The Simple Mathematics of Supply and Demand

Math always causes aniexty. Remember it is only a language to describe the world. We will go through everything that you will need to know, step by step. Let me be clear, however, that this is not intended to be a replacement for your math classes.

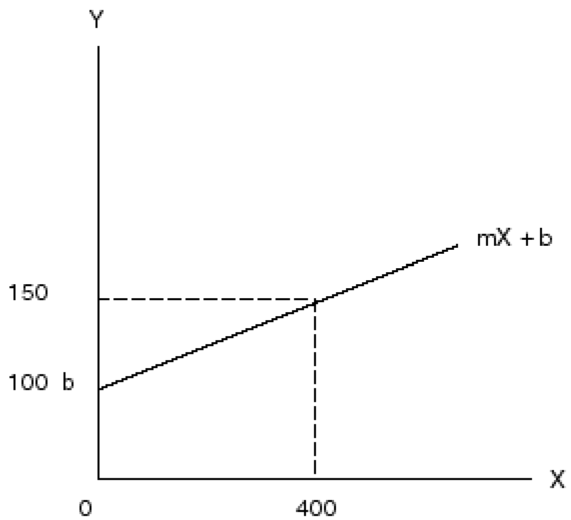
Graph of a Line

First, let's start with the equation for a line. In traditional math the equation for a line is Y = mX + b. Y is the dependent variable, X is the independent variable, m the slope of the line and b is the vertical intercept.

You may wonder what is the distinction between a dependent and independent variable. It signifies cause and effect. The independent variable causes an effect on the dependent variable. Typically, the independent variable is graphed on the horizontal axis, the dependent variable on the vertical axis. However, in economics, we put price, which is the independent variable on the vertical axis.

In the graph below, we illustrate the components of the equation Y = mX + b.

Figure 1.0



Vertical Intercept

You may wonder to what benefit is a vertical intercept. One, it sets the beginning position of a line. If it increases by 10, then every value of Y increases by 10. Note that the value of X is unaffected.

Two, it signifies how much Y exists when X is zero. Put another way, it is the amount of Y unaffected by X, the amount of Y not explained by changes in X. To some extent, it represents the amount of factors other than X that affect Y. Since this graph is only showing the relationship between Y and X, it is assuming those other factors are held constant.

Calculating Slope

To calculate slope we use the famous maxium "rise over run." In Figure 1.0 going from the b intercept to point (400, 150). In a data pair such as (400, 150), the first value, 400, represents the X co-ordinate and the second, 150, represents the Y co-ordinate. Here the rise is $\Delta Y/\Delta X$, where Δ equals change. The $\Delta Y = 150 - 100 = 50$. The $\Delta X = 400 - 0 = 400$. Note that the order of data points must be maintained, the 400 is associated with 150 and 0 with 100. This ensures the signs are correct. The slope equals 50/400 = .125

Therefore, the graph above illustrates the equation Y = .125X + 100.

Variables

In both economics and mathematics, we talk about variables. Just in case you are not exactly sure of what they are, let me provide you with a brief example. Let us say that you can earn $10 per hour baking pizza. Your income is the number of hours worked multiplied by your hourly wage of $10. Your income would be described by the following equation:

$10 * H = I where I = income, and h = hours worked. So $10 * 3 hours = $30, $10 * 4 = $40 and so on.

The h and I are placeholders. Their values can vary, hence they are called variables.

Variables are important as they help us describe mathematical relationships usually stated in equation form. In economics, our interest is not to merely describe relationships but often to use them to solve a problem such as unemployment, or to find out how to maximize profit.

Functions

This is a sidebar discussion, not directly tested on the exam but very useful in understanding mathematical relationships as well as the notation and terminology used on exams.

Quite often we speak of something being a function of something else. In the example given earlier, we said that the number of pizzas produced is dependent on labor hours. Another way of expressing that thought would be to say pizza output is a function of labor hours.

Mathematically, we could write that as follows Q = f(L) where Q is output, L is labor, and f( ) signifies that labor is the independent variable that determines output.

Note that this notation does not specify the equation that determines this relationship. In fact, Q = 5L, Q = 6 + L, Q = 10L ... could all be valid.

