a +1.0% grade and a +6.0% grade. The first column gives the station. The second column gives the intersecting grades and the locations of the BVC, P.I., and EVC. The third column gives the elevation of each point on the tangent grades, calculated as BVC elevation plus g_1x for the first tangent grade and P.I. elevation plus $g_2(x - L/2)$ for the second. The fourth column gives the offset, calculated as $rx^2/2$, with x measured from either the BVC or EVC as appropriate: since offsets are symmetrical about the P.I., however, they need be calculated only from the BVC to the P.I. The last column gives the curve elevation, which is the tangent elevation plus the offset. It should be noted that curve elevations can also be calculated by using only offsets from the g_1 tangent, and that in many cases it may be more convenient to use only one tangent.

EXAMPLE PROBLEM 4.1 A −2.5% grade is connected to a +1.0% grade by means of a 180 m vertical curve. The P.I. station is 100 + 00 and the P.I. elevation is 100.0 m above sea level. What are the station and elevation of the lowest point on the vertical curve?

Rate of change of grade:

$$r = \frac{g_2 - g_1}{L} = \frac{1.0\% - (-2.5\%)}{1.8 \text{ sta}} = 1.944\%/\text{sta}$$

Station of the low point:

At low point, $g = 0$

$g = g_1 + rx = 0$

or

$$x = \frac{-g_1}{r} = -\left(\frac{-2.5}{1.944}\right) = 1.29 = 1 + 29 \text{ sta}$$

Station of BVC = (100 + 00) − (0 + 90) = 99 + 10

Station of low point = (99 + 10) + (1 + 29) = 100 + 39

Elevation of BVC:

$$y_0 = 100.0 \text{ m} + (-0.9 \text{ sta})(-2.5\%) = 102.25 \text{ m}$$

Elevation of low point:

$$y = y_0 + g_1x + \frac{rx^2}{2}$$

$$= 102.25 \text{ m} + (-2.5\%)(1.29 \text{ sta}) + \frac{(1.944\%/\text{sta})(1.29 \text{ sta})^2}{2}$$

$$= 100.64 \text{ m}$$

Design standards for vertical curves establish their minimum lengths for specific circumstances. For highways, minimum length of vertical curve may be based on sight distance, on comfort standards involving vertical acceleration, or appearance criteria. For passenger railways (especially urban rail transit systems), minimum vertical curve lengths will often be based on vertical acceleration standards; for freight railways, much more stringent standards may be maintained to avoid undue stress on couplings

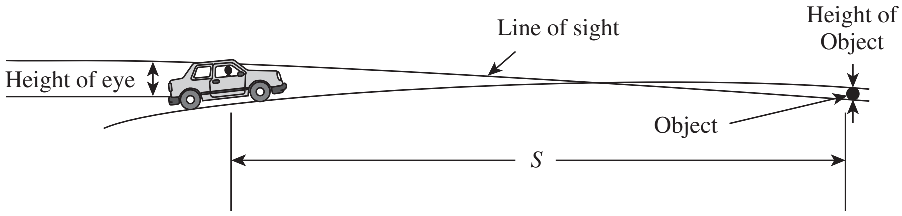


FIGURE 4.8

Stopping sight distance diagram for crest vertical curve.

in sag vertical curves. For airport runways and taxiways, minimum vertical curve lengths are based on sight distance.

In most cases, sight distance or appearance standards will govern for highways. The equations used to calculate minimum lengths of vertical curves based on sight distance depend on whether the sight distance is greater than or less than the vertical curve length. For crest vertical curves, the minimum length depends on the sight distance, the height of the driver's eye, and the height of the object to be seen over the crest of the curve, as illustrated in Figure 4.8. The minimum length is given by the formula

$$L_{\min} = \begin{cases} \dfrac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} & \text{when } S \leq L \\ 2S - \dfrac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A} & \text{when } S \geq L \end{cases} \tag{4.4}$$

where S = sight distance (from Table 3.3)
L = vertical curve length
A = absolute value of the algebraic difference in grades, in percent, $|g_1 - g_2|$
h_1 = height of eye
h_2 = height of object

For stopping sight distance, the height of object is normally taken to be 0.150 m. For passing sight distance, the height of object used by AASHTO is 1.300 m. Height of eye is assumed to be 1.070 m.

