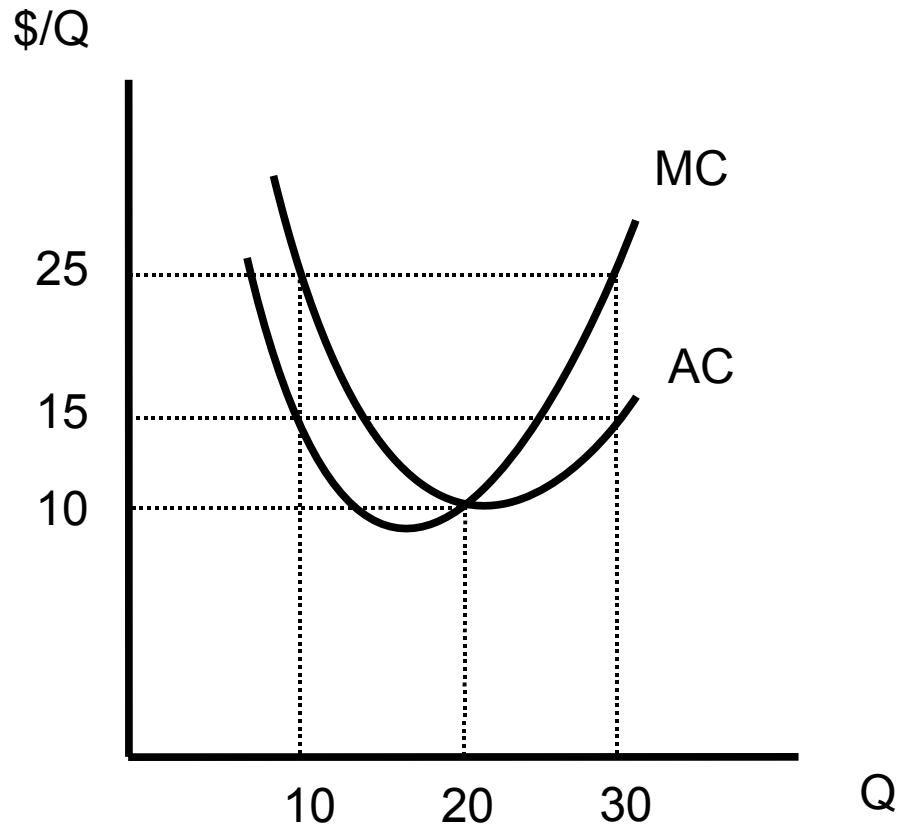


$$AC = AVC + AFC$$

$$25 = 15 + 10$$

Costs

Relationship of MC & AC Curves

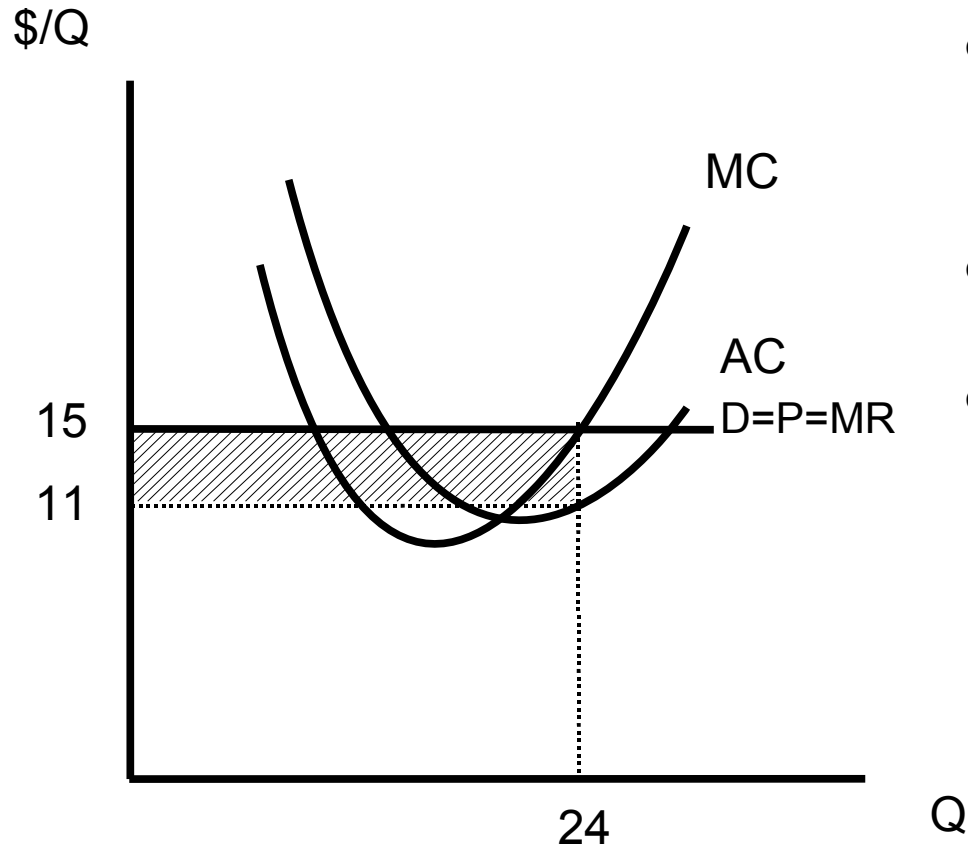


Q	MC	AC
10	15	25
20	5	15
30	15	15
40	20	16.25