By calling a mathematical relationship a function, we are imposing an important restriction, that each value of the independent variable produces only one value of the dependent variable. However, it is proper that two different values of the independent variable could produce the same value of the dependent variable. This means that not every equation is a function. Functions are important because they are usually solvable. Solvable in the viewpoint that we can find their best value.

Multiple independent variables

Not uncommoningly, the dependent variable is a function of several independent variables. In our study of demand, the quantity that folks are willing to purchase is a function of price, income, population, tastes and prices of other goods.

Let me use a home baked example to illustrate functions. Please, pardon my pun by the way. Let us say that you are baking a cake. We could express this as a function, that is, what determines the output of cakes, Q. The quantity of cakes is a function of (flour, eggs, milk, butter, sugar, baking powder, labor, pans, utensils, oven). All of these are needed to bake the cake. All are independent variables. Without any, well, don't expect me to try your cake. How I combine these is, of course, called a recipe. I could express this more formally as an equation. One cake = 3(eggs) + 2(flour) + 1(butter) + ... + 1(hourbaking). If I want two cakes, I merely double everything. Hopefully you get the idea, or at least become hungry for a cake.

Graphs

A line is a graphical representation of a relationship. Since only two variables are graphed, it is limited in its ability to show more complex relationships, which contain more variables.

For example, let's say that the quantity of shirts that I buy this week will depend on two factors: price and income. The cheaper the shirt, given by weekly paycheck, the more I can buy. Secondly, if my weekly income doubles, I can buy more shirts. Typically, I can grpah two variables, let's say price and quantity. But note that something is missing,; the effect of income. It is there but you can't see it. When we construct the graph showing the relationship between price and quantity, we assume that income is constant at only one level. If we double income, then at each price we could buy twice as many shirts compared to our original income. The price-quantity curve would shift to the right.

Very often in economics, we are interested in inputs and outputs. For example, if we have a certain amount of labor, how much output will be generated. Let's say that every hour of labor can bake 5 pizzas. Our equation describing this situation would be: Q = 5L where L = hours of labor and Q = number of pizzas baked. To bake 20 pizzas, we will need 4 hours of labor. 20 = 5L; solve for L by dividing both sides by 5, 20/5 = 4.

The advantage of a graph is that we can see the answer without calculating it. We can also understand how an equation "works" in a much quicker manner.

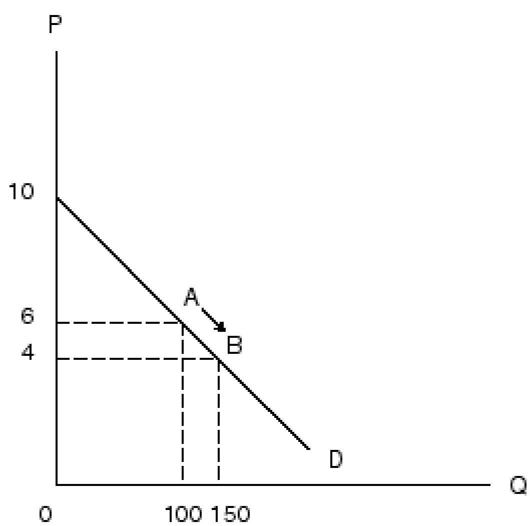
In the above graph, the relationship described above, labor input and pizza output is graphed. It is easy to see how much labor is needed for each output level.

Demand

In economics we graph the dependent variable Q on the horizontal axis and the independent variable P on the vertical axis. This is shown in Figure 2.0

Note that the vertical intercept is 10. Again, at a price of $10, zero pizzas will be wanted.

Figure 2.0



Let's say that Figure 2.0 represents the demand for pizzas at a small college on any given day. At $6, 100 pizzas are demanded, at $10 absolutely 0.

The slope of the line, rise / run is $\Delta P/\Delta Q$.

