

2.0 PRODUCTION THEORY

In this section we will examine more closely what determines supply. We will attempt to show the relationship between the physical processes of production, and the cost structure. Then based on a simple competitive market structure, we will show how costs and price interact to determine supply. Along the way we will pick up one of the most used analytical tools in business; Break-Even Point (a.k.a. Cost-Volume-Profit) Analysis.

2.1 Production Function

The role of producer is to organize production choosing a combination of technology and quantity of inputs from a universe of possible combinations of technology and inputs. This is described by the production function which is a mathematical expression relating quantities of combinations of inputs to a given level of output.

$$Q = f(K, L, A)$$

where K is capital, L is labor, and A is technology.

In the short run, labor is variable and capital is considered fixed. In the long run, capital and labor are considered variable. Technology is considered a constant (at least in this course).

Each input has a cost: labor is paid a wage, and capital is paid interest. To the extent labor is a variable input (sometimes it isn't so variable) the expense associated with labor is variable. On the other side, since capital is a fixed input, its associated costs are considered fixed.

The relationship between labor and capital may be a simple one such as two units of labor and one unit of capital such as $Q = 2L + K$. In that case, labor is independent of capital. A much more likely case is the one where the amount of labor affects capital and vice versa. An example would be a multiplicative function such as $Q = LK$. In this case, say $L = 4$ and $K = 2$ then output is 8 but increase K to 3 and output becomes 12. If we instead, increase L to 5, now output increases to 10 (5×2). But if we increase labor by one after we have increased capital by one, the increase in output is now 15 (5×3). This shows how productivity of labor is influenced by the amount of capital. The change in output due to change in one of the inputs is called the marginal product. Here we have two possible marginal products; the marginal product of labor (MPL) and the marginal product of capital (MPK). $MPL = \text{chg } Q / \text{chg } L$. The marginal product of labor (MPL), calculated at where L increased from 4 to 5 holding capital constant at 2 is $(10-8)/(5-4) = 2$. This means that adding one unit of labor increased output by 2. The marginal product of capital is 4 $(12-9)/(3-2)$. Adding one unit of capital increased output by 4.

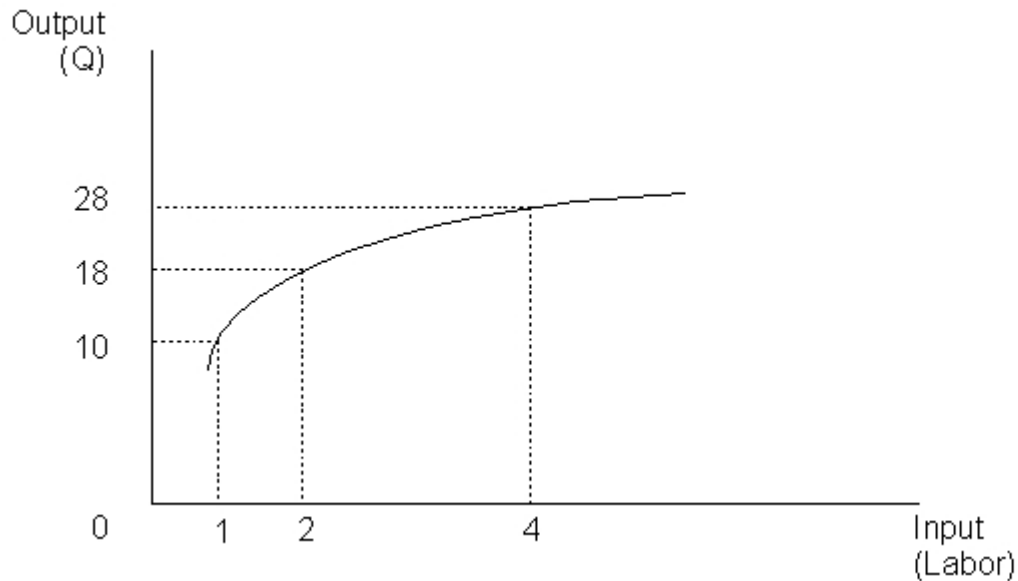
We can also calculate average products. $APL = Q/L$. Let's say that $Q=8$, $L=4$, and $K=2$. When the output is 8 and labor input is 4 then the average product of labor is 2 ($8/4$). The average product of capital is 4 ($8/2$). $APK = Q/K$.

The Relationships among Total, Average, and Marginal Product

In the following graphs, the relationships among total, average, and marginal products are illustrated. The shape of the curves can vary considerably depending on the structure of the production function. Each production process is unique, some are labor intensive, other are capital intensive, and most are a combination. Let's plot the following data for a pizzeria, assuming one oven and output is in pizza's per hour..

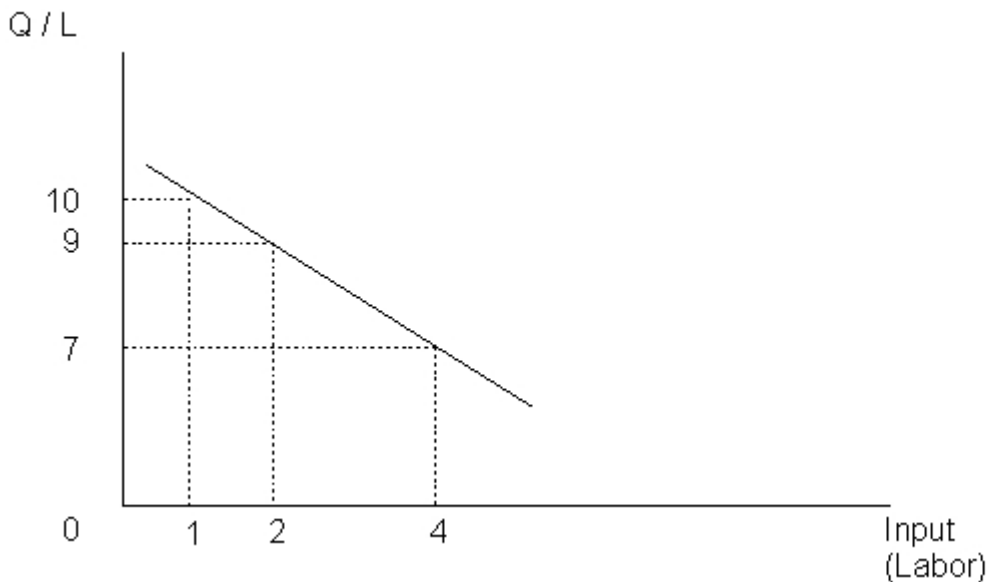
L	Q	AP	MP
0	0	-	-
1	10	10	10
2	18	9	8

3	24	8	6
4	28	7	4

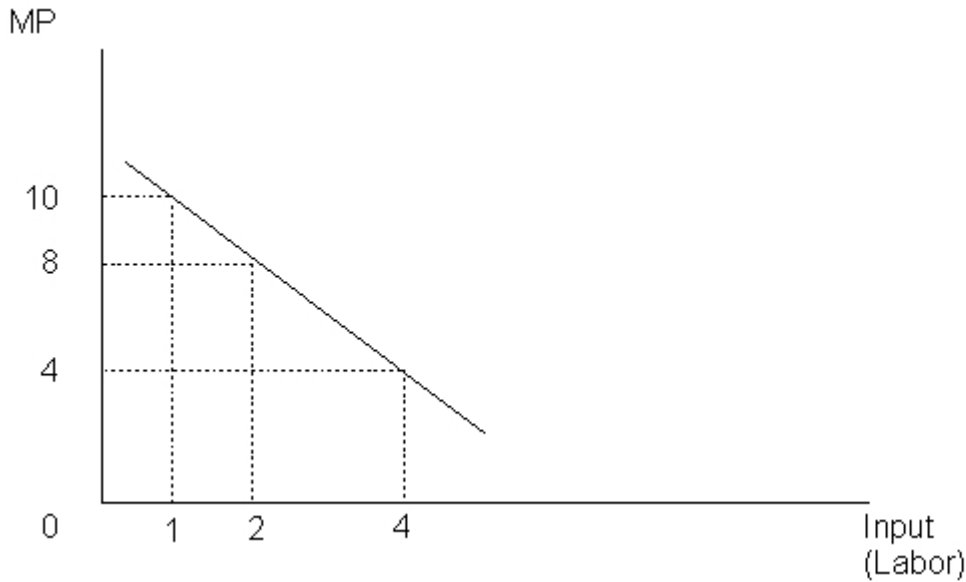


In the graph above, as labor increases, output increases but at a decreasing rate. The Marginal Product of Labor (MPL) is decreasing. The Law of Diminishing Returns applies to the relationship of input to output. It is defined as the declining addition to output from each successive unit of variable input.

In our pizzeria example, adding chefs increases output but not proportionally. With the same counter space and one oven, adding chefs increases the problem of coordination as they fight for dough and oven.



In the graph above, average product (AP) is plotted against labor. At $L=4$, $AP = 7$. Note that as input increases, AP decreases. AP decreases due to a declining MP.



Marginal product is DQ/DL or DQ/DK . Note that marginal product is declining faster than average product. Changes in MP lead changes in AP.

Isoquants

By using two generic inputs such as labor and capital, we can illustrate how they can be combined to produce a given output level. The curve generated by this combination is called an isoquant. An isoquant takes a convex shape, similar to that of an indifference curve. The slope of the isoquant indicates the Marginal Rate of Technical Substitution (MRTS) of capital for labor (or labor for capital if you prefer). As one moves along the curve, the MRTS decreases, indicating that increasing quantities of labor (capital) are needed to substitute for capital (labor) in order to keep output constant. If labor and capital are perfect substitutes, then the line will be straight. If they are perfect complements, the isoquant will be a L-shaped line.