Inserting these standard values for h_1 and h_2, Equation (4.4) may be reduced to

$$L_{\min} = \begin{cases} \dfrac{AS^2}{404} & \text{when } S \leq L \\ 2S - \dfrac{404}{A} & \text{when } S \geq L \end{cases} \tag{4.5}$$

for stopping sight distance and

$$L_{\min} = \begin{cases} \dfrac{AS^2}{946} & \text{when } S \leq L \\ 2S - \dfrac{946}{A} & \text{when } S \geq L \end{cases} \tag{4.6}$$

for passing sight distance.

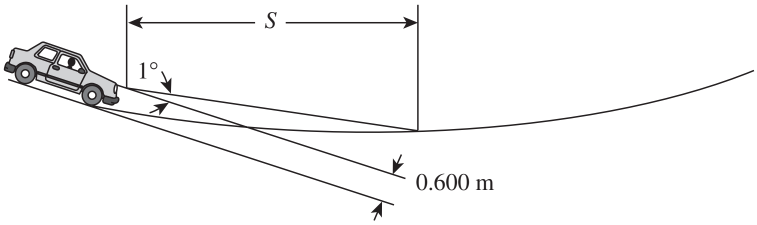


FIGURE 4.9

Stopping sight distance diagram for sag vertical curve.

For sag vertical curves, stopping sight distance is based on the distance illuminated by the headlights at night. Design standards are based on an assumed headlight height of 0.600 m and an upward divergence of the headlight beam of 1°. This is illustrated by Figure 4.9. As in the case of crest vertical curves, the formulas for minimum length of vertical curve depend on whether the length of the curve is greater or less than the sight distance. For sag vertical curves, the formula is

$$L_{\min} = \begin{cases} \dfrac{AS^2}{200[0.6 + S(\tan 1°)]} = \dfrac{AS^2}{120 + 3.5S} & \text{when } S \leq L \\ 2S - \dfrac{200[0.6 + S(\tan 1°)]}{A} = 2S - \dfrac{120 + 3.5S}{A} & \text{when } S \geq L \end{cases} \tag{4.7}$$

Design charts or tables are used to determine minimum length of vertical curve to provide stopping sight distance for both crest and sag vertical curves, and passing sight distance on crests. These may be found in the AASHTO *Policy on Geometric Design of Highways and Streets.*

In some cases, sag vertical curves with a small total grade change can be sharp enough to cause discomfort without violating sight distance standards. In this case, it is necessary to establish a comfort criterion of the form

$$r \leq \frac{a}{v^2} \tag{4.8}$$

where r is the rate of change of grade, a is the maximum radial acceleration permitted, and v is speed. There is no general agreement as to the maximum value of radial acceleration that can be tolerated without producing discomfort. AASHTO suggests a value of 0.3 m/s^2, and suggests the standard

$$L \geq \frac{AV^2}{395} \tag{4.9}$$

where L = length of vertical curve, m
$A = g_2 - g_1$, percent
V = design speed, km/h

Comfort standards for passenger railways are based on similar vertical accelerations.

For high-speed main tracks for freight railways, the specifications of the American Railway Engineering Association (AREA) are much more restrictive. The rate of change of grade is limited to 0.10 percent per station for crest vertical curves and 0.05 percent per station for sags. If the train is traveling down a grade steep enough to overcome rolling resistance, the couplings between the cars will be compressed. If the center of gravity of the train passes the low point of the curve while it is still compressed, the slack is suddenly pulled out of the couplers, and the resulting jerk may break the train apart. The design standard is intended to ensure that as the train approaches the low point, cars will be on grades too flat to overcome rolling resistance, and the slack will be pulled out of the couplers gradually.

For airports, rates of change of grade are limited to 0.33 percent per station on runways at airports used for high-speed aircraft. In addition, the Federal Aviation Administration (FAA) requires that for airports with no control tower, or at which the control tower is not always in operation, a 1.5 m object be visible from any point on the runway with a 1.5 m eye height.

