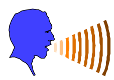
Chapter 2

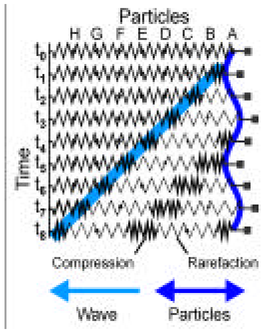
SIMPLE HARMONIC MOTION



Ch2-1

Review: Spring-Mass System

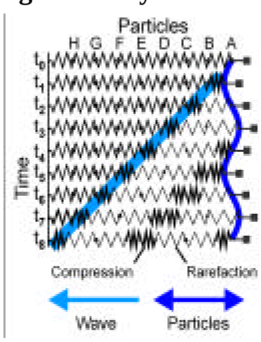
- Hooke's Law
  - Magnitude of restoring force is proportional to distance displaced
  - $F_r \propto x$
- Does the magnitude of restoring force change over time?
  - ☑ Yes. If x changes, F_r must change



Ch2-2

Review: Spring-Mass System

- Result: vibration or oscillation of system
- Magnitude of displacement (x) changes over time: 0, +, 0, -, 0, etc.
- One cycle
- Note the shape of displacement path of the mass
- System engages in simple harmonic motion



Ch2-3

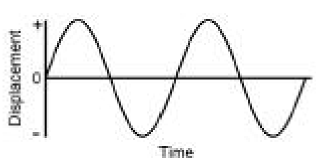
THE WAVEFORM

- Waveform shows change in <u>some quantity</u> as a function of time; e.g.,
  - Displacement (x)
  - Velocity (c)
  - Acceleration (a)
  - Force (F)
  - Pressure (p)
  - Momentum, etc. (M)

Ch2-4

THE WAVEFORM

- A plot of change in amplitude of displacement (x) over time
- The display is called the <u>time-domain waveform</u>, or <u>waveform</u>
- Air does not actually undergo this form of excursion: the waveform is a <u>representation</u>



Ch2-5

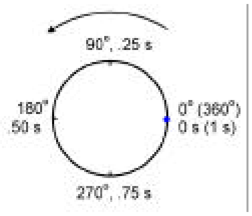
THE CONCEPT OF SIMPLE HARMONIC MOTION

- Spring-mass system undergoes simple harmonic motion
- SHM is characterized as projected uniform circular motion

Ch2-6

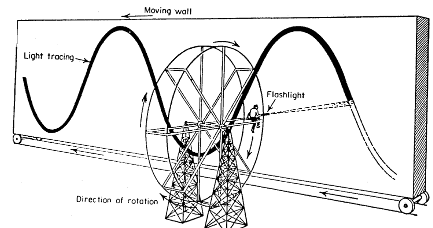
Uniform Circular Motion

- A point (dot) moves about the circumference of a circle at a constant number of degrees of rotation per second
- "Dot" engages in SHM
- One cycle equals how many degrees?
  - ☑ 360°
- But -- motion of spring-mass system is <u>rectilinear</u>, not circular



Degrees vs. Linear Displacement

- Animation F2-3
- Wall is stationary
  - Circular motion projected as rectilinear (straight-line) motion



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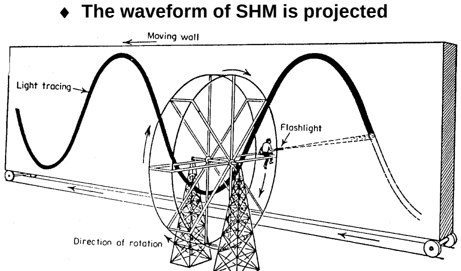
Degrees vs. Linear Displacement

- Wall moves from right to left
  - The waveform of SHM is projected



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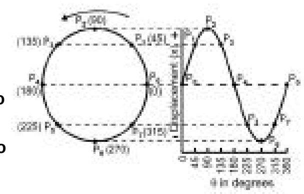
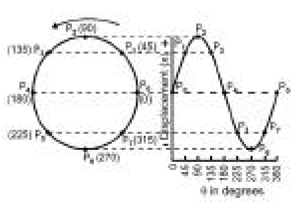
Projection of Uniform Circular Motion

- Animation F2-4
- Display is called a <u>displacement waveform</u>
- Compare what happens between 45° and 90° with what happened between 0° and 45°
  - Same # of degrees
  - Magnitude of linear displacement above baseline is different



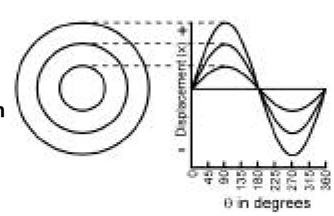
Projection of Uniform Circular Motion

- Displacement and degrees of rotation
  - 90° and 270° correspond to x_{max}
  - 0°, 180°, and 360° correspond to equilibrium
  - Rotation through 360° equals one cycle