Costs

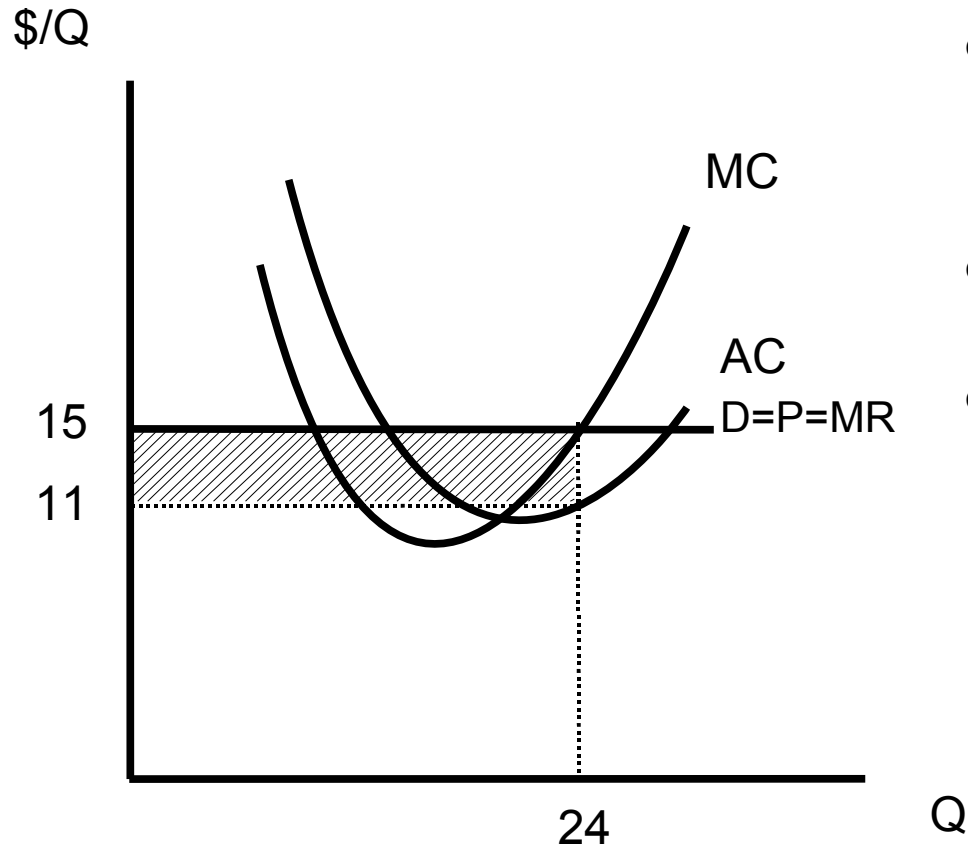
- When $MC > AC$, then AC becomes upward sloping.

Costs



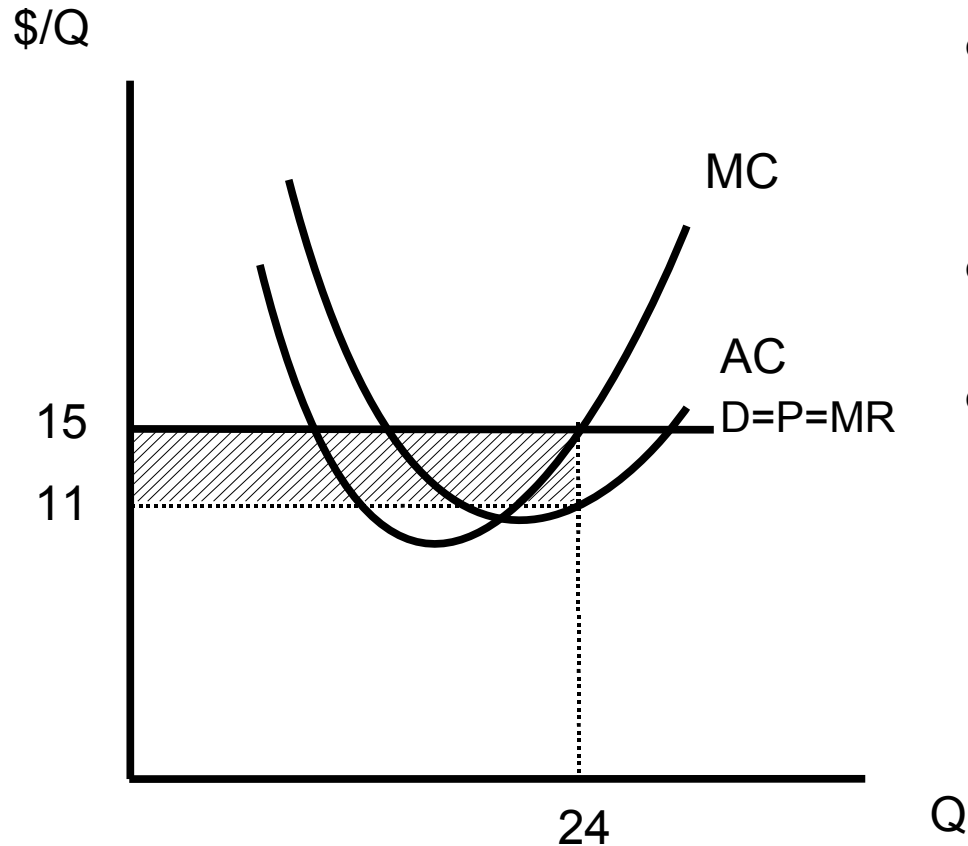
- Calculation of Total Revenue
- $P * Q = TR$
- $\$15 * 24 = \360