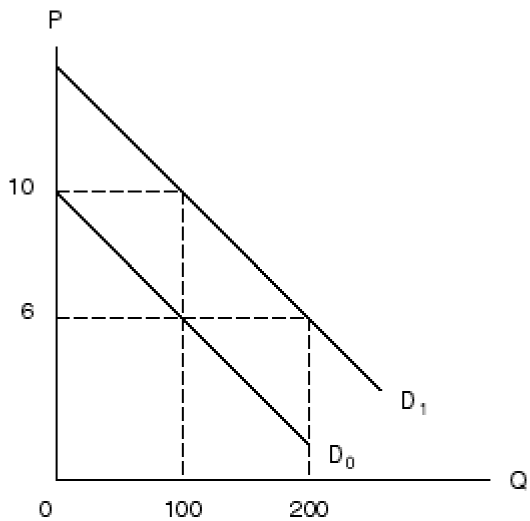
Since it is a straight line any two points will yield the same slope. From Figure 2.0, moving from point A to point B the slope is calculated as (6-4)/(100-150) = 2/-50 = -.04 Notice that we can have a negative slope. It means that to gain more quantity sold price has to be lowered. When the value of one variable increases as the value of the other variable decreases, we say that we have an inverse relationship. A slope of -.04 states that a decrease of 4 cents in price is needed to sell one more pizza.

The equation for this line is $P = 10 - .04Q$. So if you wanted to sell 80 pizzas what price would you charge? Plug it in and find out: $10 - .04(80) = \$6.80$. Could you sell more than 250 pizzas? At a price of zero these students can consume no more than 250 pizzas. What would happen if income of college students were to double? The demand line would shift to the right, parallel to the original line.

When we change price, we have a change in quantity demanded, not a change in demand. With a change in price, we merely move along the line. The line does not change.

Change in Demand

Figure 3.0



At a price of \$10 instead of zero pizzas being demanded now its 100 pizzas. Mathematically $P = 14 -.04Q$. Only the intercept has changed. The slope has not changed. A decrease in demand which cause the intercept to be less. For example, $P = 5 - .04Q$. Note that just because income doubles, it does not mean that demand will necessarily double.

A increase in the slope would be reflected in this equation: $P = 10 - .08Q$; as $.08 > .04$.

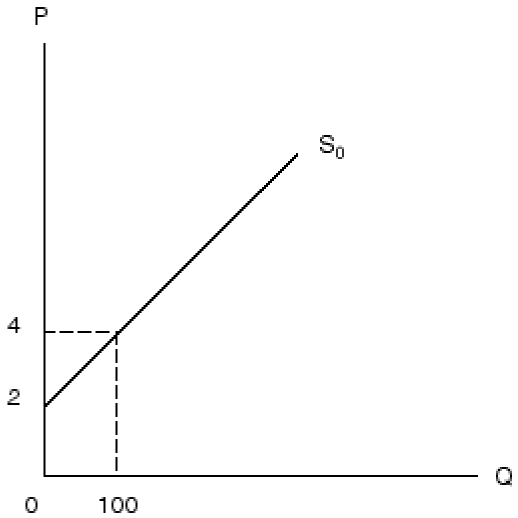
Here it would take a price decrease of 8 cents to sell one more pizza. The slope of the demand line reflects the additional price cut needed to sell one more unit of pizza. When the slope becomes steeper we say that product demand is becoming more inelastic. That is to say that changes in price have less of an effect on sales. When demand is becoming more elastic, then changes in price have a greater effect on sales.

A decrease in the slope would cause the coefficient to drop such as: $P = 10 - .02Q$

Here a 2 cent drop in price causes another pizza to be demanded.

Supply

Figure 4.0



In Figure 4.0 the slope is calculated as (4-2)/(100-0) =2/100 = .02. Notice that the slope is positive. Therefore the supply equation is:

P = 2 + .02Q

What price is required for producers to supply 300 pizzas? 2 + .02(300) = \$8. The \$8 times 300 = \$2,400. From this amount suppliers pay all of their costs and earn a normal rate of profit, otherwise they would produce something else. Accordingly the intercept can have the interpretation of the level of fixed costs. This should make some sense, if a pizzeria has a nice dining room, it has incurred an additional cost that it must recover if it is to stay in business.