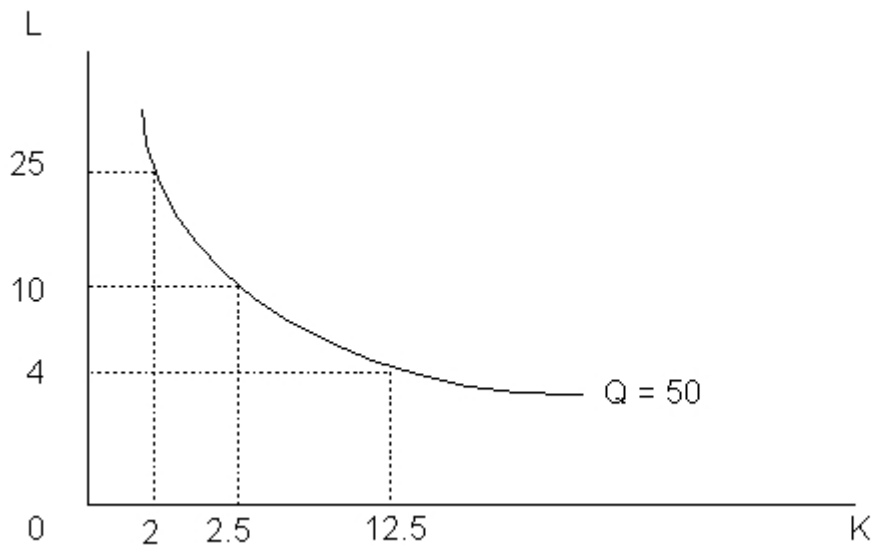
Isoquant Example

Combination	Labor	Capital	Output
A	2	25	50
B	5	10	50
C	4	12.5	50
D	2	30	60
E	4	15	60

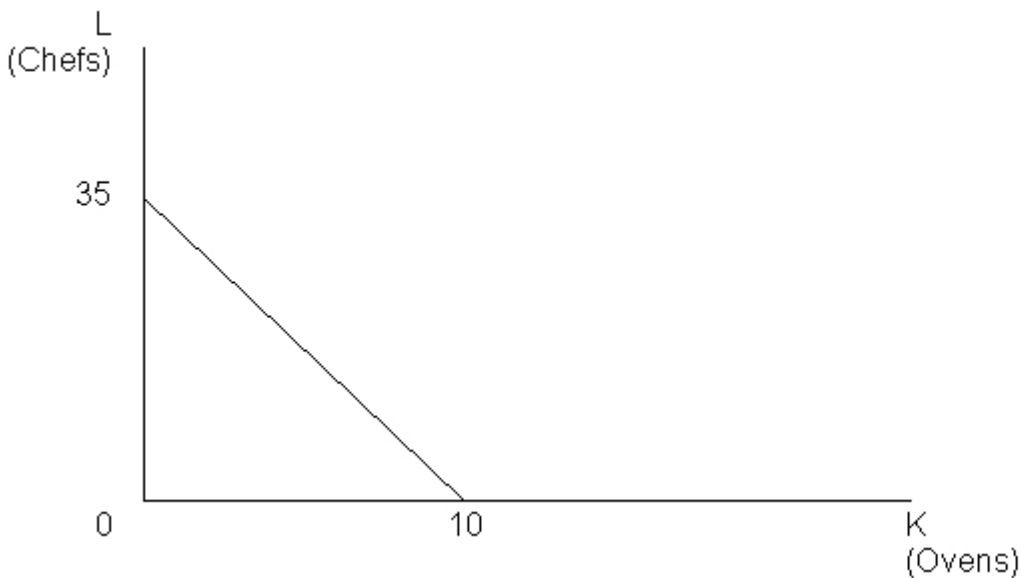
At an output level of 50 pizzas, several combinations of capital and labor are shown. At A $L=25$ and $K=2$ while at B $L = 10$ and $K = 5$. In this example, capital is easily substitutable for labor.

Isoquant Example

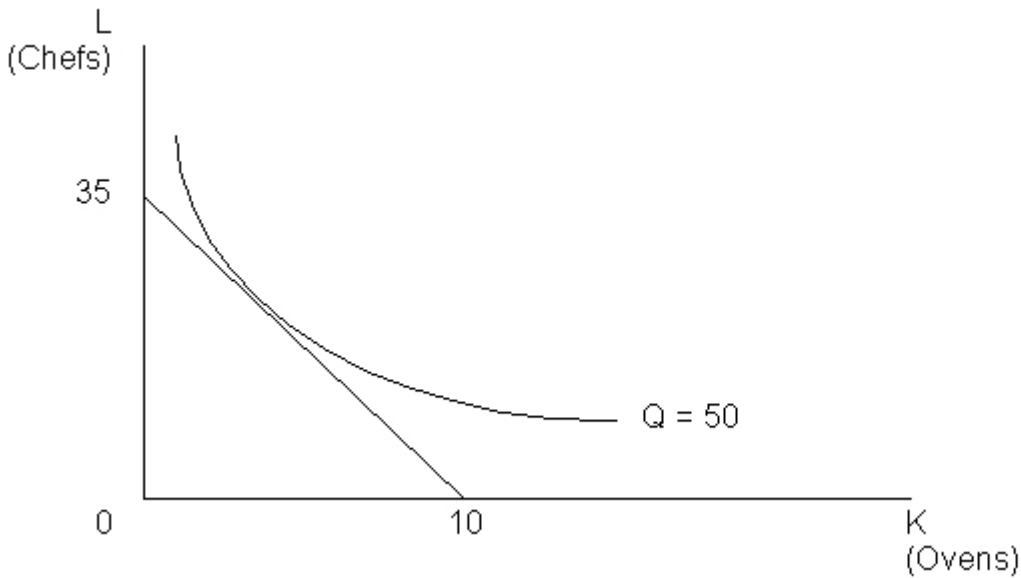
In the following example we will use a pizzeria to illustrate an isoquant. Then we will use the concept of an isocost to derive the least cost combination to attain the maximum profit for a given output level.



From the above graph, to produce 50 pizzas per day, the firm can choose combination A: 2 ovens and 25 chefs, or combination B: 5 ovens and 10 chefs. Note that these isoquants are not straight lines. Straight lines can only exist if ovens are perfect substitutes for chefs. This cannot be as ovens require chefs to load and unload let alone mixing and making the pies. Adding more ovens will increase output only to the extent chefs have time to load pizzas into the oven. If time is the only constraint, then if each chef can make and load a maximum of 100 pizzas per day then regardless of the number of ovens the pizzeria purchases, output is limited to 800 pizzas for 8 chefs or 400 for 4 chefs.



To find the minimum cost combination, we need an isocost line. This is similar to the budget line we used in consumer theory. If each chef costs \$1,000 and each oven costs \$5,000, then given a cost of \$35,000 a maximum of 35 chefs or 10 ovens can be obtained. The technique is to find the isocost curve just tangent to the isoquant. Again, just like the technique we used in consumer theory.



In this example, cost is minimized where ____ chefs are hired and ____ ovens are purchased at quantity of 50. This technique only minimizes cost given an output level. It does not determine the profit maximizing output level.

A Dismal Example of Production

In the example below, let's divide the world into six time periods. Given that we have a fixed supply of land (pre-space age) and a geometrically increasing population, our labor supply, we have output increasing arithmetically.

Time Period	1	2	3	4	5	6
Inputs:						
Land	1	1	1	1	1	1
Labor	1	2	4	8	16	32
Output:						
Total Product	1	2	3	4	5	6
Ave Product	1	1	.75	.5	.3125	.1875
Marginal Product	1	.5	.25	.125	.0625	.0312

From this picture you can understand why some have called economics the dismal science. According to this analysis we are all doomed. While output increases, it doesn't increase fast enough and each of us has less to eat. The average product is declining due to the marginal product being less than 1. The average product and marginal product are measures of productivity. To increase the standard of living, productivity must increase. In reality MP hasn't fallen such as this. The assumption of a constant technology and geometrically increasing populations hasn't held. Technology has improved vastly allowing us to feed many more with much less farm land. As more countries enter middle classdom, population growth has leveled off to replacement or zero growth. In 2000, as I write these notes, a plague known as AIDS, is starting to have a significant impact on population growth in some areas of the planet.

2.2 Production Costs

Economic costs: Explicit vs Implicit

An economic cost is the value of the alternatives that society must forgo in order to obtain those resources. In other words, its opportunity cost. Economic costs are broken down into private and external costs. Private costs are those costs borne by individuals directly involved in the production or consumption of an item. Private costs are broken down into explicit and implicit costs. Explicit costs are the market value of resources purchased by the producer. Implicit costs are the market value of self-owned, self-employed resources. For example, self constructed assets, value of time put into the business by the owner, and the cost of capital (Return On Sales, Return On Equity, or Return on Assets). Implicit costs are often not recognized in the financial statements.

External costs are those costs borne by individuals who are not directly involved in the production or consumption of an item. Often known as externalities, they could be costs imposed on the public by pollution generated by the plant.

Example

Let's illustrate these costs with an example. Let say that you are an attorney that has decided to open up a fruit stand along the side of a road. You use \$100,000 of your money to buy some refrigeration equipment and a stand from your uncle. Those assets would normally retail for \$200,000. Your accountant expects the useful service life to be 10 years. At the end of the year your accountant prepares an income statement and suggests that you have done a good job. Your consultant however, has compiled financial data that suggest that this was an awful business to be in. Who is right?