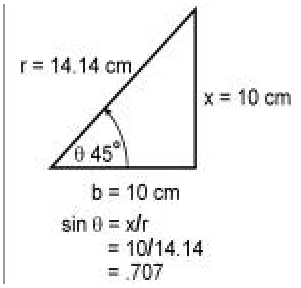
The Sine Wave

- Animation 2-5
- Projections from three wheels of different sizes
  - Each projection has the same shape
  - The unifying constant is the sine of the angle



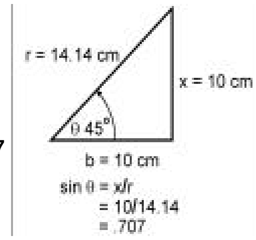
Sine of an Angle

- The ratio x / r is a <u>constant</u> for any given angle
  - x / r: the <u>sine of the angle</u>
  - b / r: the <u>cosine of the angle</u>
  - x / b: the <u>tangent of the angle</u>



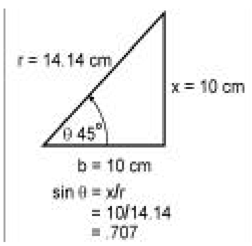
Sine of an Angle

- When x = 10 and r = 14.14, x / r = .707: sin 45^o = .707
- Suppose x = 20 and r = 28.28, but θ = 45^o: What is sin θ?
  - ☑ x / r = .707: sin 45^o = .707
- For a given angle, e.g., 45^o, sin θ is a constant regardless of the lengths of x and r



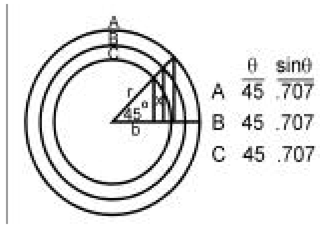
Sine of an Angle

- Suppose r remains 14.14, but x = 5.41 and θ = 22.5^o: What is sine θ?
  - ☑ x / r = .383: sin θ = .383
- Thus, sin θ is different for different angles



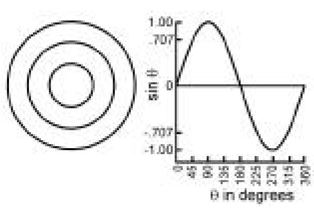
Sines of Sample Angles

<u>Angle</u>	<u>Sine</u>
0	0
45	0.707
90	1
135	0.707
180	0
225	-0.707
270	-1
315	-0.707
360	0



Displacement (x) Replaced By Sine of the Angle

- Height of each projection now is the <u>sine of the angle</u> (sin θ), not x
- Three projections now superimposed on one another. Why?
  - ☑ The ratio x / r is constant for any given angle



Construction of a Sine Wave

Table 2-1. Sines of selected angles at 11.25° intervals

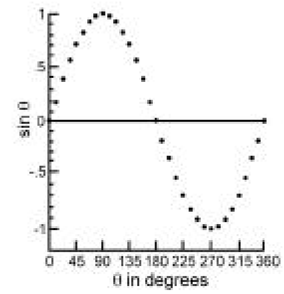
θ	sin θ	θ	sin θ	θ	sin θ	θ	sin θ
0.00	0.000						
11.25	.195	101.25	.981	191.25	-.195	281.25	-.981
22.50	.383	112.50	.924	202.50	-.383	292.50	-.924
33.75	.556	123.75	.831	213.75	-.556	303.75	-.831
45.00	.707	135.00	.707	225.00	-.707	315.00	-.707
56.25	.831	146.25	.556	236.25	-.831	326.25	-.556
67.50	.924	157.50	.383	247.50	-.924	337.50	-.383
78.75	.981	168.75	.195	258.75	-.981	348.75	-.195
90.00	1.000	180.00	.000	270.00	-1.000	360.00	0.000

- Sin θ for angles at equal intervals of 11.25^o from 0^o to 360^o

Construction of a Sine Wave

- In the next slide each value of sin θ in Table 2-1 is plotted as a function of θ: What will the function look like?
  - ☑ Sinusoidal

Construction of a Sine Wave



Summary of Sinusoidal Motion

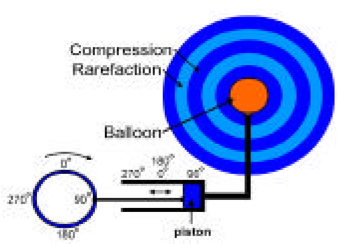
- Common element: sine of the angle
- The sine of the angle corresponds to <u>percentage of maximum displacement</u>:
  - At 22.5°, sin θ = .383: 38.3% of x_{max}
  - At 45°, sin θ = .707: 70.7% of x_{max}
  - At 90°, sin θ = 1.00: 100% of x_{max}
  - and so forth