Costs



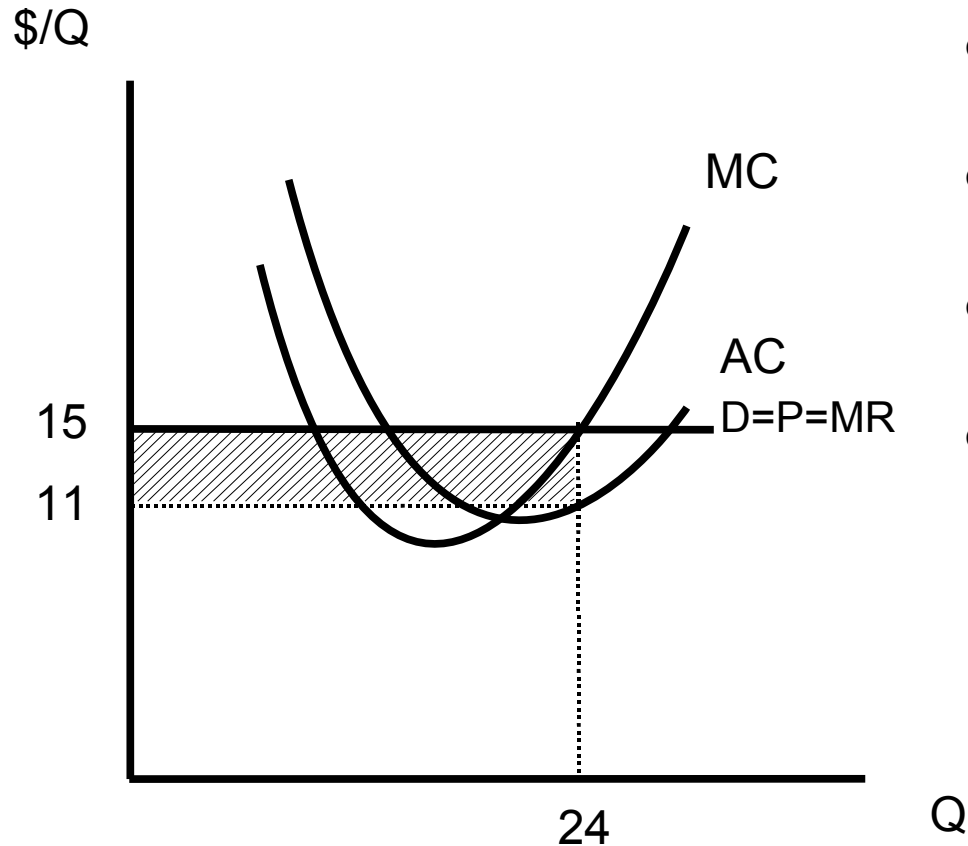
- Calculation of Total Cost
- $AC * Q = TC$
- $\$11 * 24 = \264