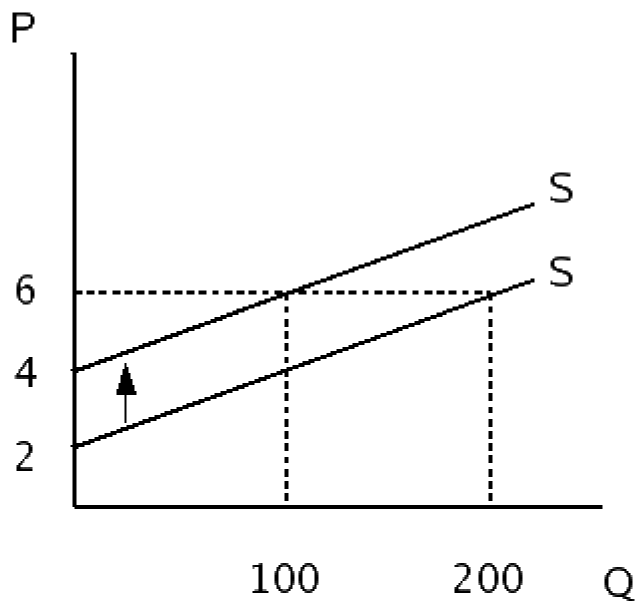
The slope can be given the interpetation of a marginal cost. In the case of pizza this would be the cost of raw materials (flour, cheese, sauce, toppings), utilities (electricity for baking) and labor for making each additional pizza. I should note here that a normal markup on cost would be included. A return on capital invested is considered an economic cost even though accountants do not consider it a cost. If pizzerias do not generate a profit they will not supply pizza.

The slope indicates the willingness to supply based on price. At a slope of .02, to increase one unit of supply would require an \$.02 increase in price: .02 * 1 = \$.02. To increase quantity supplied 100, price would need to increase \$2.00.

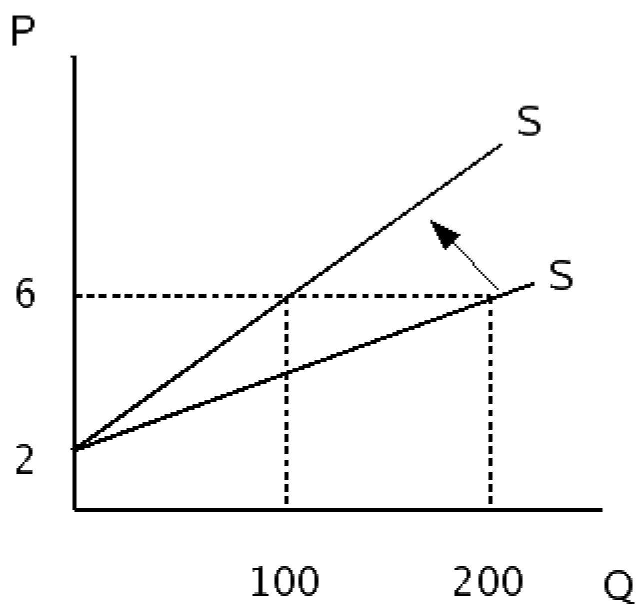
Although the supply curve is shown as touching the vertical axis, in real life it does not. Firms will only produce when the quantity expected to be sold covers all costs. For example, let's say that our pizzeria has a monthly rent of \$12,000 and that is their only fixed cost. It costs \$2 to make a pizza. At the market price of \$4 they will supply no less than 600 pizzas. Selling 600 pizzas at \$4 generates total revenue of \$2,400. Minus fixed costs of \$1,200 and variable costs of \$1,200 (600*\$2) the economic profit equals zero, this called the breakeven point (but remember that a normal operating profit has been included). At a price point of \$4 pizzerias will not supply less than 600 pies. Similar calculations can be performed at each price point.

Any change in cost structure will cause the supply curve to shift and/or rotate. For this class, we merely assume it shifts.

Notices that a parallel shift in the supply line only changes the vertical intercept.



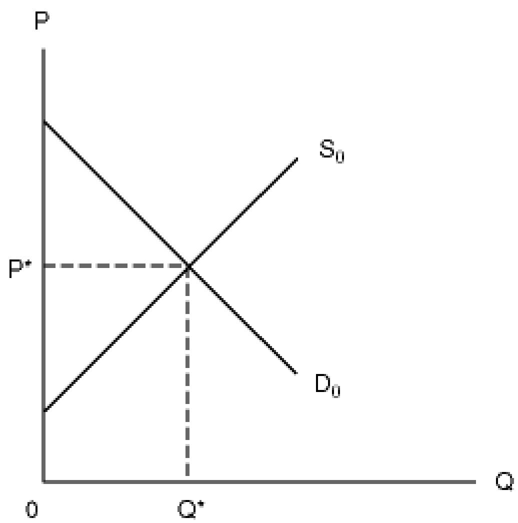
If the line rotates, the slope will change.