	Accountant	Consultant	Difference
Revenue	100,000	100,000	0
Cost of Goods Sold	50,000	50,000	0
Gross Margin	50,000	50,000	0
Marketing	1,000	1,000	0
Administrative	5,000	5,000	0
Salaries	5,000	45,000	40,000
Depreciation	10,000	20,000	10,000
Cost of Capital	0	12,000	12,000
Net Income (Loss)	\$29,000	\$(33,000)	\$62,000

The difference in salaries is due to the uncompensated owner's time put into the business. This is found by multiplying the number of hours worked times a normal wage rate the owner could have worked in another business that she is best qualified to do. When we add implicit costs such as these we say we are imputing costs to the firm.

A second difference arises over the depreciation of the fruit stand. Your accountant has calculated depreciation on a straight line basis over 10 years given the historical cost of \$100,000. Your consultant argues that you need to depreciate \$200,000 over 10 years as that reflects your true economic cost of using those assets.

Finally the \$100,000 of your money that you invested, the consultant argues, could have been invested into a stock fund earning at least \$12,000 per year. This is a cost of capital that should be recognized.

In other words, if you had sold the equipment your uncle sold you (and financed it over 10 years), worked for your former law firm, and invested your money, you would have been \$62,000 richer. Maybe the rewards of self-employment are greater than \$62,000. :)

Costs

In analyzing the profitability of firms, one would like to know the profit maximizing output level, the

profits generated at that level, and how profit changes when output varies. To do that requires a knowledge of cost behavior and structure. In our pizzeria example, the combination of ovens, chefs, and servers determines the cost structure, the relative ratios of the fixed to variable costs. Cost behavior is how costs change in regard to changes in quantity or volume. Economists make an important distinction at this point, long run and short run costs. In the short run, capacity is fixed. That means the amount of equipment and building is constant, they do not change. Consequently, the costs associated with the equipment and building are constant, that is, fixed.

Short run Production costs

Fixed costs are defined as those costs that do not vary with output. Basically, any cost that would exist at zero production. Variable costs are costs that vary with production. Total Costs (TC) are defined as the sum of Fixed Costs (FC) + Variable Costs (VC). $TC = FC + VC$. Examples of fixed costs would be depreciation, interest costs, marketing, taxes, general, and administrative costs. Variable costs are usually direct labor and materials used in producing the product or delivering the service. Some costs such as maintenance may have fixed and variable components. Fixed costs are not necessarily constant. Interest costs vary with changes in the level of interest rates. But this variation has nothing to do with production of the firm.

Per unit costs are known as average costs (AC). $AC = TC / Q$. Average costs also equal the sum of average fixed cost (AFC) and average variable costs (AVC). $AC = AFC + AVC$.

Marginal costs are defined as the change in total cost due to a change in production. $Marginal\ costs = \text{chg } TC / \text{chg } Q$. Marginal costs are not reported in the financial statements but are important as they help the firm determine the optimum production level. As production increases, marginal costs usually go up. When marginal costs exceed the marginal revenue gained from additional production then that is the point to stop additional production.

To help illustrate our discussion of production costs, consider the following data from our favorite pizzeria.

Example

Q	FC	AFC	VC	AVC	TC	AC	MC
10	200	20.0	80	8	280	28.0	
20	200	10.0	140	7	340	17.0	6.0
30	200	6.7	180	6	380	12.7	4.0
40	200	5.0	200	5	400	10.0	2.0
50	200	4.0	275	5.5	475	9.5	7.5
60	200	3.3	400	6.7	600	10.0	12.5

Let's examine the calculations where $Q = 30$.

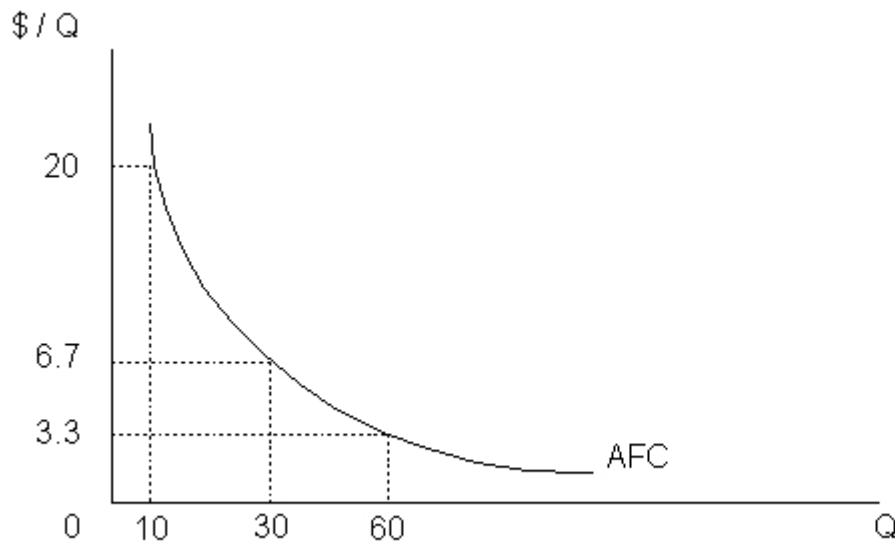
$$AFC = FC / Q; 200 / 30 = 6.7.$$

$$AVC = VC / Q; 180 / 30 = 6.$$

$$TC = FC + VC; 200 + 180 = 380.$$

$$AC = TC / Q; 380 / 30 = 12.7 \text{ or } AC = AFC + AVC; 6.7 + 6 = 12.7.$$

Fixed Costs

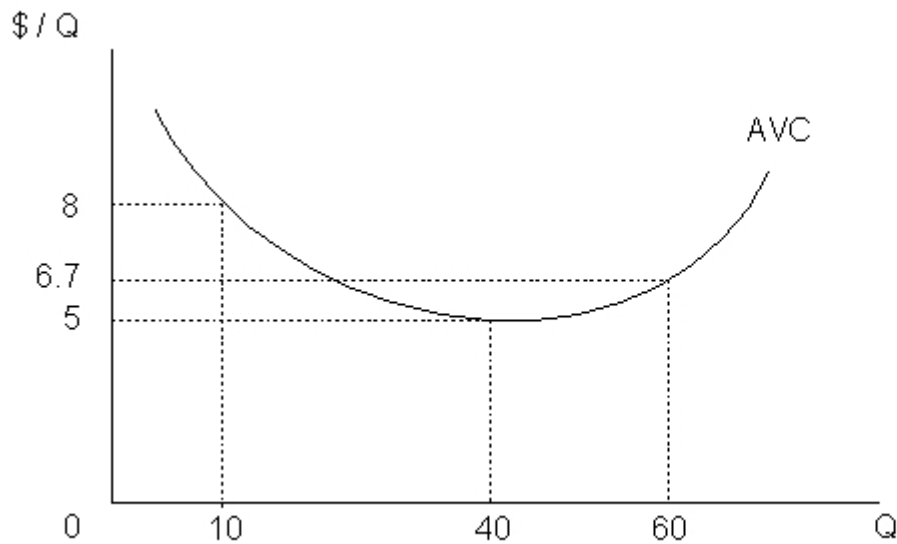


Let's examine the behavior of fixed cost. As output expands the cost remains constant as shown in the graph above. On a per unit basis, fixed cost decreases. When costs on a per unit basis decrease as output increases, we say there are economies of scale. In the example cited, fixed costs of \$200 spread out over 10 units resulted in an average fixed cost of \$20, over 30 units, \$6.70, and over 60 units, \$3.33. Note that while average fixed costs are falling, they are falling at a decreasing rate.

Fixed costs determine capacity, that is the maximum quantity firms can produce. Fixed costs often are associated with the term overhead. The cost of building and equipment are depreciated annually. Interest on the debt to finance the assets is also considered a fixed cost. It may vary due to changes in interest rates but not with respect to production, so it is considered fixed.

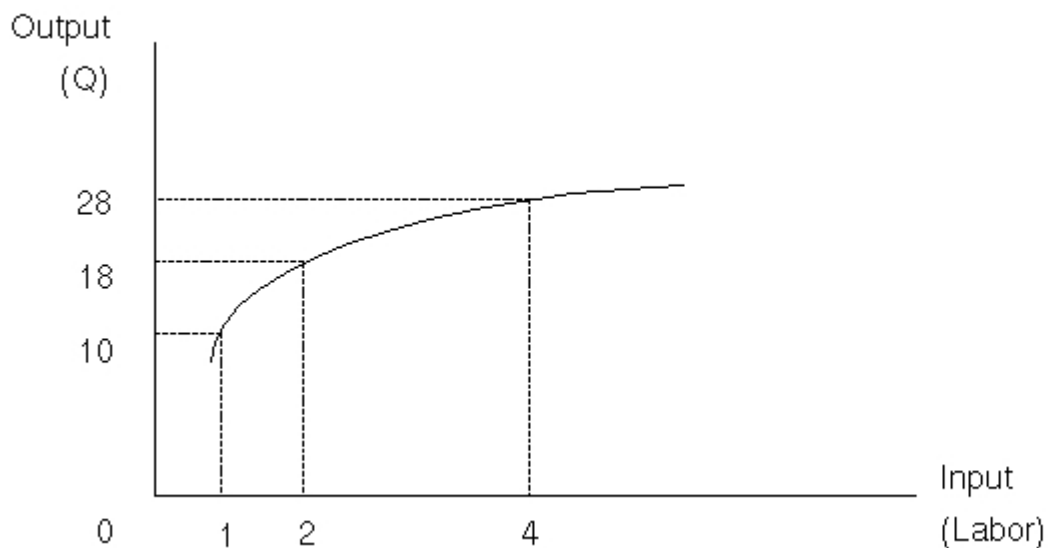
At some point in time, to increase capacity, a firm will need to incur additional fixed costs. While capital asset costs tend to be the primary source of fixed costs, other costs while they seem to be variable may actually be fixed such as labor, as management may be reluctant either to hire or fire employees.