Costs



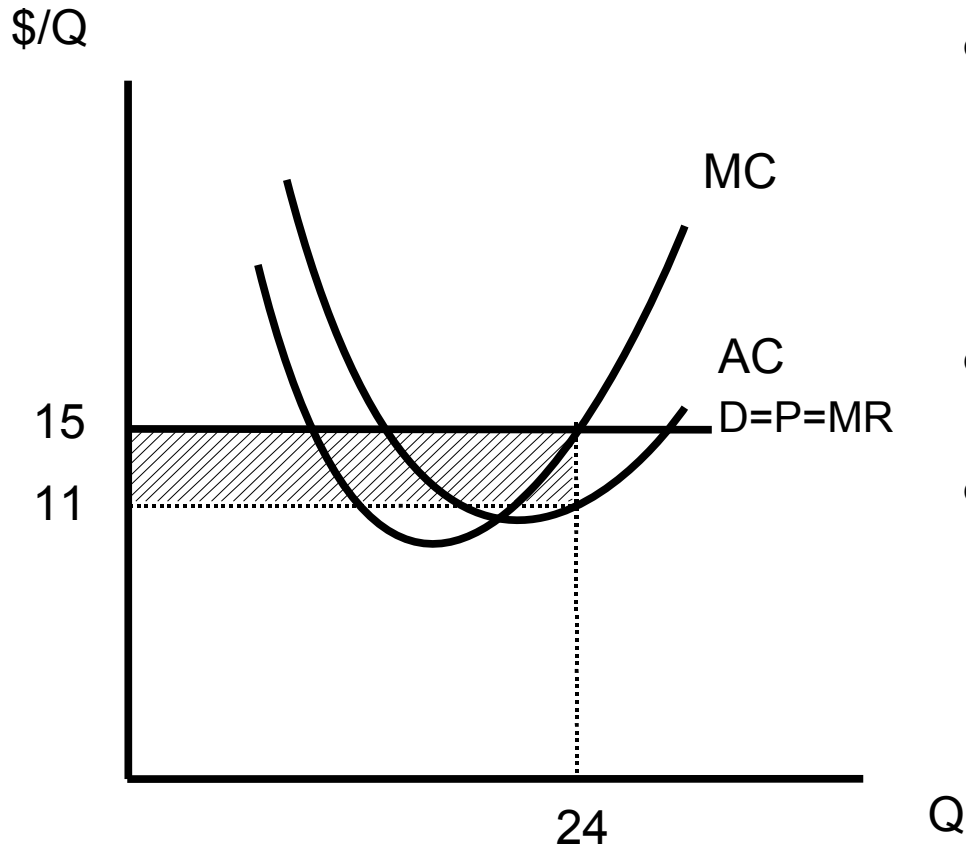
- Calculation of Profit Margin (PM)
- $P - AC = PM$
- $\$15 - 11 = \4

Costs



- Calculation of Profit
- $(P-AC)*Q = \text{Profit}$
- $(15-11)*24 = \$96$
- Note: profit is max where $MC = MR$

Costs



- Calculation of Profit (Alternative Calculation)
- $TR - TC = \text{Profit}$
- $\$360 - 264 = \96

Long Run Costs

Long Run Costs

- In the long run all costs are variable.
- Capacity is variable.
- The problem becomes to pick the optimal scale of plant.
- Note that in the short run, the problem is to optimally use the existing plant.

Long Run Costs

- Long run average costs can increase, decrease or remain constant as capacity increases.