Market Equilibrium: Supply & Demand

Mathematically we determine what the market equilibrium will be by setting supply equal to demand.

Figure 5.0



We know that in equilibrium there is only one price and one quantity. In other words the price in the demand equation equals the price in the supply equation.

Using our prior equations:

P = 20 - .04Q Demand

P = 2 + .02Q Supply

We have two unknowns, P and Q and two equations. In algebra, as long as the number of equations, we can solve this.

There is actually more than one way of doing this. Since both equations are stated in terms of one variable, P, we will use the method of substitution. The right side of the supply equation is substituted into the left side of the demand equation.

20 - .04Q = 2 + .02Q

Solve for Q by combining like variables.

20 -2 +.04Q = 2 -2 +.02Q

18 -.04Q +.04Q = .02Q +.04Q

18 = .06Q

Divide by .06 to solve for Q

300 = 18/.06 = Q

Substitute into either demand or supply to obtain P

20 -.04(300) = 8 = P Demand

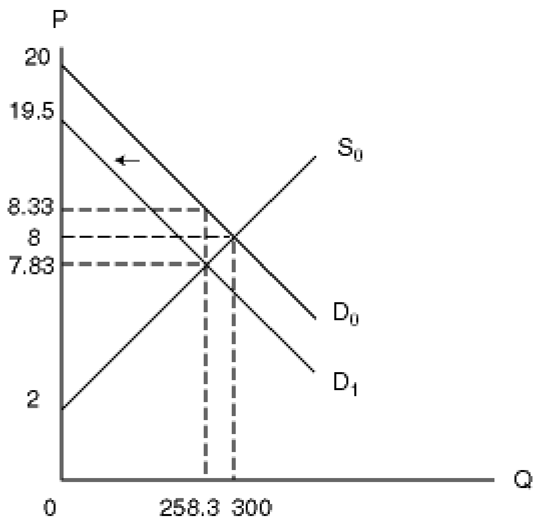
2 +.02(300) = 8 = P Supply

At market equilibrium, the price will be $8 and the quantity sold 300.

Effect of Taxes

Suppose that a sales tax of .50/pizza is levied on pizzas by the city, what will be the effect on price and quantity? Technically a sales tax is levied on consumers whereas an excise tax is levied on producers. As we will see, it does not make a difference whether it is a sales tax or an excise tax the effect will be the same. Assessing the tax on suppliers increases their cost, thereby shifting the supply curve to the left. Assessing a tax on consumers lessens demand because it removes spendable income. This causes the demand curve to shift leftward.

Figure 6.0



We start by subtracting .50 from the demand equation. Then we go through the same solution process again.

$20 - .50 - .04Q = 2 + .02Q$

$17.50 = .06Q$

$Q = 291.7$

$P = 2 + .02(291.7) = \$7.83$

Let's analyze this situation further. This price does not include tax so $7.83 plus .50 equals the price paid by consumers including tax which is $8.33. Consumers originally paid $8 times 300 units or $2,400. Since there were not taxes this was the amount received by suppliers. After taxes consumer pay $2,429.86 = (8.33 * 291.7) but suppliers receive only $2,284.01 = (7.83 * 291.7). The $145.85 = (.50 * 291.7) is paid as taxes to the city.

There are some observations that can be made from this example. One, the idea that the consumer always pays is a myth. From the law of demand, anytime a price increases, quantity demanded (i.e. sold) drops. In this example suppliers lost $115.99 in revenue. If the consumer truly had paid, this would not have happened. Secondly, if governments are to collect taxes without putting suppliers out of business they need to tax products whose demand is relatively inelastic, that is products that consumers will pay any price to have because they can't live without them. In this category fall food, gas, cigarettes and beer. Many people feel it's wrong to tax food. Since cigarette and beer consumption are considered somewhat undesirable by many segments of our society they are heavily taxed (sin taxes).