Summary of Sinusoidal Motion

- Thus, SHM can be called sinusoidal motion
- Projection of sinusoidal motion is called a sine wave, or sinusoidal wave

Rectilinear Motion Shown as Uniform Circular Motion

- How can back & forth motion be represented as uniform circular motion?
- Animation F2-9



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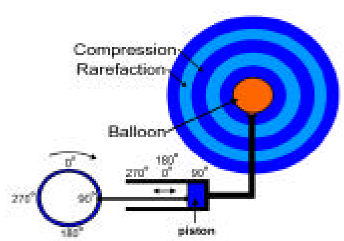
Rectilinear Motion Shown as Uniform Circular Motion

- Wheel rotates clockwise (circular motion)
  - Piston moves back and forth in cylinder (rectilinear motion)



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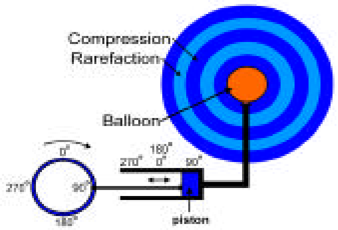
Rectilinear Motion Shown as Uniform Circular Motion

- Points along rectilinear excursion of piston labeled in degrees to correspond to points along circular excursion of wheel



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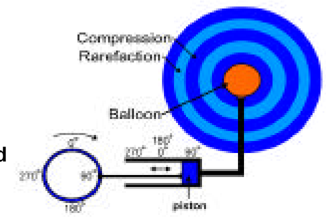
Ch2-25

Simple Harmonic Motion and Sound Waves

- At 0^o, balloon is partially inflated
- At 90^o, balloon maximally inflated
- At 270^o, balloon minimally inflated
- Compressions and rarefactions are propagated through medium
- The result is a sound wave



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Ch2-26

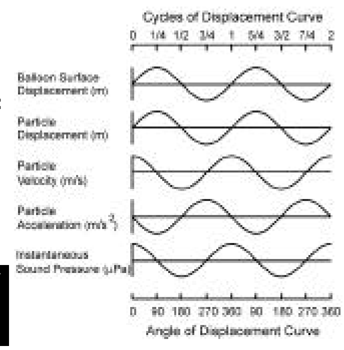
DIMENSIONS OF THE SINE WAVE

- Five dimensions of sine waves
  - Amplitude
  - Frequency
  - Period
  - Phase
  - Wavelength

Ch2-27

DIMENSIONS OF THE SINE WAVE

- (1) AMPLITUDE
- Note phasic relations among:
  - displacement
  - velocity
  - acceleration
  - pressure



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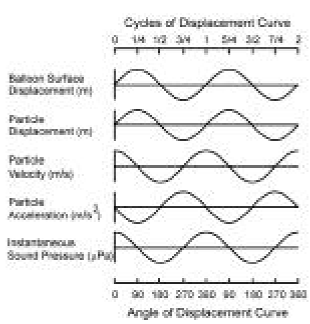
Ch2-28

Amplitude

- Particle velocity leads particle displacement by 90^o: Why?
  - ☑ c is maximal at equilibrium where x is zero; c is zero at x_{max} where motion is momentarily halted



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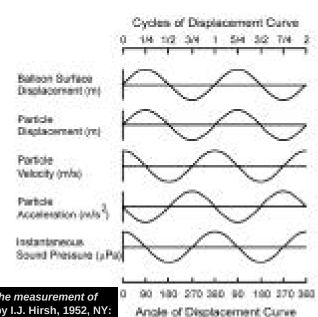
Ch2-29

Amplitude

- Particle acceleration leads particle displacement by 180^o: Why?
  - $F_i = ma$ (Newton's 2nd Law)
  - $F_r = -kx$ (Hooke's Law)
  - $F_i = F_r$ (Newton's 3rd Law)
  - $ma = -kx$



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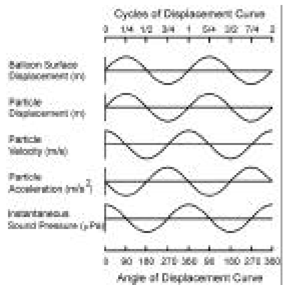
Ch2-30

Amplitude

- ♦ m and -k are constants
- ♦ a and x are variables
- ♦ a and x must be opposites (Newton's 3rd Law)
- Instantaneous sound pressure "mirrors" particle velocity and leads particle displacement by 90^{o}