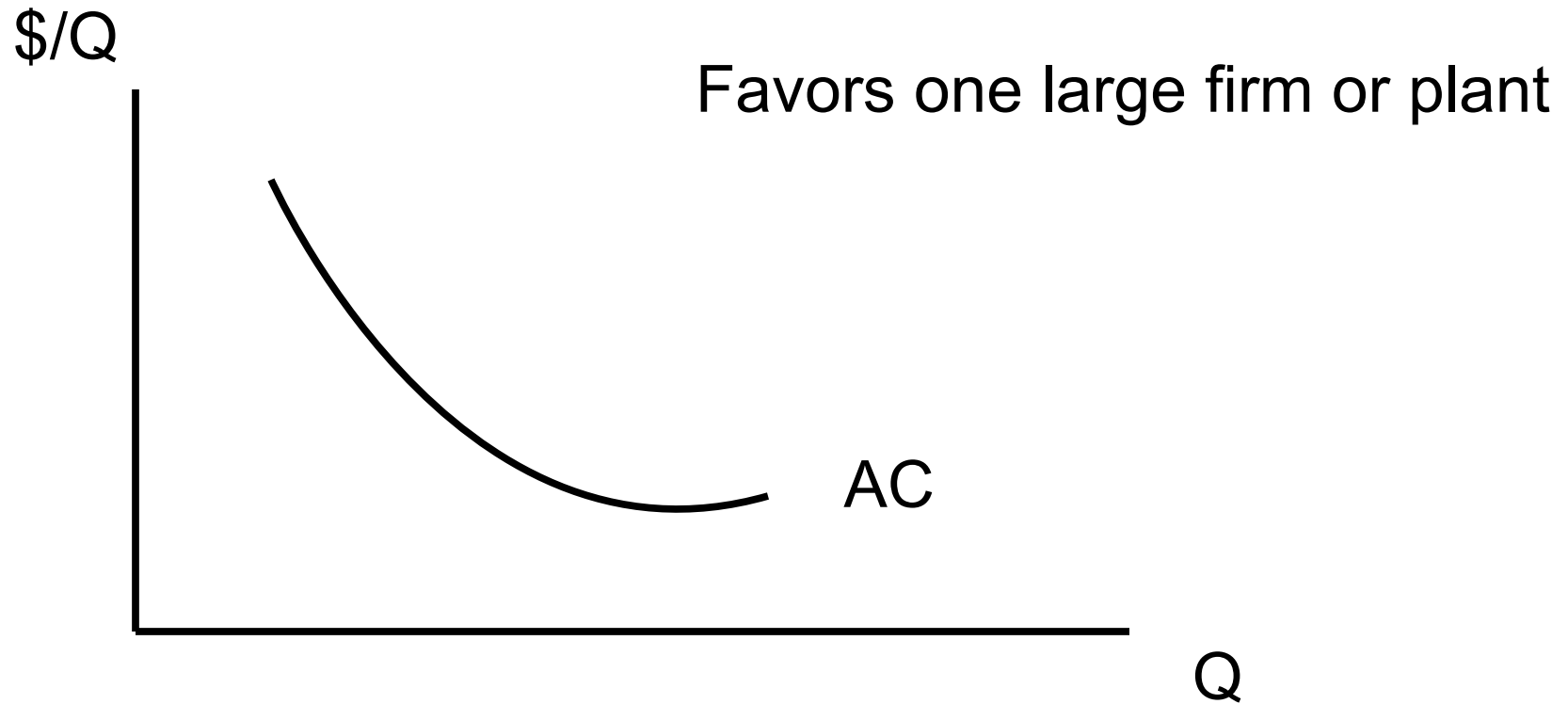
Long Run Costs

- Each point on a long run average cost represents an individual plant size complete with its own series of short run cost curves.
- A long run cost curve is also called a planning curve, or envelope curve since it envelopes SR cost curves.

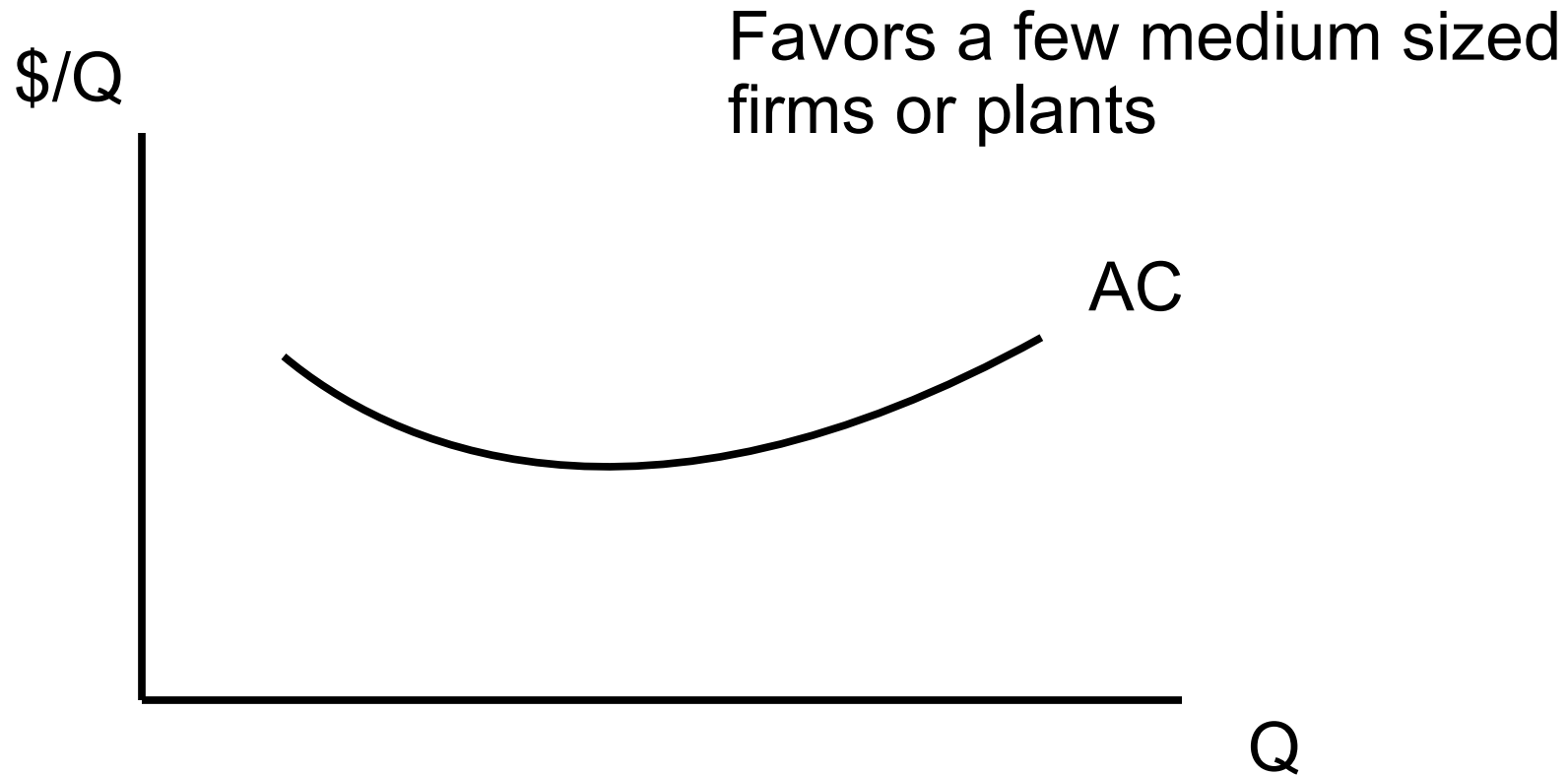
Long Run Costs

- Long run costs have a significant influence on industry or market structure.