Minimum vertical curve standards for highways may also be based on appearance. This problem arises because short vertical curves tend to look like kinks when viewed from a distance. Appearance standards vary from agency to agency. Current California standards, for instance, require a minimum vertical curve length of 60 m where grade breaks are less than 2 percent or design speeds are less than 60 km/h. Where the grade break is greater than 2 percent and the design speed is greater than 60 km/h, the minimum vertical curve is given by $L = 2V$, where L in the vertical curve length in meters and V is the design speed in km/h.

EXAMPLE PROBLEM 4.2 Determine the minimum length of a crest vertical curve between a +0.5% grade and a −1.0% grade for a road with a 100-km/h design speed. The vertical curve must provide 190-m stopping sight distance and meet the California appearance criteria. Round up to the next greatest 20 m interval.

Stopping sight distance criterion:

Assume $S \leq L$

$$L = \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{[0.5 - (-1.0)](190^2)}{200(\sqrt{1.070} + \sqrt{0.150})^2} = 134.0 \text{ m}$$

134.0 m $<$ 190 m, so $S > L$

$$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A} = 2(190) - \frac{200(\sqrt{1.070} + \sqrt{0.150})^2}{[0.5 - (-1.0)]}$$

$$= 380.0 - 269.5 = 110.5 \text{ m}$$

Appearance criterion:

Design speed = 100 km/h > 60 km/h but grade break = 1.5% < 2%. Use 60 m.

Conclusion:

Sight distance criterion governs. Use 120 m vertical curve.

EXAMPLE PROBLEM 4.3 Determine the minimum length of a sag vertical curve between a −0.7% grade and a +0.5% grade for a road with a 110 km/h design speed. The vertical curve must provide 220 m stopping sight distance and meet the California appearance criteria and the AASHTO comfort standard. Round up to the next greatest 20 m interval.

Stopping sight distance criterion:

Assume $S \leq L$

$$L = \frac{AS^2}{120 + 3.5S} = \frac{[0.5 - (-0.7)](220^2)}{120 + 3.5(220)} = 65.3 \text{ m}$$

65.3 m < 220 m, so $S > L$

$$L = 2S - \frac{120 + 3.5S}{A} = 2(220) - \frac{120 + 3.5(220)}{[0.5 - (-0.7)]}$$

$$= 440 - 741.7 = -301.7 \text{ m}$$

Since $L < 0$, no vertical curve is needed to provide stopping sight distance.

Comfort criterion:

$$L = \frac{AV^2}{395} = \frac{[0.5 - (-0.7)](110^2)}{395} = 36.8 \text{ m}$$

Appearance criterion:

Design speed = 110 km/h > 60 km/h but grade break = 1.2% < 2%. Use 60 m.

Conclusion:

Appearance criterion governs. Use 60 m vertical curve.

Finally, vertical curve lengths may be limited by the need to provide clearances over or under objects such as overpasses or drainage structures. In the case of sag vertical curves passing over objects or crest vertical curves passing under them, the required clearances establish minimum lengths; in the case of crest vertical curves passing over objects or sags passing under them, the clearances establish maximum lengths. Where clearances limit vertical curve lengths, adequate sight distance should still be provided.

In either case, the maximum or minimum length of the vertical curve may be determined by assuming that the clearance is barely met and calculating the length of the vertical curve passing through the critical point thus established. It is easiest to do this as illustrated by Figure 4.10. In the figure, C represents the critical clearance, z the horizontal distance from the P.I. to the critical point, and y' the offset between the critical point and the tangent passing through the BVC.

The equation for the offset is

$$y' = \frac{rx^2}{2} \tag{4.10}$$

where r, as before, is

$$r = \frac{g_2 - g_1}{L} = \frac{A}{L} \tag{4.11}$$

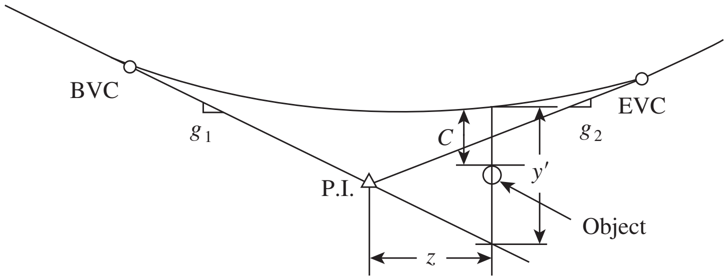


FIGURE 4.10

Calculation of lengths of vertical curves constrained by clearances.