Variable Costs

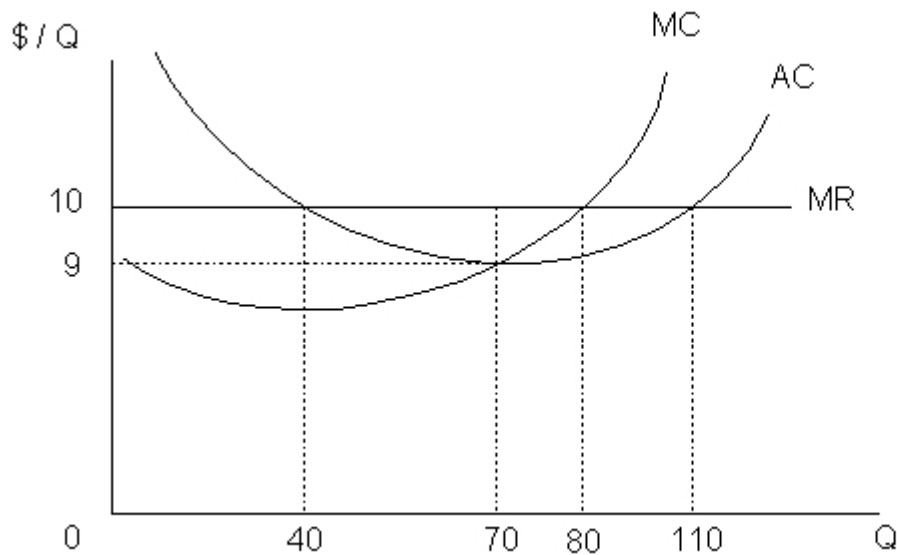


Variable costs consist primarily of labor directly involved in production (termed, direct labor as opposed to indirect labor such as administrative positions) and direct materials (as opposed to indirect materials that not part of the product itself). Traditional economics textbooks show a U shaped curve. Presumably, as output increases, cost per unit may decrease as labor learns how to do it better. However, past a certain point, overtime will be needed as well as new workers causing costs to increase per unit.

Average Costs



Adding the AFC and AVC curves give us the AC, Average Cost curve. It is the vertical summation of the two curves. In the above graph, at a quantity of 40, the AC equals \$10 and AVC equals \$5, therefore, the difference must be AFC of \$5. In other words, the addition of $AVC + AFC = AC$, $5 + 5 = \$10$. Note that the lowest point on the AVC and AC curves are not identical. This is due to the effect of AFC. The exact point is a matter of calculus, the rate of change of AFC relative to AVC to determine the lowest point on the AC curve. Not important at this level of discussion.



In the graph above, profit is maximized at $Q=80$, where $MR = MC$. Profit can be calculated several ways: $(P-AC)*Q$, $(10-9)*80$; or $(P*Q) - (AC*Q)$, $\$800 - 720 = \80 . Note that at a quantity of 70, profit is only \$70.

Marginal Costs

The importance of marginal costs cannot be overstated. Marginal cost is the change in total costs divided by the change in quantity. It is the additional cost incurred when producing one more unit. MC intersects the AC curve at the AC curve's lowest point. When marginal cost is less than average cost, then the AC curve is falling. When marginal cost is greater than average cost, then the AC curve is rising.

Referring to the earlier table, this can be seen. At a quantity of 40, AVC is 5, while MC is 2. Up to this point, AVC is falling but as output increases to 50 and 60, AVC starts to increase as MC now exceeds AVC.

Marginal costs are important because they determine the level of profit maximizing production when compared to marginal revenue. We will discuss this in greater detail later in this section.

Marginal Revenue

Let's talk about Marginal Revenue (MR). Similar to marginal cost, it is based on the change in total revenue divided by the change in quantity. It is the additional revenue gained by selling one more unit. In a competitive market, the additional revenue gained is equal to its price. This isn't true in a monopolistic market as to sell additional units the price has to be lowered. Price is lowered not just on the additional units sold but for all units sold. This results in a MR line at twice the slope of the demand curve.

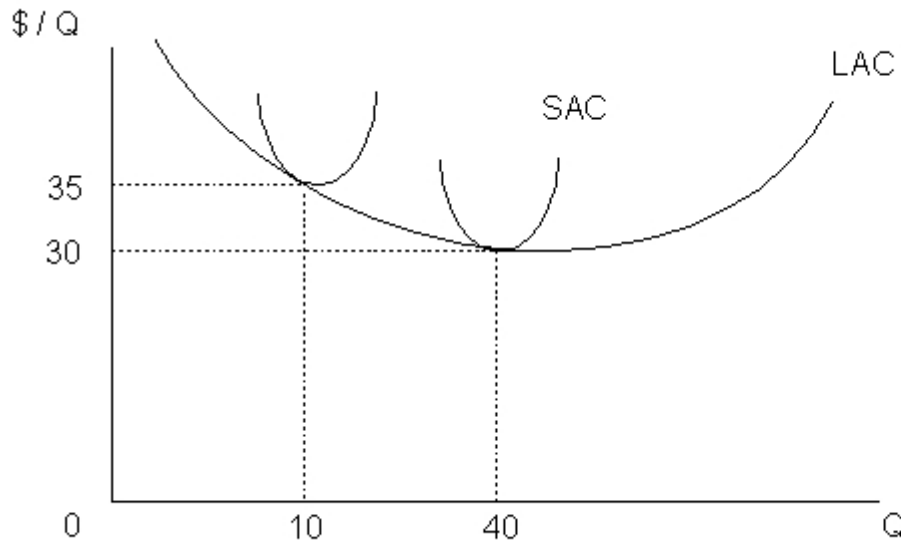
In our ongoing example, the market price is constant. Because our pizzeria is such a small player in such a large market, even if it doubles its output, it will have no effect on the market price. In such a market, each additional unit is sold at the market price. Its marginal revenue is equal to the price.

Long Run Costs

In the long run, all costs are variable. This is because capacity is fixed in the short run but in the long run any capacity can be built. On the long run average cost curve, each point represents a plant size. Consequently, each point represents a complete set of short run cost curves. Some economists call the

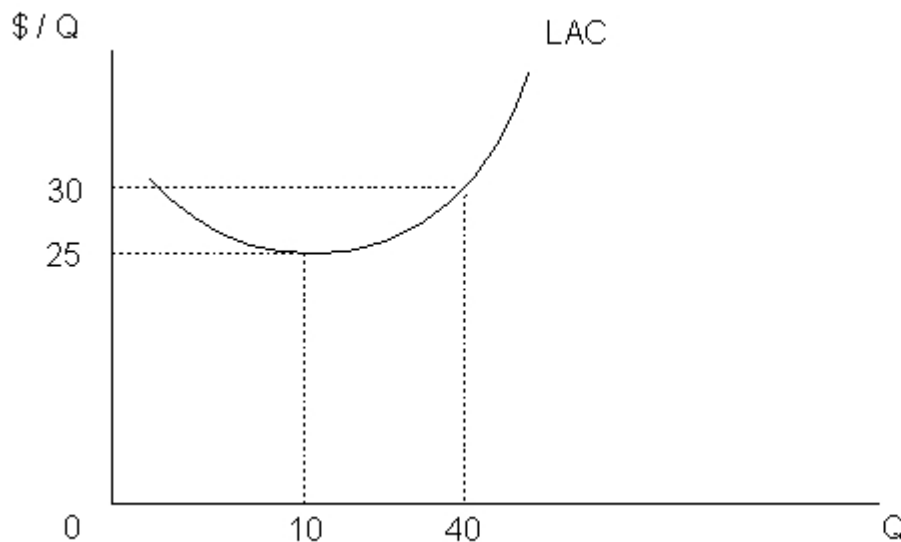
long run average cost an envelope curve because it encloses all short run cost curves. Others call it a planning curve because any point is possible but when you actually build the plant, you are in the short run.

In the graph below, costs are lowest at a quantity of 40 with an AC of \$30. Notice that the Short run Average Cost (SAC) curves are superimposed at the quantities of 10 and 40. Each point on the Long run Average Cost (LAC) curve represents the best short run average cost at that quantity.



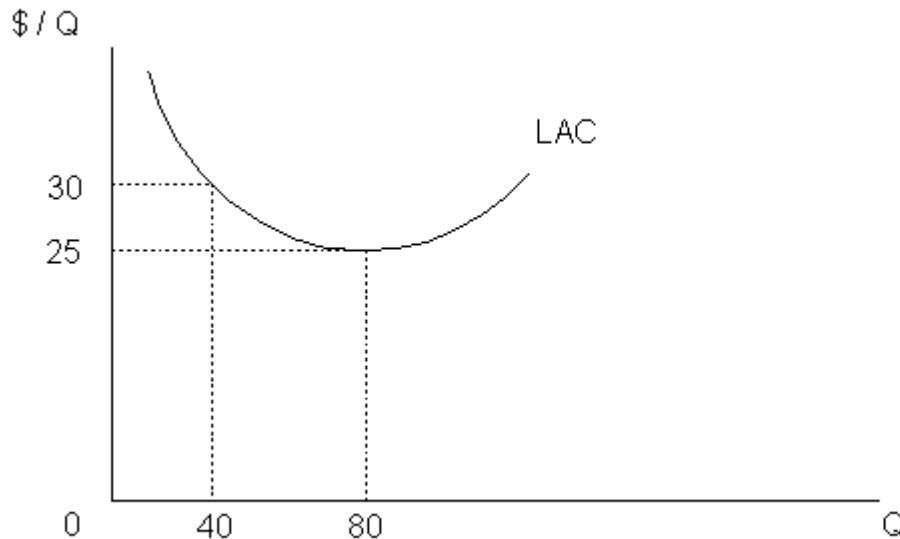
Long run cost curves reflect scales of economies. Scales of economies tell where the lowest cost per unit occur whether is in a large plant or a small one. Scale of economies influence market structures, economies of scale favor a few large producers, strong economies of scale may favor only one firm supplying the market creating a monopoly, while diseconomies of scale favor many small firms. Consequently, the selection of plant size may establish the difference between success and disaster relative to market size and location.