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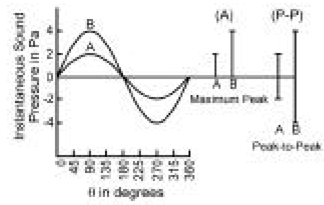
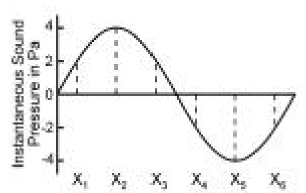
Amplitude

- a. Instantaneous Amplitude (a)
- b. Maximum Amplitude (A)
- c. Peak-to-Peak Amplitude
- d. Root-Mean-Square Amplitude (rms)



Concept of rms

- Sampling the waveform: $X_1 \ldots X_6$
- Compute standard deviation, σ
- rms is standard deviation of all instantaneous amplitudes
- rms: the square <u>root</u> (r) of the <u>mean</u> (m) of the <u>squared</u> (s) deviations of instantaneous values



Concept of rms

- rms ⟷ σ
- rms^2 ⟷ σ^2

Table 2-2. Procedures for calculating the standard deviation

X	$X - \bar{X}$	$(X - \bar{X})^2$
2.0	2.0	4.0
4.0	4.0	16.0
2.0	2.0	4.0
-2.0	-2.0	4.0
-4.0	-4.0	16.0
-2.0	-2.0	4.0

$\Sigma X = 0.0$ $\qquad \Sigma (X - \bar{X})^2 = 48.0$

$\frac{\Sigma X}{N} = 0.0$ $\qquad \sigma^2 = \frac{\Sigma (X - \bar{X})^2}{N} = 8.0$ (ms)

$\sigma = \sqrt{\frac{\Sigma (X - \bar{X})^2}{N}} = 2.828$ (rms)

Calculation of rms for a Sine Wave

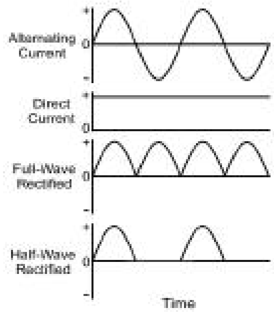
- $rms = A / \sqrt{2}$
- $rms = A / 1.414$
  $= A\,(1 / 1.414)$
- $rms = A\,(.707)$

Amplitude

- e. Mean Square
  - ♦ $rms = A / \sqrt{2}$
  - ♦ Mean Square = rms^2
  - ♦ Mean Square = $A^2 / 2$

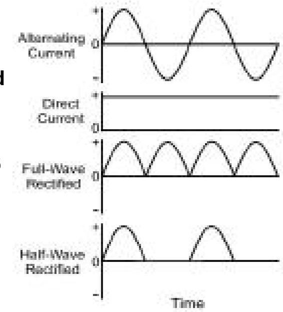
Amplitude

- f. Full-Wave Rectified Average (FW_{avg})
  - ♦ Arithmetic mean of all instantaneous amplitudes in the rectified wave
  - ♦ $FW_{avg} = 2A / \pi$
    $= 2A / 3.1416$
    $= A\ (.636)$



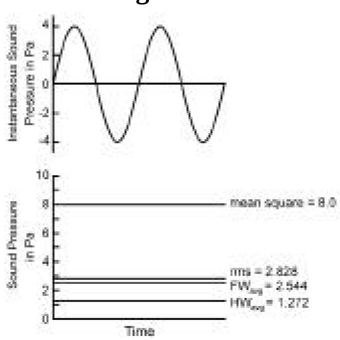
Amplitude

- g. Half-Wave Rectified Average (HW_{avg})
  - ♦ $HW_{avg} = A / \pi$
    $= A / 3.1416$
    $= A\ (.318)$



Comparison Among Metrics

- Instantaneous values vary sinusoidally over time
- Other values are time-averaged

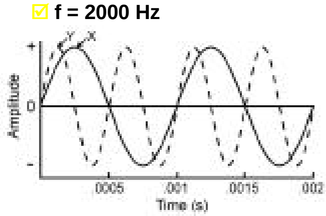


DIMENSIONS OF THE SINE WAVE

- (2) FREQUENCY (f)
  - ♦ The rate, in Hz, at which a sinusoid repeats itself
- (3) PERIOD (T)
  - ♦ The time required to complete one cycle
- $f = 1/T$; $T = 1/f$

Relation Between Frequency and Period

- T of X = .001 s: f = ?
  - ☑ f = 1000 Hz
- T of Y = .0005 s: f = ?
  - ☑ f = 2000 Hz