and

$$x = \frac{L}{2} + z \tag{4.12}$$

Substituting Equations (4.9) and (4.10) into Equation (4.8) results in

$$y' = \frac{A(L/2 + z)^2}{2L} \tag{4.13}$$

Expansion and rearrangement of Equation (4.11) leads to the quadratic equation

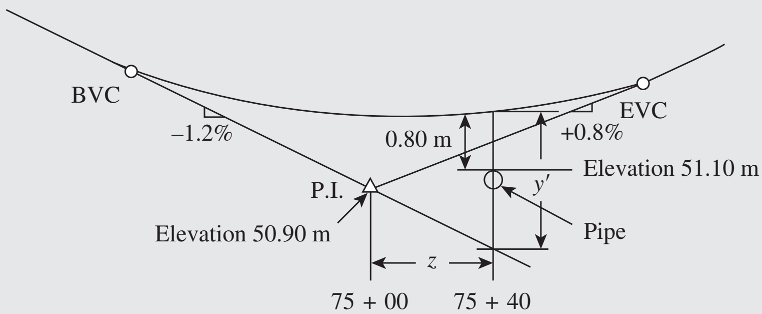
$$AL^2 + (4Az - 8y')L + 4Az^2 = 0 \tag{4.14}$$

Solving Equation (4.14) results in two roots. The smaller of these represents a vertical curve that is tangent between the P.I. and the critical point. Discarding this solution and letting $w = y'/A$ to simplify the notation, the solution for the in the larger root leads to

$$L = 4w - 2z + 4\sqrt{w^2 - wz} \tag{4.15}$$

as an expression for the maximum or minimum vertical curve length.

EXAMPLE PROBLEM 4.4 A vertical curve joins a −1.2% grade to a +0.8% grade. The P.I. of the vertical curve is at station 75 + 00 and elevation 50.90 m above sea level. The centerline of the roadway must clear a pipe located at station 75 + 40 by 0.80 m. The elevation of the top of the pipe is 51.10 m above sea level. What is the minimum length of the vertical curve that can be used?



Determine z:

$$z = (75 + 40) - (75 + 00) = 0.40 \text{ sta.}$$

Determine y'

$$\text{Elevation of tangent} = 50.90 + (-1.2)(0.4) = 50.42 \text{ m}$$

$$\text{Elevation of roadway} = 51.10 + 0.80 = 51.90 \text{ m}$$

$$y' = 51.90 - 50.42 = 1.48 \text{ m}$$

Determine w:

$$A = g_2 - g_1 = (+0.8) - (-1.2) = 2.0$$

$$w = \frac{y'}{A} = \frac{1.48}{2} = 0.74$$

Determine L:

$$L = 4w - 2z + 4\sqrt{w^2 - wz}$$

$$= 4(0.74) - 2(0.4) + 4\sqrt{0.74^2 - (0.74)(0.4)} = 4.17 \text{ sta} = 417 \text{ m}$$

Check y':

$$x = \frac{4.17}{2} + 0.4 = 2.485 \text{ sta}$$

$$r = \frac{A}{L} = \frac{2}{4.17} = 0.48$$

$$y' = \frac{rx^2}{2} = \frac{(0.48)(2.485^2)}{2} = 1.48 \qquad \textit{Check}$$

4.3 HORIZONTAL ALIGNMENT

Horizontal alignment for linear transportation facilities such as highways and railways consists of horizontal tangents, circular curves, and possibly transition curves. In the case of highways, transition curves are not always used. Figure 4.11 illustrates horizontal alignments with and without transition curves.

4.3.1 Horizontal Tangents

Horizontal tangents are described in terms of their lengths (as expressed in the stationing of the job) and their directions. Directions may be either expressed as bearings or as azimuths and are always defined in the direction of increasing station. Azimuths are expressed as angles turned clockwise from due north; bearings are expressed as angles turned either clockwise or counterclockwise from either north or south. For instance, the azimuth 280° is equivalent to the bearing north 80° west (or N80°W). Figure 4.12 illustrates azimuths and bearings.