In the graph below, long run average costs are lower for lower quantities, i.e. lower plant sizes.



When costs are lower for lower plant sizes, the market will support many firms. With many firms, the market will be competitive as firms compete for business. When costs are lower for large plant sizes, particularly if only one plant is needed to service the market, only one firm will exist. The market will be monopolistic. The firm will have some pricing power over consumers.

In the graph below, long run average costs are lower for higher quantities, i.e. higher plant sizes.



2.3 Profit Maximization

Let's discuss the goal of profit maximization. Should short run or long run profits be maximized? Profitability promotes survival. If you want to make it into the long run you have to be profitable in the short run. In the United States, pressure is put on the firm to be profitable each quarter (quarters are periods of three months, the first quarter is January through March, 2nd quarter, April through June, etc.). In other countries financial statements or data may be released semi-annually or annually. It has been argued that this allows management flexibility to pursue projects that have greater long term profitability by sacrificing short term results. The global trend seems to be leaning toward quarterly reporting.

One may argue that some firms do not need to maximize profit. It is in the charter of some organizations to be non-profit. But even in these cases, firms that fail to earn an excess over costs will eventually fail.

2.3.1 Role of Firms

Firms maximize profit, that is their goal. But in fulfilling that goal they fill some vital economic roles, one, to reduce transactions cost, and secondly, to perform entrepreneurial functions.

Companies are constantly evolving, seeking ways to make a profit while serving the customer. Because of constant competition from existing and new firms, companies improve profits by lowering costs. Ultimately, lower costs become lower prices. Firms that provide lower prices to consumers win. The operational objective is to minimize the distance between product creation and customer consumption. In other words, minimize transactions costs. With the advent of the internet, it is possible to order directly from the factory.

In contrast, the other major economic role is entrepreneurship. Just spelling this word is a major challenge. It means taking risks developing new products and markets. In class, I often quote the

statistic that 80% of all new businesses fail in the first five years. The point is the considerable risk in entrepreneurship.

2.3.2 Economic Costs and Profits

The difference between revenue and economic cost is economic profit. If the revenue is greater than the sum of economic costs, then we have economic profits. Generally accepted accounting principles (GAAP) do not recognize the implicit cost of capital as an expense. In competitive markets, economic profit tends to be close to zero, as markets experiencing economic profit attract entrants, those with losses incur shrinkage as firms exit.

Cost of Capital

To recognize the cost of capital, we must impute it to the firm. This can be done in a number of ways. Many are variations on a theme. A number of such approaches take the market value of assets, multiply that number by an expected return and any profit beyond that will be economic profit. An approach called Economic Value Added relies on this as does a very similar approach called residual income.

For example, if PepsiCo uses \$20,000 million of assets to generate \$1,000 million of income and those assets could be expected to generate a return of 3% (by the way, this is called return on assets) then the \$1,000 million minus \$600 million ($\$20,000 * 3\%$, for the imputed cost of capital) equals \$400 million economic profit. Presumably, this indicates if the firm is bringing value to the shareholders. A mutual fund was operating in the 90's buying stock based on this concept. From what I understand it did well for a few years, and then didn't. While as an investment concept it may not generate superior returns this technique, I think is useful as one of several tools to evaluate management.

There are alternative measures of profitability that are widely used in evaluating a firm's performance. One of the most popular is called Return On Sales (ROS), calculated as net income/sales. ROS is also known as net profit margin. The advantage of ROS is that it provides a measure of the firm's ability to control expenses.

Another profitability measure is Return On Assets (ROA). Calculated as net income/assets, ROA provides insight as to how well the firm manages assets.

As shown below, profit is equal to total revenue minus total costs.

$$\text{Profit} > = \text{TR} - \text{TC}$$

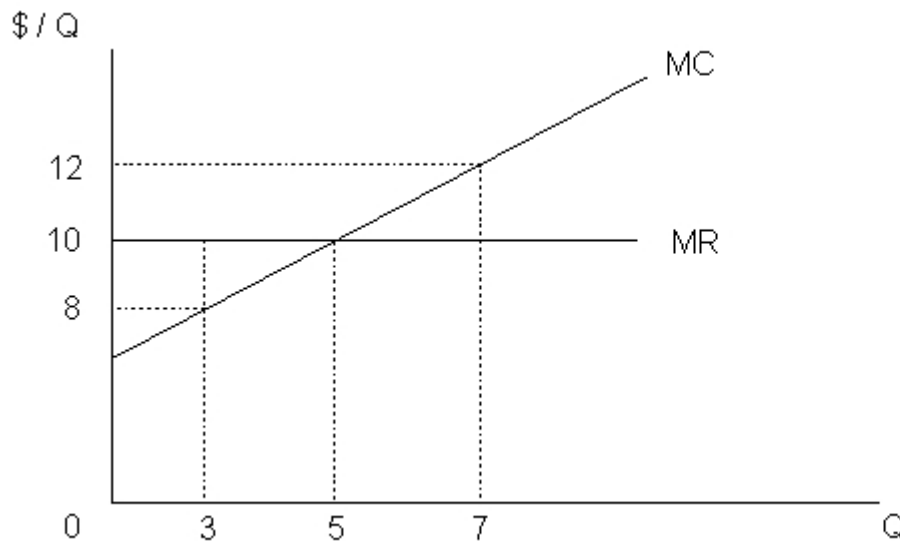
$$\text{Profit} = P * Q - \text{TC}$$

$$\text{Profit} = P * Q - (\text{FC} + \text{VC})$$

$$\text{Profit} = P * Q - (\text{FC} + \text{AVC} * Q)$$

2.3.3 Rules for profit maximization

Produce quantity where $\text{MC} = \text{MR}$. In the graph below we show MR as a constant, implying a competitive market.



In the graph above, if the firm produces to the right of $Q=5$ at $Q=7$, then the firm loses \$2 for each additional unit it produces. By producing less it would increase profitability by \$2 since it is eliminating losses. If the firm is producing at $Q=3$, it is earning a profit of \$2 per unit, it can increase profitability by increasing output. At $Q=5$, the firm maximizes profit as moving either to the right or the left decreases profit.

In the short run as long as Price is greater than or equal to AVC, continue production even if incurring losses. Variable costs are usually labor and raw materials. Suppliers and workers quickly stop if payments cease. Fixed costs include depreciation, which is a non-cash expense, and debt service, where payment can be delayed for a short while before foreclosure proceedings commence. Short run could exist for only a few months to a year or two depending on the severity of loss and cyclical nature of the industry.

Profit maximization rules differ in the long run in that the firm should select the plant size that minimizes average cost. As long as economic profit is greater than or equal to zero, then the firm should remain in business. Note that at the optimal plant size the $MR = MC$ rules still applies.

2.3.4 Break-Even Analysis (Cost-Volume-Profit Analysis)

Definition

Volume needed to breakeven i.e. no profit or loss. Ratio of total fixed costs to variable costs (Operational Leverage). There are some important assumptions needed to establish validity of this technique. The main assumption is that of constant prices and average variable cost. A constant average variable cost means that it is equal to marginal cost.

Formula

$$BEP = TFC / (P - AVC)$$

Note BEP is the same as C-V-P. The difference can be thought of as adding a profit margin in either the top as a target profit or as target profit percentage in the bottom. The BEP can be expressed in dollars or units. The formula given above expresses the result in units. Just multiply it by the price to convert to dollars.

$(P - AVC)$ is the contribution margin per unit. It can be expressed as a percentage by using this formula: $(P - AVC) / P$. The advantage of using the contribution margin as a percentage is the ability to

handle heterogeneous goods, in other words, different products. A retail store sells many different items with different prices. Retail managers know the average mark-up on retail. This helps them to set sales quotas for the store each month.

