A better undertanding to Poisson process
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Problem
If on average in an hour 5 chinese and 3 english men walk into the store with a poisson process.

1. What's the pdf of the people arriving?

   Since we know than on average 8 people show up in an hour. This is another poisson process with

$$f(n,t) = e^{-8t}\frac{(8t)^n}{n!}$$

2. If a person show up, what's the probability that he is chinese.

   We know that in one hour 5 chinese and 3 english will averagely show up. The probability that a chinese showing up is $\frac{5}{8}$. In general we have the expression.

$$P(X_{\lambda_i}|1\ showup) = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

   so now if we have n sources of poisson process, we would have:

$$P(X_{\lambda_i}|1\ showup) = \frac{\lambda_1}{\lambda_1 + \lambda_2 + ... + \lambda_n}$$

3. If 10 people show up, what's the probability that 3 is chinese

   we know that this is a binomial RV with N = 10 and p = 5/8

$$P(3\ chinese) = \binom{10}{3}(5/8)^3(3/8)^7$$

4. Now the probability that a customer is a women is p, what is the pdf of women customers showing up?

   We know that originally averagely 8 customers show up in an hour. Now we have 8p women customers showing up in an hour. So now we have a poisson process with 8p = λ, so we have the function.

$$f(n,t) = e^{-8pt}\frac{(8pt)^n}{n!}$$

5. Now if we be more specific, and say that the probability that a chinese is a women is p_1 and english is a women is p_2. What's the new pdf of the arrival of women?

   We know that the new mean is

$$\lambda = \lambda_1 p_1 + \lambda_2 p_2$$

   so the new pdf would be

$$f(n,t) = e^{-\lambda t}\frac{(\lambda t)^n}{n!}$$

6. What is the waiting time until the 4th person shows up?

   If we use the original equation where

$$f(n,t) = e^{-8t}\frac{(8t)^n}{n!}$$

   we know that the inter-arrival time is $\frac{1}{\lambda}$. In this case, it would be $\frac{1}{8}$. If we multiply this by 4, this would be 1/2, or half an hour. This is the amount of time it takes on average for the forth person to show up.

   However, to prove the original equation, is a real pain. We know that:

$$f(m,t) = \sum_{n=0}^{m}\left[e^{-5t}\frac{(5t)^n}{n!}\right]\left[e^{-3t}\frac{(3t)^{m-n}}{n}\right]$$

   this is because at a giving time, for a given amount of people to show up, you must sum through the entire permutation of n and (m-n). This sum could re re-written as:

$$f(m,t) = \sum_{n=0}^{m} e^{-8t}\frac{(5t)^n(3t)^{m-n}}{(m-n)!n!}$$

   we can take the exponential out and multiply m! in

$$f(m,t) = \frac{e^{-8t}}{m!}\sum_{n=0}^{m}\frac{m!}{(m-n)!n!}(5t)^n(3t)^{m-n}$$

   notice how this is a binomial distribution

$$f(m,t) = \frac{e^{-8t}}{m!}\sum_{n=0}^{m}\binom{m}{n}(5t)^n(3t)^{m-n}$$

   the binomial sum is

$$f(m,t) = \frac{e^{-8t}}{m!}(5t+3t)^m$$

   or

$$f(m,t) = e^{-8t}\frac{(8t)^m}{m!}$$

   now if we use this equation, we would get the same expectated time

$$\frac{1}{2}\ hour$$