Rework this problem adding the excise tax to the supplier side. Note that the price now includes the tax. The results will be the same unless you have made a mistake.

Price Controls

Using mathematics we can more accurately analyze the effects of price ceilings and price floors. Let's start with the case of the price ceiling and assume that pizzerias have convinced Congress to mandate a pizza price of $10. Using our original equations and substituting in the new price we can determine the quantity demanded (Q_d)and quantity supplied (Q_s).

$10 = 20 - .04Q_d$

$250 = Q_d$

$10 = 2 + .02Q_s$

$400 = Q_s$

Note that quantity demanded decreased from the 300 pizzas at equilibrium to 250. This is to be expected as the price is higher. Quantity supplied increases from 300 to 400. A higher price encourages pizzerias to bake more pizzas. Since $Q_d < Q_s$ there is a surplus, which is 150 pizzas in this example.

If 150 pizzas remain unsold the pizzerias would need to lower prices to sell them which is contrary to the original intent of the price control. Government will have to act as a buyer to clear the market of this excess inventory. Buying 150 pizzas at $10 will cost $1,500 to the taxpayers. So consumers pay more for less pizza and pay higher taxes.

Suppose pizza is considered an essential nutrient (isn't it?). Congress, wishing to increase pizza consumption, passes a law fixing the price of pizza at $6. Let's calculate the new quantities demanded and supplied.

$\$6 = 20 - .04Q_d$

$350 = Q_d$

$6 = 2 + .02\ Q_s$

$200 = Q_s$

Now we have a shortage of 150 units. As long as the law is effective no more needs to be said. However let's assume that a black market arises. "Transactions costs" average $7 a pizza (everyone wants a slice of the pie). Going back to our original equations:

$20 - .04Q = 2 + .02Q + 7$

$11 = .06Q$

$183.3. = Q$

$\$12.67 = P$

Note how much higher the black market price is over the free market price. Someone is rolling in the dough. :)

Elasticity (Microeconomics Only)

In managing a business (or any organization for that matter) one would like to know what would happen if ... prices were increased 5% ... competitors cut their prices 10% ... how much will sales increase if it is a good year ... and so on. Notice that these changes are usually expressed in terms of percentages. Elasticity is a simple tool capable of providing quick answers. The generic formula for elasticity is:

$\varepsilon_Z = \%\Delta Q / \%\Delta Z$

where Z can represent variables such as price (p), income (i), competitor's price (p_y), price of a complementary product (p_y) or any other variable that may influence sales such as advertising. As a number calculated, elasticity is not a percentage nor does it imply a magnitude of any kind such as quantity, dollars, weight, etc.

For example, let's say that a 10% increase in price causes sales to decline 20%. Price elasticity of demand equals -20%/10% = -2, the negative 20% indicates that sales decreased. If prices drop 20% then sales should increase 50%. To see this set up the elasticity equation and solve for sales. (This is simple and you can do it.)

$\varepsilon_P = \%\Delta Q / \%\Delta P$

$-2 = \%\Delta Q / -20\%$

$-20\% * -2 = \%\Delta Q$

$40\% = \%\Delta Q$

If the elasticity and the independent variable (ie. p, i p_y...) are known, then the dependent variable %ΔQ can be calculated by just simple multiplication. If Ford wanted to predict the effect on the number of cars sold when income rises 15%, they would multiply their income elasticity (let's assume it is 3.0) by %Δ income to equal 45%. Ford can only dream that their income elasticity were that high. For income inferior items, income elasticity is negative. In fact, when a price elasticity exceeds 1.0 (or -1.0 in the other direction) we say demand is elastic. When elasticity is less than 1.0 (or -1.0) then demand is said to be inelastic.

An important elasticity is called the cross price elasticity of demand. Calculated as the percentage change in quantity divided by the percentage change in another product's price. $\%\varepsilon Y.X = \%\Delta Q / \%\Delta P$.

For example, let's say that GM increases its auto prices 4%, the cross elasticity of demand is 3.0. Ford will see a 12% increase in their sales, 4% * 3. A positive elasticity indicates a substitute good, a negative elasticity indicates a complementary good. If coffee prices increase 6%, then with a -2.5 cross elasticity of demand creamer sales will drop -15% (6% * -2.5).