Let me derive the formula, as the exercise will show you the reasoning behind it.

$$\text{Profit} = P * Q - FC - AVC * Q$$

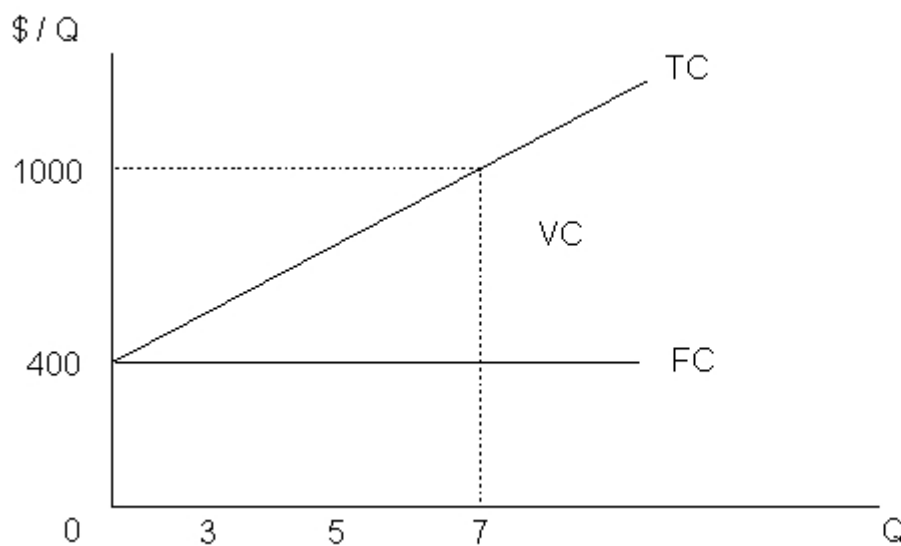
$$\text{Since profit} = 0, \text{ then } P * Q - FC - AVC * Q = 0$$

Solving for Q:

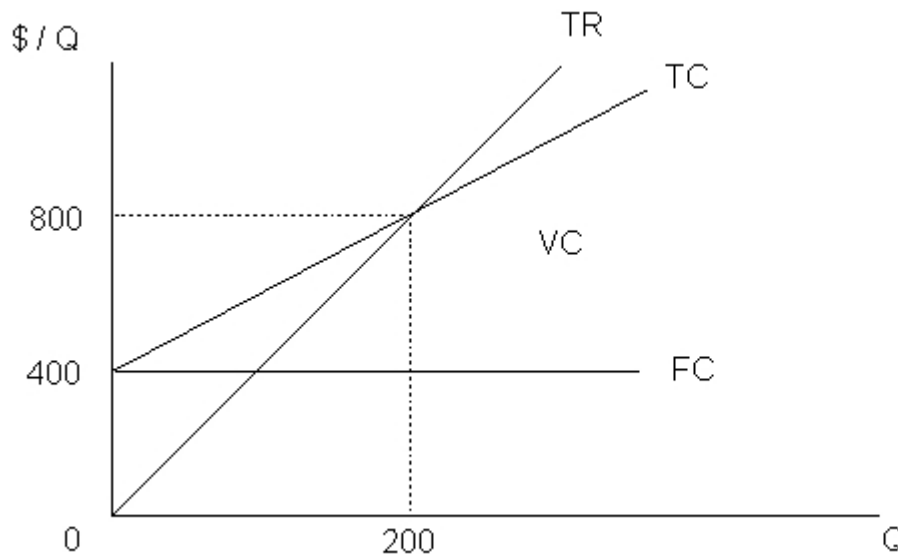
$$(1) Q * (P - AVC) = FC$$

$$(2) Q = FC / (P - AVC)$$

Graphing



Let's say fixed cost is \$400 and the variable cost per unit is \$2. The graph above shows the level of costs at each quantity. For example, at 300 units total cost equals \$1,000, fixed cost equals 400 and variable costs equal \$600. The slope of the TC line is equal to 2 which is the AVC (Average Variable Cost). The vertical intercept is equal to the fixed cost, \$400.



The second graph shows the breakeven point as well as loss and profitability at different volumes. Given a price of \$4, the TR (Total Revenue) at a quantity of 200 would be \$800. Costs at 200 units would be \$800. This is the BEP. At a quantity of 100, a loss of \$200 is incurred due to TC (Total Cost), \$600 which are greater than TR of \$400. While at a quantity of 300, TR \$1,200 exceeds \$1,000 TC for a profit of \$200. TC is calculated as $\$400 + 2(300)$. The slope of the TR line is \$3, the price.

Observations

As firm becomes more highly leveraged their fixed costs increase relative to variable costs. Break-even analysis has a number of uses: It can give an indication of economic feasibility. It gives an idea of when a firm should shut down. At various volumes it can give an indication of potential profitability.

Examples

- Let's say that you have a company called Widget Manufacturing. Each widget can be sold for \$3.00, the fixed cost is \$10,000 and the average variable cost is \$1.00. What is the break even point in units?
Answer: $10,000 / (3 - 1) = 5,000$ units
- Who wants to break even? Let's say that you want to earn at least a \$6,000 profit. How many units must you sell?
Answer: $(10,000 + 6,000) / (3 - 1) = 8,000$ units
- More often than not, we want to earn a target profit expressed as a percentage of sales. This is called Return On Sales (ROS) but is also known as Net Profit Margin (NPM). Let's say that our investors demand a 15% ROS. How many units must we sell now?
Answer: $10,000 / (3 - 1 - .45) = 6,452$ (always round up on BEP calculations)
- Let's go back to our original problem. We need to advertise to attract customers. Let's spend \$5,000 on marketing. How many units must we sell to break even?
Answer: $(10,000 + 5,000) / (3 - 1) = 7,500$ units
- Sometimes advertising is budgeted for as a percentage of sales. Let's use 10%. How many units must we sell to break even now?

Answer: $10,000 / (3 - 1 - .3) = 5,883$ units

6. Let's put everything together. We want to advertise at 10% of sales and make a target ROS of 15%. How many units must we sell?

Answer: $10,000 / (3 - 1 - .3 - .45) = 8,000$ units

Appendixes for C-V-P

Degree of Operating Leverage

This is slightly more advanced material but something that many will find useful. Operating leverage, which is called the degree of operating leverage (DOL), can be calculated as $1 + FC/OCF$ where OCF is operating cash flow. Higher DOL's result in higher breakeven points.

Relationship of ROS & CM

Let's say you have the following data: $P = \$10$, $AVC = \$4$, $FC = \$12,000$

$BEP = FC / CM\% = \text{Sales } (\$)$

Note: $CM\% = (\text{Sales} - TVC) / \text{Sales}$ or $(P - AVC)/P$

$CM\% = (10 - 4)/10 = 60\%$

$12,000/60\% = \$20,000$

Now let's add a 20% ROS requirement.

The procedure is to subtract the 20% ROS requirement from the CM%

$12,000/40\% = \$30,000$

Another viewpoint is to see that it takes a 60% markup on retail to generate a 20% ROS. Of the original price at 100%, 40% is the original cost, which we assume here to be variable cost, an additional 40% consists of fixed costs with the remainder 20% profit.

Sample Question

You rent some space at the new Polk Towne Center at a monthly rate of \$2,000. Your staffing costs are expected to average \$10,000. Your store, CellsR'Us is expected to have gross sales of \$36,000 per month. To earn a 15% Return On Sales, what will be the required retail margin (markup on retail)?

Solution:

$12,000/x = \$36,000$; solving for x, the CM%, $x = 33\%$

Add back the 15% to the 33% to get the required CM% of 48%.

Sample Question

If you sell 20% of goods marked down 60%, what must be the original markup of retail necessary to generate an average 50% markup of retail?

Solution:

Using the algebra that many of you love to hate and that many think that is absolutely useless ;)

$.2x + .8y = .5$, where x is the discounted ROS and y is the normal ROS, this equation would calculate the average.

$x = y - .6$, reflecting the 60% discount we are taking

$.2(y - .6) + .8y = .5$, substituting $y - .6$ for x

$.2y - .12 + .8y = .5$

$y = .62$, which means the markup on retail is 62%

and $.62 - .60 = .02$ markup on the 60% discounted goods

let's prove it, $(20\% * 2\%) + (80\% * 62\%) =$ an average markup of 50%

The point of these section is to illustrate the uses of some simple key microeconomics concepts. Approximately 25% of Polk County's employment is in retail. All businesses use these profit concepts to some degree depending on the skill level of the managers. Considering that 80% of all businesses fail in their first five years and that approximately 25% of college graduates will eventually be self-employed and I think you see my point.

Study Guide

Fill-in-the-Blank

Production Function

1. Marginal Product = $\frac{\text{_____}}{\text{_____}}$.
2. Average Product = $\frac{\text{_____}}{\text{_____}}$.
3. Production function usually assumes that output is a function of _____ and _____ holding technology constant.
4. Marginal Productivity of Labor = $\frac{\text{_____}}{\text{_____}}$.
5. Marginal Productivity of Capital = $\frac{\text{_____}}{\text{_____}}$.
6. As output increases, the MPL tends to _____.
7. As output increases, the MPK tends to _____.
8. Improvements in technology shift the marginal product curves _____.
9. Production functions show the relationship between _____ and _____.
10. Inputs are grouped into main categories of _____, _____, and _____.
11. As input increases, output rises, but a(n) _____ rate.

Production Costs

12. Total Cost = _____ + _____.
13. _____ = Variable Cost / Quantity.
14. Average Cost = Total Cost / _____.
15. Average Cost = _____ + _____.
16. Marginal Cost = $\frac{\text{_____}}{\text{_____}}$.
17. As output rises, in the short run, marginal costs tend to _____.
18. Output is determined at the point where _____ = _____.
19. Marginal Revenue = $\frac{\text{_____}}{\text{_____}}$.
20. Total Revenue = _____ * _____.
21. Economic profit exists when economics costs are _____ than revenue.
22. Economics costs include a _____.

23. Variable costs are typically _____ and _____.
24. As output increases, average fixed costs _____.
25. _____ costs are those costs borne directly by those involved in either consumption or production.
26. _____ costs are those costs not borne directly by those involved in either consumption or production.
27. In the long run, the firm should pick the plant size that _____ production costs.

Profit Maximization

28. Increasing output beyond the optimum causes profits to _____.
29. Decreasing output below the optimum causes profits to _____.
30. Contribution Margin per unit = Price _____ Average Variable Costs.
31. Break Even Point (in units) = _____ / _____.
32. Break Even Point in dollars = _____ * _____.
33. Break Even Point assumes constant a _____.
34. As fixed costs increase, the BEP _____.
35. As prices increase, the BEP _____.
36. As average variable costs increase, the BEP _____.
37. Operational leverage occurs when _____ are a significant proportion of total costs.
38. The advantage of operational leverage is that as _____ increases, _____ increases.
39. Advertising can be included into the C-V-P formula as either a _____ or _____.
40. Profit can be included into the C-V-P formula as either a _____ or _____.
41. Return On Sales = _____ / _____.
42. _____ / Contribution Margin = _____.

