1

Chapter 2

SIMPLE HARMONIC MOTION

2 **Review: Spring-Mass System**

- **Hooke's Law**
  - **Magnitude of restoring force is proportional to distance displaced**
  - **$F_r \propto x$**
- **Does the magnitude of restoring force change over time?**
  - ☑ **Yes. If x changes, F_r must change**

3 **Review: Spring-Mass System**

- **Result: vibration or oscillation of system**
- **Magnitude of displacement (x) changes over time: 0, +, 0, -, 0, etc.**
- **One cycle**
- **Note the shape of displacement path of the mass**
- **System engages in simple harmonic motion**

4 **THE WAVEFORM**

- **Waveform shows change in <u>some</u> <u>quantity</u> as a function of time; e.g.,**
  - **Displacement (x)**
  - **Velocity (c)**
  - **Acceleration (a)**
  - **Force (F)**
  - **Pressure (p)**
  - **Momentum, etc. (M)**

5 **THE WAVEFORM**

- **A plot of change in amplitude of displacement (x) over time**
- **The display is called the <u>time-domain waveform</u>, or <u>waveform</u>**
- **Air does not actually undergo this form of excursion: the waveform is a <u>representation</u>**

6 **THE CONCEPT OF SIMPLE HARMONIC MOTION**

- **Spring- mass system undergoes simple harmonic motion**
- **SHM is characterized as projected uniform circular motion**

7 **Uniform Circular Motion**

- **A point (dot) moves about the circumference of a circle at a constant number of degrees of rotation per second**
- **"Dot" engages in SHM**
- **One cycle equals how many degrees?**
  - ☑ **360^o**
- **But -- motion of spring-mass system is <u>rectilinear</u>, not circular**

8 **Degrees vs. Linear Displacement**

- Animation F2-3
- Wall is stationary
  - Circular motion projected as rectilinear (straight-line) motion

9 **Degrees vs. Linear Displacement**

- Wall moves from right to left
  - The waveform of SHM is projected

10 **Projection of Uniform Circular Motion**

- Animation F2- 4
- Display is called a <u>displacement waveform</u>
- Compare what happens between 45^{o} and 90^{o} with what happened between 0^{o} and 45^{o}
  - Same # of degrees
  - Magnitude of linear displacement above baseline is different

11 **Projection of Uniform Circular Motion**

- Displacement and degrees of rotation
  - 90^{o} and 270^{o} correspond to x_{max}
  - 0^{o}, 180^{o}, and 360^{o} correspond to equilibrium
  - Rotation through 360^{o} equals one cycle

12 **The Sine Wave**

- Animation 2- 5
- Projections from three wheels of different sizes
  - Each projection has the same shape
  - The unifying constant is the sine of the angle

13 **Sine of an Angle**

- The ratio x / r is a <u>constant</u> for any given angle
  - x / r: the <u>sine of the angle</u>
  - b / r: the <u>cosine of the angle</u>
  - x / b: the <u>tangent of the angle</u>

14 **Sine of an Angle**

- When x = 10 and r = 14.14, x / r = .707: sin 45^{o} = .707
- Suppose x = 20 and r = 28.28, but $\theta = 45^{o}$: What is sin θ?
  - ☑ x / r = .707: sin 45^{o} = .707
- For a given angle, e.g., 45^{o}, sin θ is a constant regardless of the lengths of x and r

15 **Sine of an Angle**

- Suppose r remains 14.14, but x = 5.41 and $\theta = 22.5^{o}$: What is sine θ?
  - ☑ x / r = .383: sin θ = .383
- Thus, sin θ is different for different angles

16 **Sines of Sample Angles**

Angle	Sine
0	0
45	0.707

90	1
135	0.707
180	0
225	-0.707
270	-1
315	-0.707
360	0

17 **Displacement (x) Replaced By Sine of the Angle**

- Height of each projection now is the sine of the angle (sin θ), not x
- Three projections now superimposed on one another. Why?
  - ☑ The ratio x / r is constant for any given angle

18 **Construction of a Sine Wave**

- Sin θ for angles at equal intervals of 11.25^o from 0^o to 360^o

19 **Construction of a Sine Wave**

- In the next slide each value of sin θ in Table 2-1 is plotted as a function of θ: What will the function look like?
  - ☑ Sinusoidal