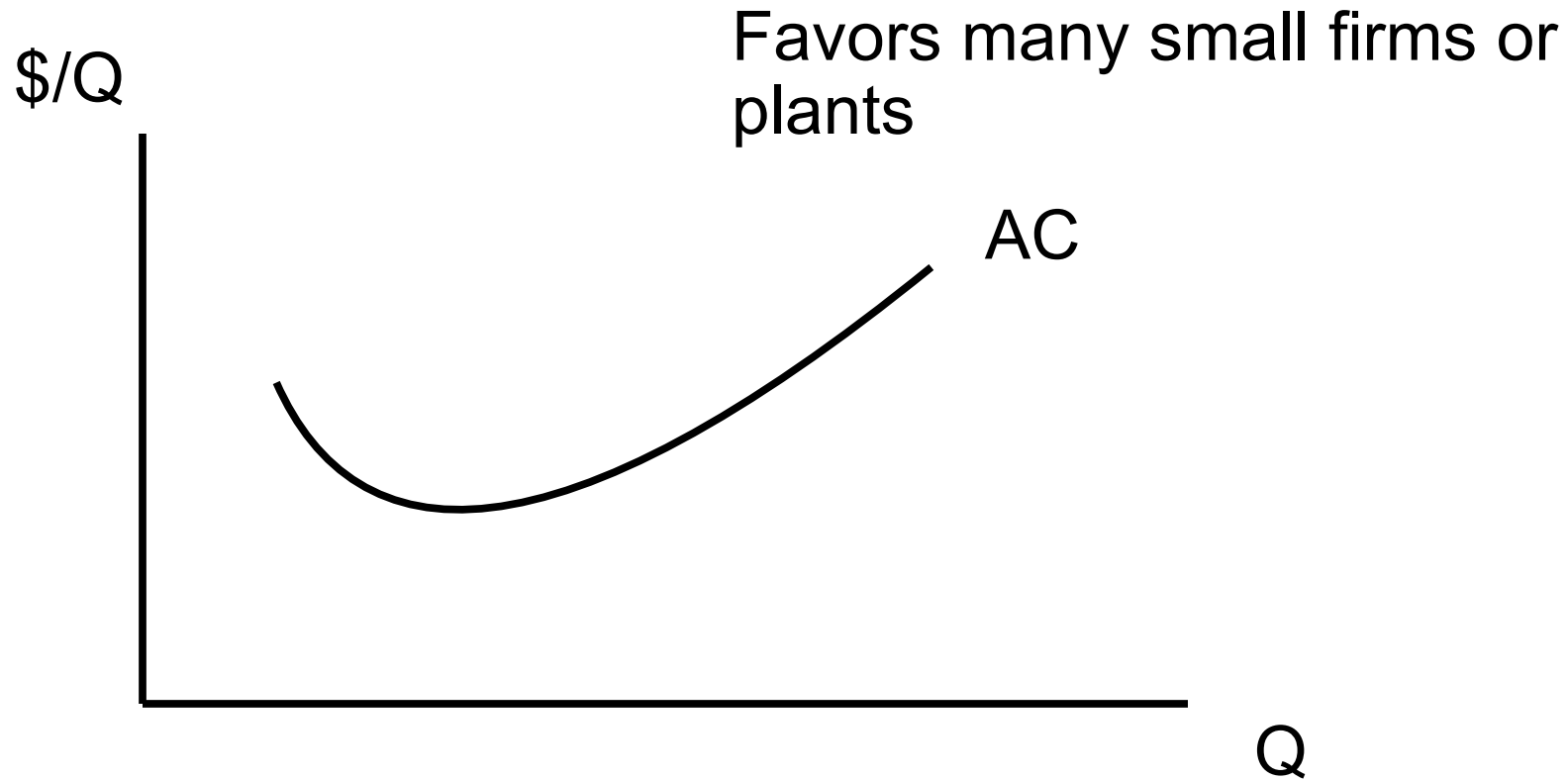
Long Run Costs



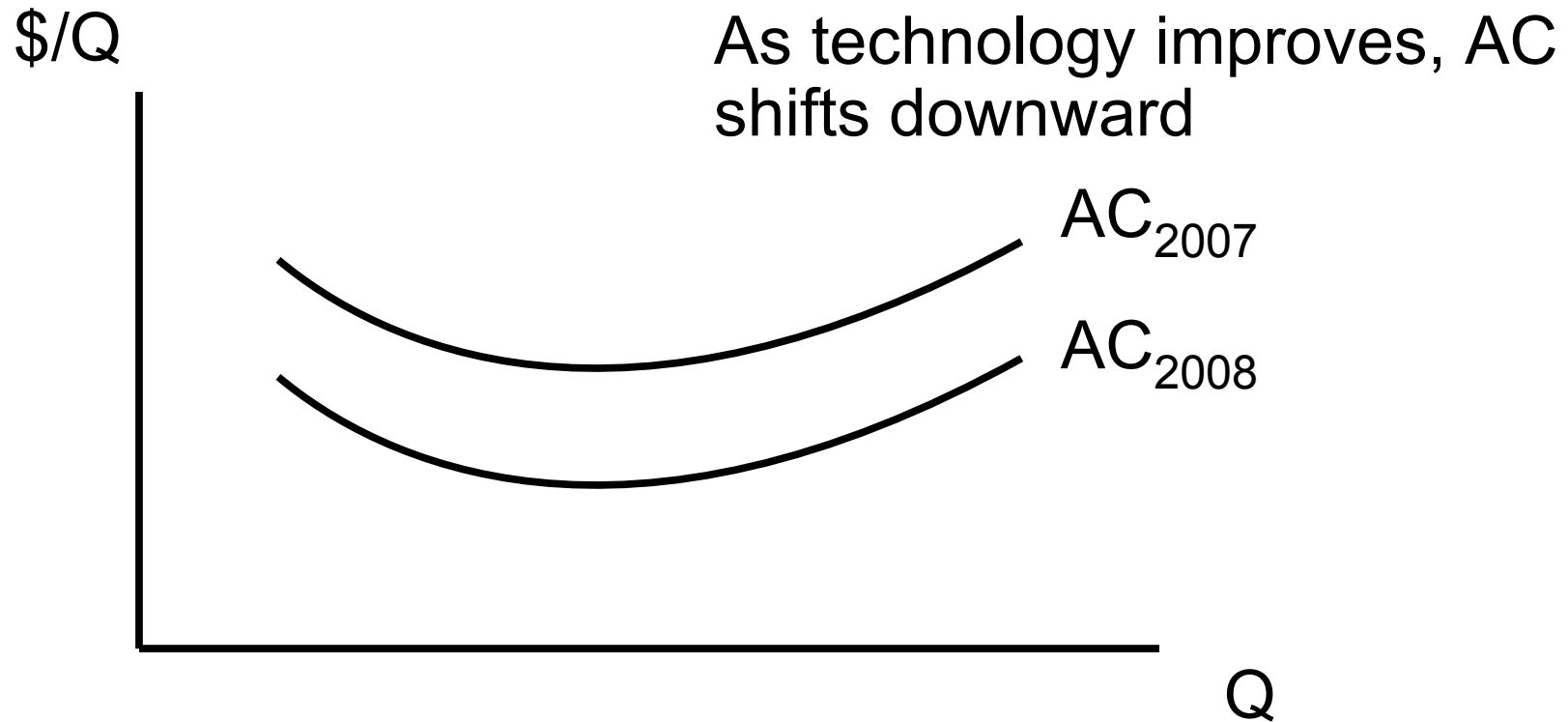
Long Run Costs



Long Run Costs



Long Run Costs



Long Run Costs

Input Costs and Markets

- Long run costs apply to input costs as well.
- In the input market, if LR AC rises as output