FIB Solutions

Production Function

1. Change in Total Product, change in input [labor]
2. Total Product, input
3. capital, labor
4. change in output, change in labor
5. change in output, change in capital
6. decrease
7. decrease
8. upward
9. input, output
10. capital, labor, technology
11. decreasing

Production Costs

12. Fixed Cost, Variable Cost
13. Average variable cost
14. Quantity
15. Average Variable Cost, Average Fixed Cost
16. change in total cost, change in output
17. rise
18. marginal costs, marginal revenue
19. change in total revenue, change in output
20. price, quantity

21. less
22. normal operating profit [to pay return on capital]
23. materials, labor
24. decrease
25. Private
26. External
27. minimizes

Profit Maximization

28. decrease
29. decrease
30. minus
31. fixed costs, contribution margin [per unit]
32. BEP in units, price
33. price
34. increases
35. decreases
36. increases
37. fixed costs
38. output, profit
39. fixed cost, variable cost per unit
40. fixed cost [consider a normal profit as an economic cost], variable cost per unit
41. net income, sales [total revenue]
42. Fixed Costs, BEP \$

True/False

Production Function

1. The Marginal Product of labor declines as input increases.
2. The Marginal Product of capital increases as input increases.
3. The Average Product of labor increases as input increases.
4. The Average Product of capital decreases as input increases.
5. Marginal product of labor represents the output generated by one additional unit of labor.
6. Marginal product of capital represents the average productivity of capital.
7. Technology generally lowers the productivity of labor.

Production Costs

8. Explicit costs are the market value of purchased resources.
9. Implicit costs are transactions costs.
10. In the short run, all costs are variable.
11. Total costs consist of fixed and variable costs, in the short run.
12. Average costs multiplied by quantity equals total costs.
13. As long as $P > AVC$, then in the short run, the firm should continue production.
14. In the long run, as long as profit is greater than or equal to zero, the firm should continue production.
15. Average fixed cost equals total fixed cost divided by quantity.
16. Average cost equals average variable cost plus average fixed cost.
17. Total variable cost equals average variable cost times quantity.
18. Average cost is at its lowest point when it is equal to marginal cost.
19. Average fixed cost is greater than average cost when marginal cost is greater than average

variable cost.

20. Average variable cost decreases as output increases.
21. Marginal costs are constant as output increases, in the standard model.
22. Average costs represent cost per unit.
23. Fixed costs increase as output increases.
24. Average fixed costs increase as output increases.
25. Direct labor and materials are the primary variable costs.
26. Return On Sales is an implicit cost.
27. Rent is an implicit cost
28. Operating expenses are an explicit cost.
29. Depreciation is an implicit cost.
30. Cash Revenues are imputed.

Profit Maximization

31. Break Even Point assumes constant variable costs per unit.
32. Firms exist to reduce transactions costs and perform entrepreneurial functions.
33. As operational leverage increases, variable costs increase relative to fixed costs.
34. Contribution margin per unit is price minus average variable cost.
35. Contribution margin per unit multiplied by fixed costs equals break-even point.
36. BEP in units divided by price equals BEP in dollars.
37. Increasing the contribution margin, decreases profitability.
38. Total Revenue divided by Total Costs equals profit.
39. Return On Sales is the percentage profit of sales.
40. Profit maximization is achieved at the output level where $MR = MC$.
41. Return on Sales is calculated as cost divided by sales.
42. Net profit margin represents the percent of sales that is profit.
43. Increasing productivity lowers costs and increases profitability.
44. Output should be decreased when marginal revenue $>$ marginal cost.
45. Profit is maximized at the point where average cost is the lowest.

T/F Solutions

Production Function

1. True
2. False, it decreases.
3. False, it decreases.
4. True
5. True
6. False, MP is additional productivity, AP is average productivity.
7. False, It increases MPL.

Production Costs

8. True
9. False, implicit costs are costs not purchased on open market.
10. False, in the long run all costs are variable, in the short run fixed costs exist.
11. True
12. True
13. True
14. True
15. True

16. True
17. True
18. True
19. False, AFC can never be greater than AC.
20. False, it can increase.
21. False, in the standard model we study, it increases.
22. True
23. False, they remain constant.
24. False, they decrease.
25. True
26. True, Return On Sales is an implicit cost. It is not an operating expense and is not recorded by accountants.
27. False, As long cash is paid for rent, it is an explicit cost.
28. True
29. False, Depreciation is a recorded cost by accountants.
30. False, Cash Revenues are NOT imputed.

Profit Maximization

31. True
32. True
33. False, fixed costs increase relative to variable costs.
34. True
35. False, $\text{Fixed Costs} / \text{CM} = \text{BEP}$.
36. False, $\text{BEP in units} \times \text{Price} = \text{BEP in dollars}$.
37. False, it increases profitability.
38. False, $\text{TR} - \text{TC} = \text{Profit}$.
39. True
40. True
41. False, $\text{ROS} = \text{Net Income} / \text{Sales}$
42. True
43. True
44. False, increased.
45. False, profit is maximizes where $\text{MR} = \text{MC}$.

Problems & Exercises

1. Fill in the missing numbers in the table below.

Figure 1.0

Labor	Capital	Output	APL	MPL
10	2	600	(c)	60
20	2	1,000	50	(d)
(a)	2	1,200	40	20
40	2	(b)	30	10

2. Let's say that a deal to make widgets is presented to you. Fixed costs are estimated at \$10,000; Price per widget will be set at \$5.00 with an estimated production cost of \$3.00 per widget.
 - a. What is the break even point in units?
3. What is the break even point in dollars?

4. What is the contribution margin?
5. If 8,000 widgets are sold, what is the ROS?
6. Another offer to make Gizmos is presented, Fixed costs are \$20,000; price \$10, AVC \$5, required ROS is 15% and the advertising budget is estimated at 10% of sales. What is the break even point?

7. Figure 2.0

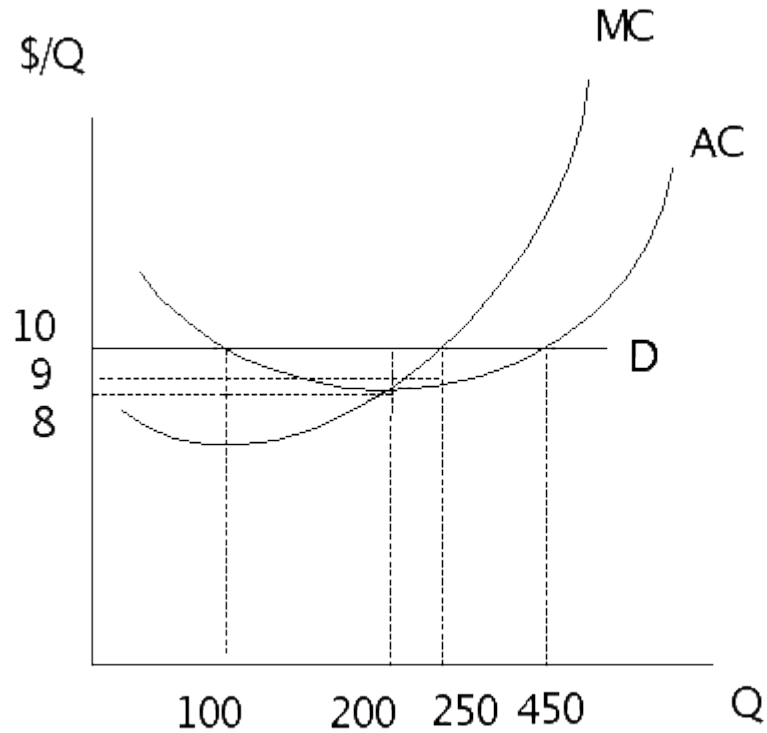
Combination	Labor	Capital	Output
A	2	6	60
B	3	6	70
C	3	7	90
D	4	8	110

- a. Given labor costs \$4 and capital \$6, what is the cost of input combination A?
8. What is the marginal product of labor going from combination A to B?
9. What is the average product of labor at combination C?
10. What is the APK at combination D?
11. What is the MPK from B to C?

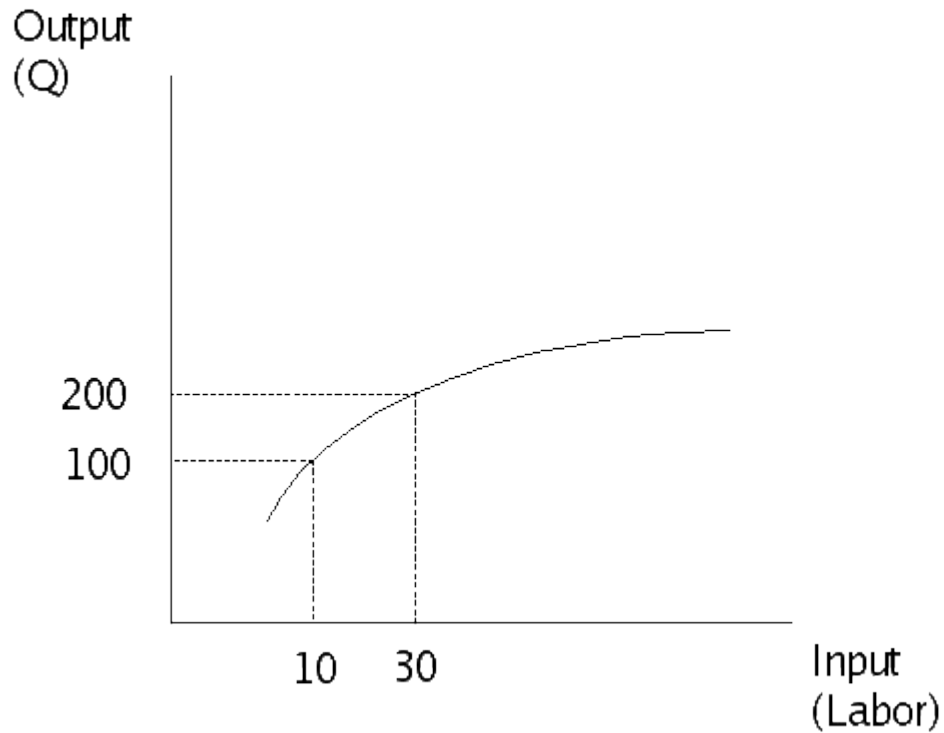
12. Fill in the missing numbers in the table below.