20 **Construction of a Sine Wave**

21 **Summary of Sinusoidal Motion**

- Common element: sine of the angle
- The sine of the angle corresponds to percentage of maximum displacement:
  - At 22.5^o, sin θ = .383: 38.3% of x_{max}
  - At 45^o, sin θ = .707: 70.7% of x_{max}
  - At 90^o, sin θ = 1.00: 100% of x_{max}
  - and so forth

22 **Summary of Sinusoidal Motion**

- Thus, SHM can be called sinusoidal motion
- Projection of sinusoidal motion is called a sine wave, or sinusoidal wave

23 **Rectilinear Motion Shown as Uniform Circular Motion**

- How can back & forth motion be represented as uniform circular motion?
- Animation F2- 9

24 **Rectilinear Motion Shown as Uniform Circular Motion**

- Wheel rotates clockwise (circular motion)
  - Piston moves back and forth in cylinder (rectilinear motion)

25 **Rectilinear Motion Shown as Uniform Circular Motion**

- Points along rectilinear excursion of piston labeled in degrees to correspond to points along circular excursion of wheel

26 Simple Harmonic Motion and Sound Waves

- **At 0^o, balloon is partially inflated**
- **At 90^o, balloon maximally inflated**
- **At 270^o, balloon minimally inflated**
- **Compressions and rarefactions are propagated through medium**
- **The result is a sound wave**

27 DIMENSIONS OF THE SINE WAVE

- **Five dimensions of sine waves**
  - ♦ **Amplitude**
  - ♦ **Frequency**
  - ♦ **Period**
  - ♦ **Phase**
  - ♦ **Wavelength**

28 DIMENSIONS OF THE SINE WAVE

- **(1) AMPLITUDE**
- **Note phasic relations among:**
  - ♦ **displacement**
  - ♦ **velocity**
  - ♦ **acceleration**
  - ♦ **pressure**

29 Amplitude

- **Particle velocity leads particle displacement by 90^o: Why?**
  - ☑ **c is maximal at equilibrium where x is zero; c is zero at x_{max} where motion is momentarily halted**

30 Amplitude

- **Particle acceleration leads particle displacement by 180^o: Why?**
  - ♦ **F_i = ma (Newton's 2nd Law)**
  - ♦ **F_r = -kx (Hooke's Law)**
  - ♦ **$F_i = F_r$ (Newton's 3rd Law)**
  - ♦ **ma = -kx**

31 Amplitude

  - ♦ **m and -k are constants**
  - ♦ **a and x are variables**
  - ♦ **a and x must be opposites (Newton's 3rd Law)**
- **Instantaneous sound pressure "mirrors" particle velocity and leads particle displacement by 90^o**

32 Amplitude

- **a. Instantaneous Amplitude (a)**
- **b. Maximum Amplitude (A)**
- **c. Peak-to-Peak Amplitude**

- d. Root-Mean- Square Amplitude (rms)

33 ## Concept of rms

- Sampling the waveform: $X_1 \ldots X_6$
- Compute standard deviation, σ
- rms is standard deviation of all instantaneous amplitudes
- rms: the square root (r) of the mean (m) of the squared (s) deviations of instantaneous values

34 ## Concept of rms

- rms σ
- rms^2 σ^2

35 ## Calculation of rms for a Sine Wave

- rms = $A / \sqrt{2}$
- rms = A / 1.414
  = A (1 / 1.414)
- rms = A (.707)

36 ## Amplitude

- e. Mean Square
  - rms = $A / \sqrt{2}$
  - Mean Square = rms^2
  - Mean Square = $A^2 / 2$

37 ## Amplitude

- f. Full-Wave Rectified Average (FW_{avg})
  - Arithmetic mean of all instantaneous amplitudes in the rectified wave
  - $FW_{avg} = 2A / \pi$
    = 2A / 3.1416
    = A (.636)

38 ## Amplitude

- g. Half-Wave Rectified Average (HW_{avg})
  - $HW_{avg} = A / \pi$
    = A / 3.1416
    = A (.318)