The elasticities calculated thus far relate to demand but supply elasticities can also be calculated. Let's calculate a supply elasticity for labor. Suppose for a 5% increase in wages that the quantity supplied of labor hours increases 4%. The elasticity of supply for labor would be 4% / 5% or .8.

In business we generally want to know what happens when we change from our current price. We use a point elasticity formula. This is useful for small changes, but it is not considered accurate for large changes. For larger changes the arc formula is more appropriate. To understand the difference let's look at the individual formulas. The key is calculating the percent change %Δ. To calculate a percent change we divide change by a base. Under a point elasticity the base is the current value, which of course is known. Using the arc formulation the base is calculated as an average of the beginning and ending value. The results will be different.

An example would be helpful.

Figure E1

Starting at point A moving to point B, we drop our price 33.33% ((9-6)/9) while increasing quantity 66.67% ((150-90)/90) consequently the point elasticity is 66.67/33.33 = -2.

Now let's calculate this using the arc formulation.

$\%\Delta P = 3 / ((9+6)2) = 3/7.5$

$\%\Delta Q = 60 / ((90+150)/2) = .5$

$= .5 / 3/7.5 = -1.25$

Notice that these calculated elasticities are not the same, a point elasticity of -2 vs. an arc elasticity of -1.25. Note that moving backwards from point B to point A produces a point elasticity of -.8, direction does matter in calculating point elasticity. In the calculating of arc elasticity direction does not matter. That is one reason why some people prefer the arc formulation of elasticity.

In fact along a linear line each point has a different elasticity. Moving from point A to point B produced a point elasticity of -2. Moving from point B to point C produces a point elasticity of -.8

Figure E2

As demand shifts so changes the elasticities. Again referring to our example in figure E2, demand shifts from D to D' by 90 units. Moving from point A' to B' calculating the point elasticity; $\%\Delta Q = 33.33\%$, $\%\Delta P = 33.33\%$, the price elasticity of demand equals 1.0.

Shifting to the right causes elasticities to decrease. That is due to a decreasing $\%\Delta Q$ which results from ΔQ being divided by an ever larger base of quantity.

Figure E3

Rotating the demand line also changes the elasticity. In figure E3 moving from line D to D' the elasticity from point A to B' is calculated as $\%\Delta Q = 30/90 = 33.33\%$ while the $\%\Delta P$ 3/0 = 33.33% remains the same to yield a price elasticity of demand equal to -1.0 compared to -2.0 originally. Making a line steeper decreases the elasticity and consequently more inelastic.

Some textbooks do use price elasticity as an absolute value meaning that it doesn't carry the negative sign. This was done to avoid confusion for students. I argue it increased the confusion.

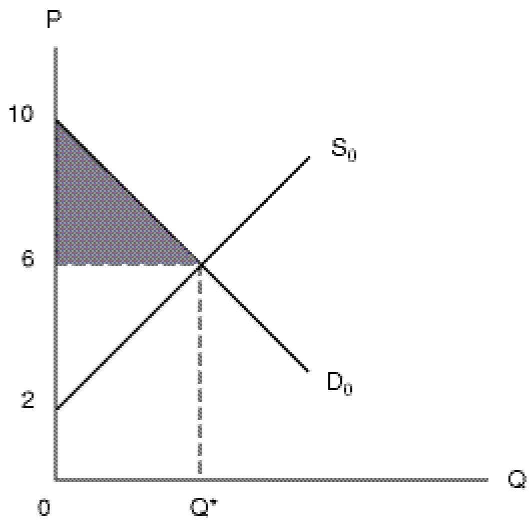
A very practical way of understanding elasticity is to note that as demand becomes more inelastic customers are affected less by price changes consequently they will pay any price to obtain the product they desire. Conversely, as demand becomes more elastic customers become more price sensitive. Increasing the price a little will cause sales to dcrease. It is important for organizations to know if their product's demand is elastic or inelastic. Inside the marketing department for many large firms is an area called market research. Here they hire economists to find elasticities as an aid in determining pricing policy. Even small businesses usually have an idea if demand is elastic or inelastic although they may not be able to quantify it.

To summarize, elasticity is an important tool in establishing pricing policy for firms, and in predicting macroeconomic policy effects.