Figure 3.0

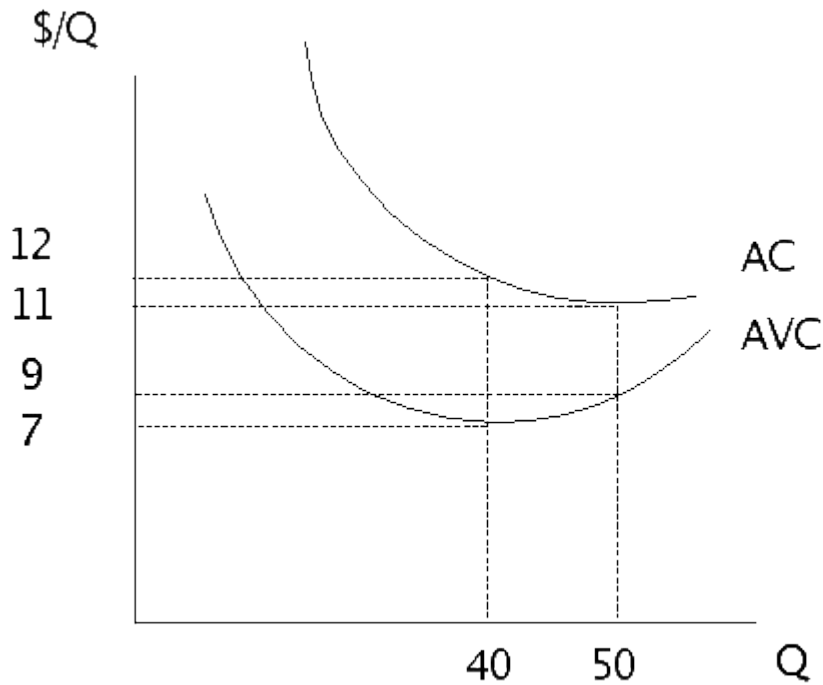
Q	FC	AFC	VC	AVC	TC	ATC	MC
200	(b)	5.0	600	(d)	1,600	8.0	-
(a)	1,000	(c)	800	2.0	(e)	4.5	1.0
600	1,000	1.67	900	1.5	(f)	(g)	.5
800	1,000	1.25	1,600	2.0	2,600	3.3	2.5



13. Answer the following questions from the graph above.
- a. What is the profit maximizing level of production?
14. At a quantity of 200, what is the total cost and profit?
15. What is the marginal revenue?
16. What is the marginal cost at $Q = 250$?
17. What is the AFC at $Q = 200$, given $FC = 400$?
18. What is the AVC at $Q = 200$, given $FC = 400$?
19. What is the profit margin, at $Q = 250$?
20. What are the two breakeven (per unit) points?



21. Answer the following questions from the graph above.
 - a. What is the output with 30 units of labor?
22. What is the marginal product of labor at labor = 30?
23. What is the average product of labor at $L = 30$?
24. Given 2 units of capital, what is the average product of capital at $L = 30$?
25. Given 2 units of capital, what is the average product of capital at $L = 20$?



26. Answer the following questions from the graph above.
- What is the average variable cost (AVC) at $Q = 40$?
27. What is the total variable cost (TVC) at $Q = 40$?
28. What is the average cost (AC) at $Q = 50$?
29. What is the total cost at $Q = 50$?
30. What is the marginal cost in increasing output from 40 to 50?

Problems and Exercises: Solutions

1. Figure 1.0

- $Q = Q/APL = 1,200/40 = 30$
 - $Q = APL * Q = 40 * 30 = 1,200$
 - $APL = Q/L = 600/10 = 60$
 - $MPL = \text{chg } Q/\text{chg } L = (1,000 - 600)/(20 - 10) = 40$
- $10,000 / (5-3) = 5,000$ units
 - $5,000 * \$5 = \$25,000$
 - $\$5 - 3 = \2
 - $((\$2 * 8000) - \$10,000) / (\$5 * 8000) = 15\%$
 - $20,000 / (10 - 5 - 1 - 1.5) = 8,000$ units

- a. $(2 * \$4) + (6 * \$6) = \$44$
 6. $(70 - 60) / (3 - 2) = 10$
 7. $90 / 3 = 30$
 8. $110 / 8 = 13.75$
 9. $(70-90)/(6-7) = 20 / 1 = 20$

10. Figure 3.0

- (a) $Q = VC/AVC = 800/2.0 = 400$
 (b) $FC = TC - VC = 1,600 - 600 = 1,000$
 (c) $AFC = FC/Q = 1,000/400 = 2.5$
 (d) $AVC = VC/Q = 600/200 = 3.0$
 (e) $TC = FC + VC = 1,000 + 800 = 1,800$
 (f) $TC = FC + VC = 1,000 + 900 = 1,900$
 (g) $AC = TC/Q = 1,900/600 = 3.3$

- a. Where $Q = 250$
 11. $TC = \$1400 (7*200)$, Profit = \$600
 12. $MR = 10$
 13. $MC = 10$ at point of profit maximization.
 14. $FC / Q = AFC$; $400 / 200 = \$2$
 15. $AVC = AC - AFC$; $\$8 - 2 = \6
 16. Profit Margin = $P - AC$; $\$10 - 9 = \1
 17. $Q = 100$ and $Q = 450$, where $AC = D$

- a. 200
 18. $(200-100)/(30-10) = 5$
 19. $APL = Q / L = 200 / 30 = 6.7$
 20. $APK = Q / K = 200 / 2 = 100$
 21. $100 / 2 = 50$

- a. At $Q = 40$: $AVC = \$7$
 22. $TVC = \$280 (40 * \$7)$
 23. \$11
 24. $\$11 * 50 = \550
 25. $((11*50)-(12*40))/(50-40) = 70$

Activities and Discussion

1. Estimate the BEP for Ford Motor Company.
2. Calculate the ROS for Ford Motor Company.
3. Estimate the degree of operating leverage for Ford Motor Company.
4. Project the profitability for Ford Motor Company if sales increased 5%.

Selected Definitions

Average Cost (AC)

$AC = TC / Q$, cost per unit.

Average Fixed Cost (AFC)

$AFC = TFC / Q$, fixed cost per unit.

Average Product

$AP = TP / \text{Input}$ such as labor. It is a measure of average productivity of that input.

Average Product of Capital (APK)

$APK = Q / K$.

Average Product of Labor (APL)

$APL = Q / L$.

Average Variable Cost (AVC)

$AVC = VC / Q$, variable cost per unit.

Break Even Point (BEP)

Volume needed to breakeven, that is there is no profit or loss.

Contribution Margin (CM)

The amount $P - AVC$. When the contribution margin (CM) is divided by P , it is the CM%. Very useful for calculating break even volumes for different goods.

Cost-Volume-Profit (C-V-P)

$C-V-P = FC / (P - AVC)$.

Economic Costs

Addition of explicit and implicit costs, total of costs incurred in production (and marketing, admin, etc). Includes cost of capital.

Economic Profit

Any profit in excess of economic costs.

Explicit Costs

Costs from inputs that are purchased.

External Costs

Costs incurred by someone who is not consuming nor producing the good. Example health care expenditures from pollution.

Fixed Cost (FC)

Costs that do not vary with production. Generally includes costs such as marketing, interest, insurance, taxes, and administrative.

Implicit Costs

Costs from self-owned resources.

Imputed Costs

Costs not originally recorded by accountants but calculated and added as they reflect a usage of resource. Example, cost of capital such as Return on Assets.

Isocost

Combinations of input that produce the same level of costs.

Isoquant

Combinations of input that produce the same level of output.

Marginal Cost (MC)

$MC = \text{Change in total cost} / \text{change in quantity}$. It is the additional cost incurred by producing one more unit.

Marginal Product

$MP = \text{Chg } Q / \text{Chg input}$.

Marginal Product of Capital (MPK)

$MPK = \text{Chg } Q / \text{Chg } K$. Where K is capital.

Marginal Product of Labor (MPL)

MPL Chg Q / Chg L Where L is labor.

Marginal Revenue (MR)

MR = Chg TR / Chg Q.

Operating Leverage

Determined by the ratio of fixed costs to variable costs.

Private Costs

Costs incurred by producer.

Return On Assets (ROA)

Net income/Assets. Many authors also call this return on investment or return on capital.

Return On Equity (ROE)

Net income/Equity. Just to make matters confusing, this really is the return on capital from a stockholders viewpoint.

Return On Sales (ROS)

ROS = Net Income / Sales.

Total Cost (TC)

Sum of fixed and variable costs.

Variable Cost (VC)

Direct labor and materials. Costs that vary with production