rises, then for the firm as its output rises, it will have to increase price.

Long Run Costs

Input Costs and Markets

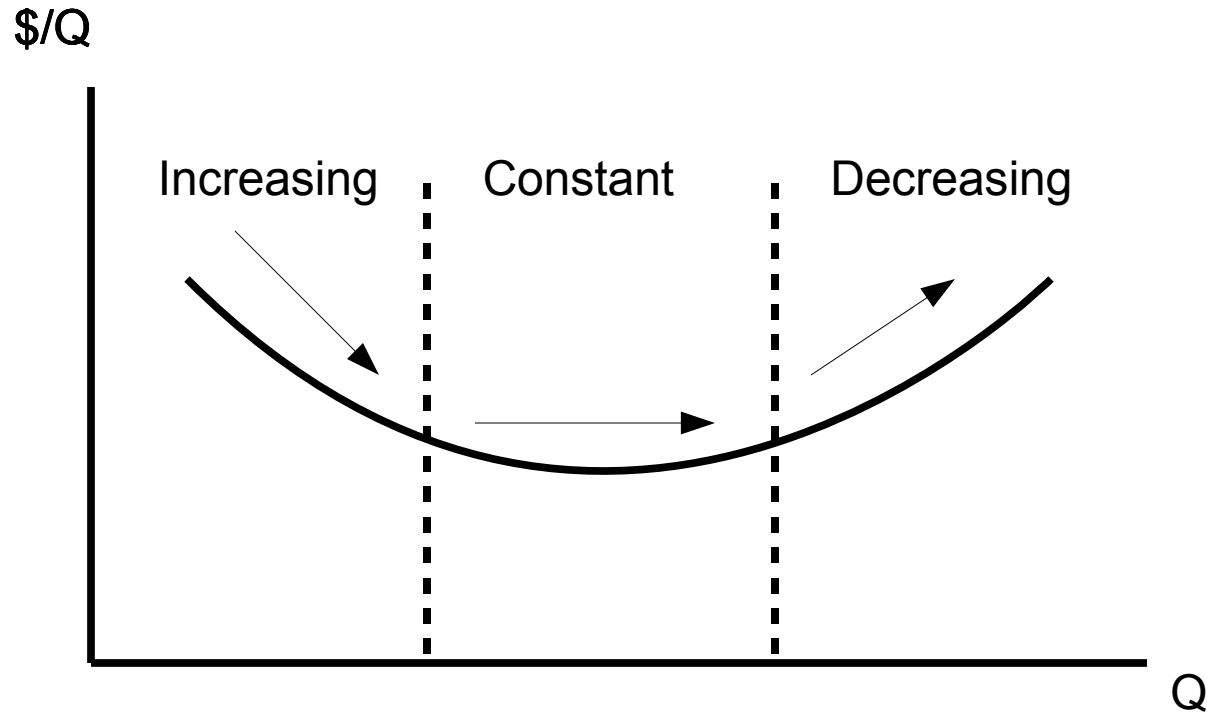
- In the input market, if LR AC rises as output rises, then for the firm as its output rises, it will have to increase price.