Units of Measure For Frequency and Period

- Frequency (f)
  - ♦ Hz to kHz: Divide by 1,000
  - ♦ kHz to Hz: Multiply by 1,000
- Period (T)
  - ♦ s to ms: Multiply by 1,000
  - ♦ ms to s: Divide by 1,000
- $f = 1/T$ and $T = 1/f$

Table 2-3. Standard units of measure for frequency and period

FREQUENCY		MULTIPLIER	PERIOD		MULTIPLIER
Hertz	(Hz)	1	second	(s)	1
Kilohertz	(kHz)	1,000	millisecond	(ms)	.001
Megahertz	(MHz)	1,000,000	microsecond	(µs)	.000001
Gigahertz	(GHz)	1,000,000,000	nanosecond	(ns)	.000000001

Determinants of Frequency

- Frequency depends on <u>properties of the source</u> of sound
- Spring-mass system: <u>mass</u> (m) and <u>stiffness</u> (s) of system

Natural Frequency (f_{nat})

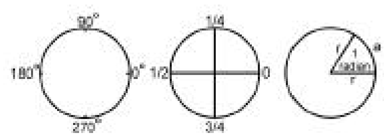
- The frequency with which a system oscillates freely (f_{nat})
- $f_{nat} = \sqrt{s/m}$
- What are the proportional relations of f_{nat} with s and m?
  - ☑ $f_{nat} \propto \sqrt{s}$
  - ☑ $f_{nat} \propto 1/\sqrt{m}$

f_{nat} of Vibrating String

- $f = 1/2L\sqrt{t/m}$
- What are the proportional relations of f_{nat} with L, t, and m?
  - ☑ $f_{nat} \propto 1/2L$
  - ☑ $f_{nat} \propto \sqrt{t}$
  - ☑ $f_{nat} \propto 1/\sqrt{m}$

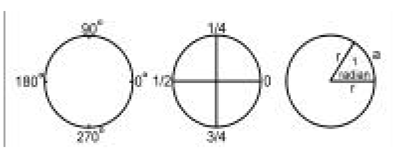
Angular Velocity (ω)

- Alternative ways to express frequency
  - degrees / s; circle divided into 360 equal parts
    >> 1 Hz = 360°/ s; 10 Hz = 3600°/ s
  - quarter cycles / s; circle divided into 4 equal parts
    >> 1 Hz = 4 quarter cycles / s;
    10 Hz = 40 quarter cycles / s



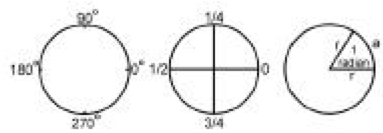
Angular Velocity (ω)

- The measure of choice
  - 2π radians / s; circle divided into 2π (6.2832) equal parts
    >> 1 Hz = 2π radians / s
- An angle = 1 radian when intersection of 2 sides of angle with C yields arc with length = to radius



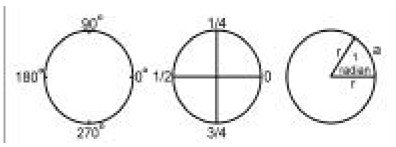
Comments on the Radian

- 1 radian = 57.3°
  - 360° / 57.3° = 2π
  - Thus, 360° = 2πr
- Snip a circle: unroll it
  - For circles of all sizes:
  - Length = 2π (6.2832) times radius of circle (2πr)



Comments on the Radian

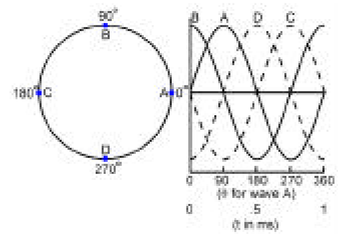
- One cycle = 360°
  - 360° = 2π radians
  - 360° / s = 2π radians / s
- ω = 2πf



Ch2-49

DIMENSIONS OF THE SINE WAVE

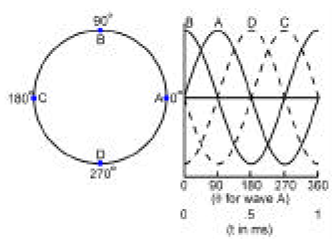
- (4) PHASE
  - Four reference points: A, B, C, & D



Ch2-50

Phase

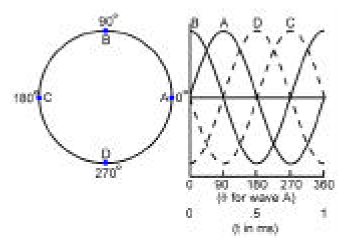
- At moment rotation begins, what is displacement in degrees for each of four points?
  - ☑ A = 0
  - ☑ B = 90
  - ☑ C = 180
  - ☑ D = 270