39 ## Comparison Among Metrics

- Instantaneous values vary sinusoidally over time
- Other values are time-averaged

40 ## DIMENSIONS OF THE SINE WAVE

- (2) FREQUENCY (f)

- ♦ The rate, in Hz, at which a sinusoid repeats itself
- (3) PERIOD (T)
  - ♦ The time required to complete one cycle
- $f = 1/T$; $T = 1/f$

41 **Relation Between Frequency and Period**

- T of X = .001 s: f = ?
  - ☑ f = 1000 Hz
- T of Y = .0005 s: f = ?
  - ☑ f = 2000 Hz

42 **Units of Measure For Frequency and Period**

- <u>Frequency</u> (f)
  - ♦ Hz to kHz: Divide by 1,000
  - ♦ kHz to Hz: Multiply by 1,000
- <u>Period</u> (T)
  - ♦ s to ms: Multiply by 1,000
  - ♦ ms to s: Divide by 1,000
- $f = 1/T$ and $T = 1/f$

43 **Determinants of Frequency**

- Frequency depends on <u>properties of the source</u> of sound
- Spring-mass system: <u>mass</u> (m) and <u>stiffness</u> (s) of system

44 **Natural Frequency (f_{nat})**

- The frequency with which a system oscillates freely (f_{nat})
- $f_{nat} = \sqrt{s/m}$
- What are the proportional relations of f_{nat} with s and m?
  - ☑ $f_{nat} \propto \sqrt{s}$
  - ☑ $f_{nat} \propto 1/\sqrt{m}$

45 **f_{nat} of Vibrating String**

- $f = 1/2L \sqrt{t/m}$
- What are the proportional relations of f_{nat} with L, t, and m?
  - ☑ $f_{nat} \propto 1/2L$
  - ☑ $f_{nat} \propto \sqrt{t}$
  - ☑ $f_{nat} \propto 1/\sqrt{m}$

46 **Angular Velocity (ω)**

- Alternative ways to express frequency
  - ♦ degrees / s; circle divided into 360 equal parts
    - >> 1 Hz = 360^o / s; 10 Hz = 3600^o / s
  - ♦ quarter cycles / s; circle divided into 4 equal parts
    - >> 1 Hz = 4 quarter cycles / s; 10 Hz = 40 quarter cycles / s

47 ## Angular Velocity (ω)

- The measure of choice
  - 2π radians / s; circle divided into 2π (6.2832) equal parts
    - >> 1 Hz = 2π radians / s
- An angle = 1 radian when intersection of 2 sides of angle with C yields arc with length = to radius

48 ## Comments on the Radian

- 1 radian = 57.3^o
  - $360^o / 57.3^o = 2\pi$
  - Thus, $360^o = 2\pi r$
- Snip a circle: unroll it
  - For circles of all sizes:
  - Length = 2π (6.2832) times radius of circle ($2\pi r$)

49 ## Comments on the Radian

- One cycle = 360^o
  - $360^o = 2\pi$ radians
  - 360^o / s = 2π radians / s
- $\omega = 2\pi f$

50 ## DIMENSIONS OF THE SINE WAVE

- (4) PHASE
  - Four reference points: A, B, C, & D

51 ## Phase

- At moment rotation begins, what is displacement in degrees for each of four points?
  - ☑ A = 0
  - ☑ B = 90
  - ☑ C = 180
  - ☑ D = 270

52 ## Starting Phase

- That defines the starting phase; the angle, in degrees, at the moment rotation begins

53 ## Starting Phase

- Starting phase relations
  - B leads A by?
    - ☑ 90^o
  - C leads B by?
    - ☑ 90^o
  - C leads A by?
    - ☑ 180^o
  - D leads B by?
    - ☑ 180^o
  - B lags C by?
    - ☑ 90^o

54 ## Instantaneous Phase

- Angle of rotation at some specified moment in time
- What are phase angles at t = .5 ms?
  - ☑ A = 180^o
  - ☑ B = 270^o
  - ☑ C = 0^o, 360^o
  - ☑ D = 90^o