Consumer and Producer Surplus (Optional)

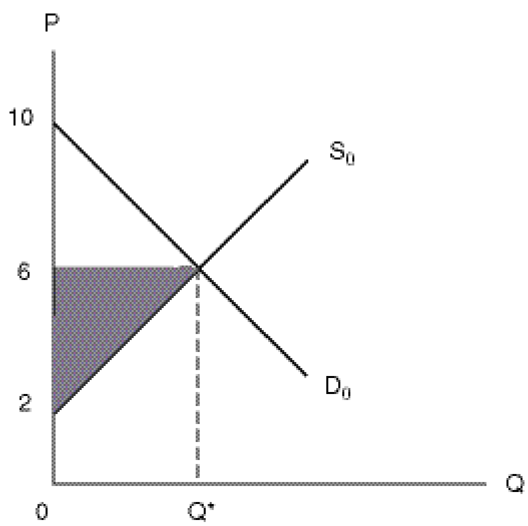
Consumer surplus represents the consumer's willingness to pay as shown by the area under the demand curve above the equilibrium price in figure E4. These customers are willing to pay more than the market price of $6.00 per unit but do not thereby saving funds that can be used for other purposes.

Figure E4



Producer surplus represents the producer's willingness to supply goods and services at lesser prices. This is shown in figure E5 in the shaded area between the supply line and the equilibrium price of $6 per unit.

Figure E5



The sum of consumer and producer surplus is called social surplus. When government projects and programs are evaluated in terms of economic efficiency, they attempt to measure the change in social surplus. If markets meet the critera for the competitive model, then a case can be made that social surplus is maximized. This issue is more critically analyzed in courses in public economics or welfare economics.

Consumer and Producer Surplus: Pizzas

P	Q_d	Q_s
15	500	3,000
14	900	2,700
13	1,500	2,400
12	2,000	2,000
11	2,800	1,800
10	4,000	1,600

To compute consumer surplus, take the difference between the price the consumer was willing to pay and the price actually paid, and multiply by the units purchased. Repeat the process at each price point.

P=15: (15-12) * 500 units = 1,500
P=14: (14-12) * 400 units = 800
P=13: (13-12) * 600 units = 600
Total Consumer Surplus $2,900

To compute producer surplus, take the difference between the price the supplier actually receives and the price the supplier was willing to take, and multiply by the number of units. Repeat the process at each price point.

P=11: (12-11) * 200 units = 200
P=10: (12-10) * 1,600 units = 3,200
Total Producer Surplus $3,400

Total social surplus: $2,900 + 3,400 = $6,300

A Review of Some Elementary Algebra

Using the classical slope equation Slope = Rise / Run, I would like to demonstrate some elementary algebraic concepts. In supply and demand, the rise is equivalent to the change in price while the run is the change in quantity. Slope = ΔP / ΔQ where Δ signifies change.

To solve an equation with one unknown is easy, just combine and rearrange the numbers to one side and the unknown variable to the other. Let's say you know the slope is 5, and ΔP = 3 but you need to find out the change in quantity. Set up the equation: 5 = 3 / X where X is the unknown change in quantity.

Multiply both sides by X to get 5X = 3.

Divide by 5; X = 3/5 = .6, which is the answer, the change in quantity is .6.

Let's try it again with slope = 10, ΔQ = 20 but we need to find the ΔP.

Construct our equation: 10 = X / 20

This is a little easier since we need only multiply each side by 20.

10 * 20 = X = 200. Therefore, the ΔP is 200. Check that this true, 200 / 20 = 10.

Let's try a different example to show addition and subtraction. Let us say that Profit = Revenue - Cost. Suppose that we know Profit is 500 and Revenue is $2,500. What we need is the cost. Solving this is easy. First set up the equation with what we know: 500 = $2,500 - X where X is the unknown cost.

Add X to each side: 500 + X = $2,500 - X + X, the X's cancel out to get 500 + X = $2,500.

Now subtract 500 from each side to solve: 500 + X - 500 = $2,500 - 500 to get X = $2,000. Costs are $2,000.

Professors are accused of devious things when we state equations in a slightly different form. If I stated the following: Cost + Profits = Revenue, it really isn't any different than Revenue - Cost = Profit. Being able to recognize different forms of the same equation is important.