Ch2-51

Starting Phase

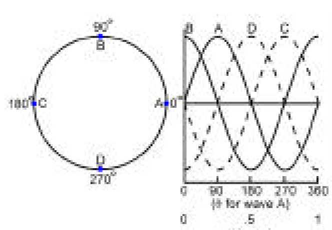
- That defines the starting phase; the angle, in degrees, at the moment rotation begins



Ch2-52

Starting Phase

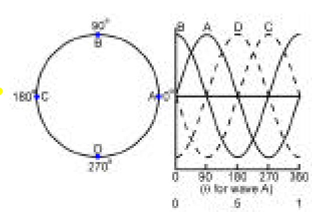
- Starting phase relations
  - B leads A by?
    - ☑ 90°
  - C leads B by?
    - ☑ 90°
  - C leads A by?
    - ☑ 180°
  - D leads B by?
    - ☑ 180°
  - B lags C by?
    - ☑ 90°



Ch2-53

Instantaneous Phase

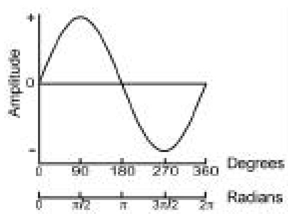
- Angle of rotation at some specified moment in time
- What are phase angles at t = .5 ms?
  - ☑ A = 180°
  - ☑ B = 270°
  - ☑ C = 0°, 360°
  - ☑ D = 90°



Ch2-54

Phase Angles in Radians

- Radians replace degrees on abscissa
- $360^{o} = 2\pi$ radians
- 0^{o} = ?
  - ☑ 0 radians
- 90^{o} = ?
  - ☑ $\pi / 2$ radians
- 180^{o} = ?
  - ☑ π radians
- 270^{o} = ?
  - ☑ $3\pi / 2$ radians

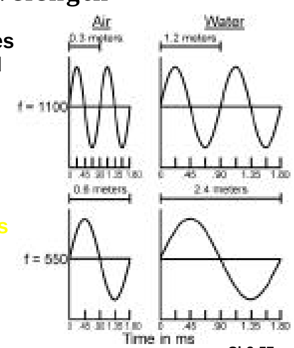


DIMENSIONS OF THE SINE WAVE

- (5) WAVELENGTH (λ)
- Two quantities are measured with respect to <u>time</u>
  - Frequency (f)
  - Speed of sound (s)

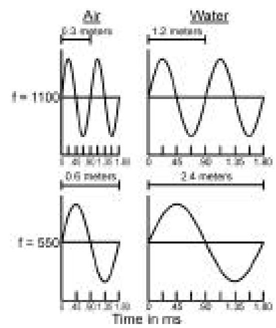
Wavelength

- Wavelength (λ) relates frequency and speed of sound
- λ = distance traveled during one period
- $\lambda = s / f$
- What are the proportional relations of λ with s and f?
  - ☑ $\lambda \propto s$
  - ☑ $\lambda \propto 1/f$



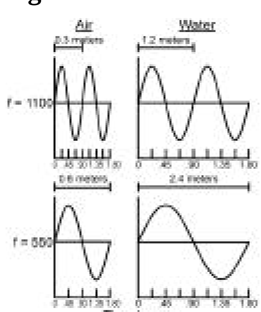
Wavelength

- Examples
  - In air: f = 1100 Hz, s = 340 m / s; λ = ?
    - ☑ $\lambda = 340 / 1100 = .3$ m
  - In air: f = 550 Hz, s = 340 m / s; λ = ?
    - ☑ $\lambda = 340 / 550 = .6$ m



Wavelength

- In water: f = 1100 Hz, s = 1360 m / s; λ = ?
  - ☑ $\lambda = 1360 / 1100 = 1.2$ m
- In water: f = 550 Hz, s = 1360 m / s; λ = ?
  - ☑ $\lambda = 1360 / 550 = 2.4$ m

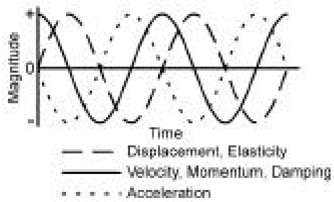


DAMPING

- Oscillating systems encounter opposition to motion: <u>friction</u>, or <u>frictional resistance</u>
- Friction limits velocity

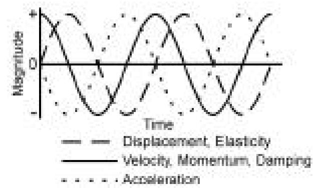
Review of SHM and Important Phasic Relations

- Displacement (Elasticity)
- Velocity (Momentum; Damping)
- Acceleration
- What are the phasic relations?