55 ## Phase Angles in Radians

- Radians replace degrees on abscissa
- $360^o = 2\pi$ radians
- 0^o = ?
  - ☑ 0 radians
- 90^o = ?
  - ☑ $\pi / 2$ radians
- 180^o = ?
  - ☑ π radians
- 270^o = ?
  - ☑ $3\pi / 2$ radians

56 ## DIMENSIONS OF THE SINE WAVE

- (5) WAVELENGTH (λ)
- Two quantities are measured with respect to <u>time</u>
  - ♦ Frequency (f)
  - ♦ Speed of sound (s)

57 ## Wavelength

- Wavelength (λ) relates frequency and speed of sound
- λ = distance traveled during one period
- $\lambda = s / f$
- What are the proportional relations of λ with s and f?
  - ☑ $\lambda \propto s$
  - ☑ $\lambda \propto 1/f$

58 ## Wavelength

- Examples
  - ♦ In air: f = 1100 Hz, s = 340 m / s; λ = ?
    - ☑ λ = 340 / 1100 = .3 m
  - ♦ In air: f = 550 Hz, s = 340 m / s; λ = ?
    - ☑ λ = 340 / 550 = .6 m

59 ## Wavelength

- ♦ In water: f = 1100 Hz, s = 1360 m / s; λ = ?
  - ☑ λ = 1360 / 1100 = 1.2 m

- In water: f = 550 Hz, s = 1360 m / s; λ = ?
  - ☑ λ = 1360 / 550 = 2.4 m

60 **DAMPING**

- **Oscillating systems encounter opposition to motion: <u>friction</u>, or <u>frictional resistance</u>**
- **Friction limits velocity**

61 **Review of SHM and Important Phasic Relations**

- **Displacement (Elasticity)**
- **Velocity (Momentum; Damping)**
- **Acceleration**
- **What are the phasic relations?**

62 **Review of SHM and Important Phasic Relations**

- **Learned previously that**
  - **c leads x by 90°**
  - **a leads c by 90°,**

    and
  - **a leads x by 180°**

63 **Review of SHM and Important Phasic Relations**

- **In addition**
  - **E is in phase with x; Hooke's Law**
  - **M is in phase with c**
  - **Damping also in phase with c**

64 **Effects of Friction on Vibratory Motion**

- **Friction limits velocity**
- **Amplitude of vibration diminishes over time**
- **Vibrations are damped**

65 **Effects of Friction on Vibratory Motion**

- **In SHM, damping varies sinusoidally over time: it is <u>in phase with velocity</u>**
- **As velocity increases, kinetic energy is transformed to thermal energy: system is damped**

66 **The Magnitude of Damping**

- **Magnitude of displacement depends on force applied**
- **Duration of vibration depends on magnitude of damping re: force applied**

67 **The Magnitude of Damping**

- In Figure,
  - Panel A: lossless system
  - Panel B: low-damped system
  - Panel C: high-damped system

68 **The Damping Factor**

- Ratio of amplitudes of any <u>two consecutive cycles</u> is a constant
- $A_1 / A_2 \ = A_2 / A_3 \qquad = A_N / A_{N+1}$
- $d_f = \ln (A_1 / A_2)$
- From panels A-D, d_f increases from 0.69 to 2.77
- Panel E: critical damping

69 **Examples of Damped Systems**

- Shock absorbers (nearly critically damped)
- VU meter (nearly critically damped)
- What would happen if they were nearly undamped?
  - ☑ Excessive oscillation

70 **ACOUSTIC IMPEDANCE**

- System engages in SHM: it vibrates <u>freely</u> at its natural frequency (f_{nat})
- $f_{nat} = \sqrt{s/m}$
- What are the proportional relations of f_{nat} with s and m?
  - ☑ $f_{nat} \propto \sqrt{s}$
  - ☑ $f_{nat} \propto \sqrt{1/m}$

71 **ACOUSTIC IMPEDANCE**

- Forces exist that <u>oppose</u>, or <u>impede</u>, motion: Impedance (Z)
- Total impedance has <u>two components</u>:
  - resistance R
  - reactance X
    - >> mass reactance X_m
    - >> compliant reactance X_c

72 **Resistance (R)**

- Friction, or frictional resistance, occurs: kinetic energy is transformed to thermal energy
- Resistance measured in ohms (Ω)
- Resistance is <u>independent of frequency</u>!