Long Run Costs

Input Costs and Markets

- Example: if lumber prices rise as their output rises in response to demand for lumber via demand for homes rise, then costs for homes will rise necessitating a rise in home prices.

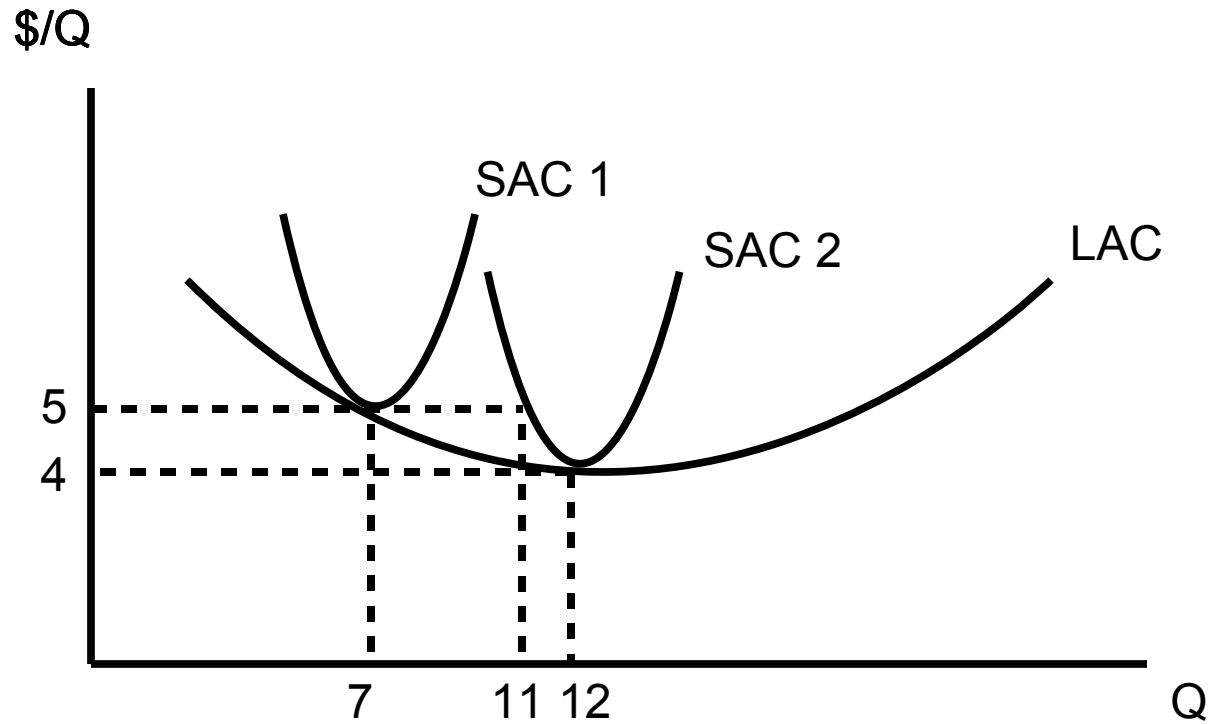
Long Run Cost Curve



Long Run Cost Curve

- Increasing economies of scale occur as output / capacity increases, the average cost, or cost per unit, decreases.
- Constant economies of scale occur as output / capacity increases, the average cost, or cost per unit, remains constant.
- Decreasing economies of scale occur as output / capacity increases, the average cost, or cost per unit, increases.

Long Run Cost Curve



Long Run Cost Curve

Notes to graph

- Plant 1 minimizes its average cost at an output level of 7 million
- Plant 2 minimizes its average cost at an output level of 12 million
- Plant 2 can produce widgets at a lower average cost than Plant 1

Long Run Cost Curve

Notes to graph

- If market size is close to 7 million then Plant 1 may be a better choice, as the average costs of Plant 2 will be higher than that of Plant 1
- If market size is above 7 million than Plant 2 may be a better choice

The End