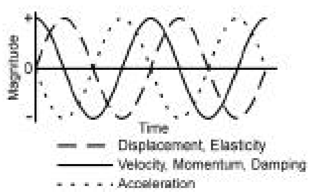
Review of SHM and Important Phasic Relations

- Learned previously that
  - c leads x by 90^{o}
  - a leads c by 90^{o}, and
  - a leads x by 180^{o}



Review of SHM and Important Phasic Relations

- In addition
  - E is in phase with x; Hooke's Law
  - M is in phase with c
  - Damping also in phase with c

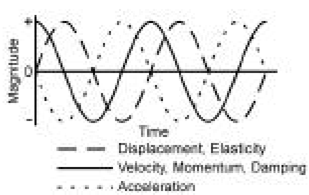


Effects of Friction on Vibratory Motion

- Friction limits velocity
- Amplitude of vibration diminishes over time
- Vibrations are damped

Effects of Friction on Vibratory Motion

- In SHM, damping varies sinusoidally over time: it is <u>in phase with velocity</u>
- As velocity increases, kinetic energy is transformed to thermal energy: system is damped

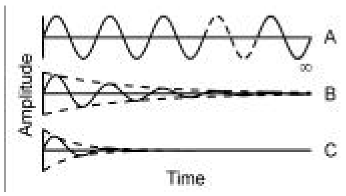


The Magnitude of Damping

- Magnitude of displacement depends on force applied
- Duration of vibration depends on magnitude of damping re: force applied

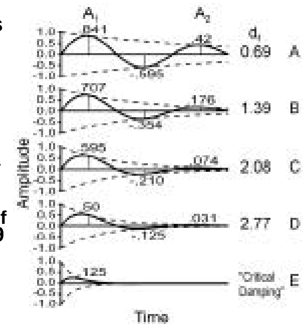
The Magnitude of Damping

- In Figure,
  - Panel A: lossless system
  - Panel B: low-damped system
  - Panel C: high-damped system



The Damping Factor

- Ratio of amplitudes of any two consecutive cycles is a constant
- $A_1 / A_2 = A_2 / A_3 = A_N / A_{N+1}$
- $d_f = \ln (A_1 / A_2)$
- From panels A-D, d_f increases from 0.69 to 2.77
- Panel E: critical damping



Examples of Damped Systems

- Shock absorbers (nearly critically damped)
- VU meter (nearly critically damped)
- What would happen if they were nearly undamped?
  - ☑ Excessive oscillation

ACOUSTIC IMPEDANCE

- System engages in SHM: it vibrates freely at its natural frequency (f_{nat})
- $f_{nat} = \sqrt{s/m}$
- What are the proportional relations of f_{nat} with s and m?
  - ☑ $f_{nat} \propto \sqrt{s}$
  - ☑ $f_{nat} \propto \sqrt{1/m}$

ACOUSTIC IMPEDANCE

- Forces exist that oppose, or impede, motion: Impedance (Z)
- Total impedance has two components:
  - resistance R
  - reactance X
    - >> mass reactance X_m
    - >> compliant reactance X_c

Resistance (R)

- Friction, or frictional resistance, occurs: kinetic energy is transformed to thermal energy
- Resistance measured in ohms (Ω)
- Resistance is independent of frequency!

Reactance (X)

- Forces that oppose motion in a frequency selective way: <u>frequency dependent</u>
- With R, <u>energy is dissipated</u>
- With X, <u>energy is stored as PE</u>

Two Components of Impedance

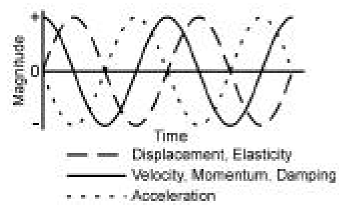
- 1. Energy-dissipating: What is it?
  - ☑ Resistance (R), which is independent of frequency
- 2. Energy-storage: What is it?
  - ☑ Reactance (X), which is dependent on frequency
- Impedance: Complex sum of R & X

Reactance

- Reactance depends on mass and compliance of the system
- Both mass and compliance oppose, or impede, motion
  - ♦ But in opposite ways
  - ♦ Can understand the difference by review of certain phasic relations

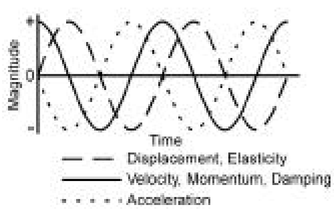
Crucial Phasic Relations

- Opposition to motion from Resistance is <u>in phase</u> with <u>velocity</u>
  - ♦ Resistance: in phase with c, M, and damping