73 **Reactance (X)**

- Forces that oppose motion in a frequency selective way: <u>frequency dependent</u>
- With R, <u>energy is dissipated</u>

- With X, energy is stored as PE

74 ## Two Components of Impedance

- 1. Energy-dissipating: What is it?
  - ☑ Resistance (R), which is independent of frequency
- 2. Energy-storage: What is it?
  - ☑ Reactance (X), which is dependent on frequency
- Impedance: Complex sum of R & X

75 ## Reactance

- Reactance depends on mass and compliance of the system
- Both mass and compliance oppose, or impede, motion
  - ♦ But in opposite ways
  - ♦ Can understand the difference by review of certain phasic relations

76 ## Crucial Phasic Relations

- Opposition to motion from Resistance is in phase with velocity
  - ♦ Resistance: in phase with c, M, and damping

77 ## Crucial Phasic Relations

- Opposition to motion from Compliance is in phase with elasticity; lags Resistance by 90^o
  - ♦ Compliance: in phase with E and x

78 ## Crucial Phasic Relations

- Opposition to motion from Mass is in phase with acceleration;
  - ♦ leads resistance by 90^o
- Opposition to motion from Mass is 180^o out of phase with opposition to motion from Compliance

79 ## Two Components of X: X_m and X_c

- When one reactance component stores energy, the other gives up energy
- They are 180^o out of phase with one another
- They act in opposition to one another

80 ## Mass Reactance: X_m

- Also measured in ohms (Ω)
- $X_m = 2\pi fm$
- X_m is directly proportional to frequency
- Negligible at low frequencies
- For every octave (2:1) increase in f, X_m doubles
- X_m

f

81 **Mass Reactance: X_m**

- At low frequencies,
  - ♦ X_m negligible; larger amplitude of vibration
- At high frequencies,
  - ♦ X_m large; smaller amplitude of vibration
- Can demonstrate with low-pass filter

82 **Compliant Reactance: X_c**

- Also measured in ohms (Ω)
- $X_c = 1/2\pi fc$
- X_c is <u>inversely proportional</u> to frequency
- Large at low frequencies
-

X_c

f

83 **Compliant Reactance: X_c**

- At low frequencies,
  - ♦ X_c large; smaller amplitude of vibration
- At high frequencies,
  - ♦ X_c negligible; larger amplitude of vibration
- Can demonstrate with high-pass filter

84 **Mass Reactance (X_m) and Compliant Reactance (X_c)**

- What if $X_m = X_c$?
  - ♦ If $X_m = X_c$, $X = 0$
  - ♦ $Z = R$
  - ♦ Impedance is minimal
  - ♦ Amplitude of vibration is largest
  - ♦ f_{nat}

85 **Mass Reactance (X_m) and Compliant Reactance (X_c)**

- $f < f_{nat}$
  - ♦ Z increases
  - ♦ Amplitude of vibration decreases
  - ♦ Compliance dominant ($X_c = 1/2\pi fc$)

86 **Mass Reactance (X_m) and Compliant Reactance (X_c)**

- $f > f_{nat}$
  - ♦ Z increases
  - ♦ Amplitude of vibration decreases
  - ♦ Mass dominant ($X_m = 2\pi fm$)

87 **Impedance (Z)**

- R causes energy to be dissipated
- X causes energy to be stored as PE

- X_m leads R by 90^o
- X_c lags R by 90^o
- X_m leads X_c by 180^o

88 **Impedance**

- $X_m = X_c$
- X_m, X_c, & R are vector-like quantities
- Called <u>phasor quantities</u>, or <u>phasors</u>
  - $Z = R$
  - f_{nat}

89 **Impedance**

- Panel A: ($X_m > X_c$)
  - mass dominant
  - $Z > R$
- Panel B: ($X_m < X_c$)
  - compliance dominant
  - $Z > R$

90 **Impedance**

- $Z = \sqrt{R^2 + X_m^2}$
- $Z = \sqrt{R^2 + X_c^2}$
- $Z = \sqrt{R^2 + (X_m - X_c)^2}$