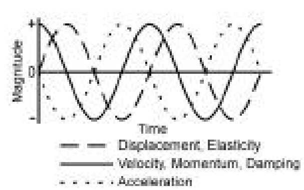
Crucial Phasic Relations

- Opposition to motion from Compliance is <u>in phase</u> with <u>elasticity</u>; <u>lags</u> Resistance by 90^o
  - ♦ Compliance: in phase with E and x



Crucial Phasic Relations

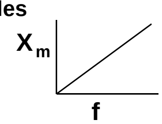
- Opposition to motion from Mass is <u>in phase</u> with <u>acceleration</u>;
  - ♦ leads resistance by 90^o
- Opposition to motion from <u>Mass</u> is 180^o <u>out of phase</u> with opposition to motion from <u>Compliance</u>



Two Components of X: X_m and X_c

- When one reactance component stores energy, the other gives up energy
- They are 180^o out of phase with one another
- They act in opposition to one another

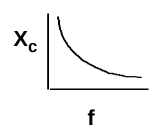
Mass Reactance: X_m

- Also measured in ohms (Ω)
- $X_m = 2\pi fm$
- X_m is directly proportional to frequency
- Negligible at low frequencies
- For every octave (2:1) increase in f, X_m doubles
- 

Mass Reactance: X_m

- At low frequencies,
  - ♦ X_m negligible; larger amplitude of vibration
- At high frequencies,
  - ♦ X_m large; smaller amplitude of vibration
- Can demonstrate with low-pass filter

Compliant Reactance: X_c

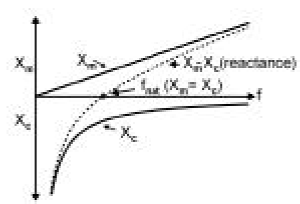
- Also measured in ohms (Ω)
- $X_c = 1/2\pi fc$
- X_c is inversely proportional to frequency
- Large at low frequencies
- 

Compliant Reactance: X_c

- At low frequencies,
  - ♦ X_c large; smaller amplitude of vibration
- At high frequencies,
  - ♦ X_c negligible; larger amplitude of vibration
- Can demonstrate with high-pass filter

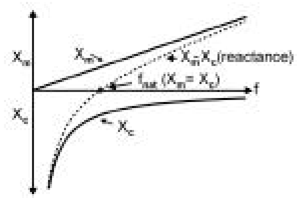
Mass Reactance (X_m) and Compliant Reactance (X_c)

- What if $X_m = X_c$?
  - ♦ If $X_m = X_c$, X = 0
  - ♦ Z = R
  - ♦ Impedance is minimal
  - ♦ Amplitude of vibration is largest
  - ♦ f_{nat}



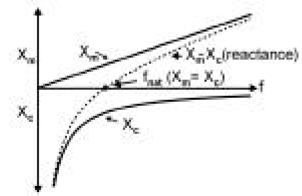
Mass Reactance (X_m) and Compliant Reactance (X_c)

- $f < f_{nat}$
  - Z increases
  - Amplitude of vibration decreases
  - Compliance dominant ($X_c = 1/2\pi fc$)



Mass Reactance (X_m) and Compliant Reactance (X_c)

- $f > f_{nat}$
  - Z increases
  - Amplitude of vibration decreases
  - Mass dominant ($X_m = 2\pi fm$)

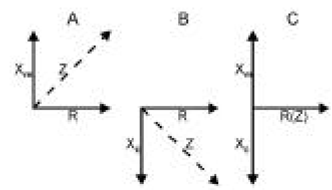


Impedance (Z)

- R causes energy to be dissipated
- X causes energy to be stored as PE
  - X_m leads R by 90°
  - X_c lags R by 90°
  - X_m leads X_c by 180°

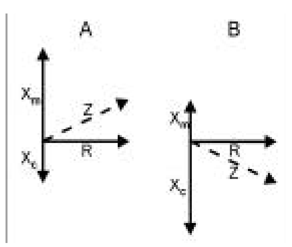
Impedance

- $X_m = X_c$
- X_m, X_c, & R are vector-like quantities
- Called <u>phasor quantities</u>, or <u>phasors</u>
  - Z = R
  - f_{nat}



Impedance

- Panel A: ($X_m > X_c$)
  - mass dominant
  - Z > R
- Panel B: ($X_m < X_c$)
  - compliance dominant
  - Z > R



Impedance

- $Z = \sqrt{R^2 + X_m^2}$
- $Z = \sqrt{R^2 + X_c^2}$
- $Z = \sqrt{R^2 + (X_m - X_c)^2}$