

THE CONCEPT OF LOGARITHMS AND ANTILOGARITHMS

● Equivalent problems

- ◆ $\text{Antilog}_2 2 = 2^2 = 2 \times 2 = 4$
- ◆ $\text{Antilog}_2 3 = 2^3 = 2 \times 2 \times 2 = 8$
- ◆ $\text{Antilog}_{10} 2 = 10^2 = 10 \times 10 = 100$
- ◆ $\text{Antilog}_{10} 3 = 10^3 = 10 \times 10 \times 10 = 1,000$

● Thus, $\text{antilog}_x n = x^n$

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Interval (Linear) and Ratio (Exponential, or Logarithmic) Scales

- [Linear] Size of interval between adjacent numbers is known and is a constant
- [Linear] Size of interval is called the BASE
- [Linear] Successive units formed by adding (or subtracting) base to each number
- [Log] One unit on scale is so many times greater or less than another
- [Log] Successive units formed by multiplying (or dividing) each number by the BASE
- [Log] Successive units differ by a constant ratio, which is the BASE
- OR, The scale of numbers can be represented as the BASE raised to some power, or exponent

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Examples

● Base = 2

- ◆ $2^0 = 1$
- ◆ $2^1 = 2$
- ◆ $2^2 = 4$
- ◆ $2^3 = 8$

● The numbers on the scale differ by a constant ratio, and

- They can be expressed as the base, 2 in this case, raised to progressively higher powers

$$\text{◆ } 2^0; 2^1; 2^2; 2^3; 2^4$$

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Ratio Scale

● Defining equation for exponential series:

$$\text{◆ } X^n = \#$$

- That means the BASE x is to be used n times in multiplication

Table 3-3. Base = 2 raised to powers of 3, 4, and 5

X^n	Operation	Result
2^3	$2 \times 2 \times 2$	8
2^4	$2 \times 2 \times 2 \times 2$	16
2^5	$2 \times 2 \times 2 \times 2 \times 2$	32

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Important Facts

● Any BASE to the zero power (X^0) equals 1

$$\text{◆ } X^0 = 1 \quad \text{ie. } 2^0 = 1, 10^0 = 1, 3^0 = 1, \text{ etc}$$

● Any BASE to the first power (X^1) equals the BASE

$$\text{◆ } X^1 = X \quad \text{ie. } 2^1 = 2, 10^1 = 10, 3^1 = 3, \text{ etc}$$

● Multiplication

$$\text{◆ } x^n = \# \quad \text{ie. } 5^4 = 5 \times 5 \times 5 \times 5 = 625$$

● Division

$$\text{◆ } x^{-n} = 1 / x^n \quad \text{ie. } 2^{-2} = 1 / 2^2 = 1 / 4$$

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Laws of Exponents

● 1) Law 1: $(x^a) \cdot (x^b) = x^{a+b}$

$$\text{◆ } 2^4 \times 2^2 = 2^{4+2} = 2^6 = 64$$

$$\text{◆ } 2^{1.4} \times 2^{3.6} = 2^{1.4+3.6} = 2^5 = 32$$

● 2) Law 2: $x^a / x^b = x^{a-b}$

$$\text{◆ } 2^6 / 2^4 = 2^{6-4} = 2^2 = 4$$

$$\text{◆ } 2^{5.5} / 2^{3.5} = 2^{5.5-3.5} = 2^2 = 4$$

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Laws of Exponents

- 3) Law 3: $(x^a)^b = x^{ab}$

◆ $(10^2)^3 = (10)^{2 \cdot 3} = 10^6$

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- 1) Law 1: $(x^a) \cdot (x^b) = x^{a+b}$
 - 2) Law 2: $x^a / x^b = x^{a-b}$
 - 3) Law 3: $(x^a)^b = x^{ab}$

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Antilogs

- Form two exponential series: Base = 2 and Base = 10

Table 3-5. Two exponential series of numbers for base = 2 and base = 10

Base = 2		Base = 10	
X ⁿ	Answer	X ⁿ	Answer
2 ⁰ = ?	1	10 ⁰ = ?	1
2 ¹ = ?	2	10 ¹ = ?	10
2 ² = ?	4	10 ² = ?	100
2 ³ = ?	8	10 ³ = ?	1,000
2 ⁴ = ?	16	10 ⁴ = ?	10,000

- We ask: Xⁿ = ?

- Thus,

◆ Xⁿ = ? is the same as antilog_x n = ?

◆ 2⁵ = 32 and antilog₂ 5 = 32

◆ 10³ = 1,000 and antilog₁₀ 3 = 1,000

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Sample Problems

- Antilog₂ 6 = ?
 64
- Antilog₁₀ 5 = ?
 100,000
- Antilog₃ 2 = ?
 9

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Sample Problems

- Antilog₁₀ -2 = ?
 ◆ x⁻ⁿ = 1/xⁿ
 ◆ antilog_x -n = 1/antilog_x n
 ◆ 1/10² = .01
- Antilog₁₀ 0 = ?
 1 (x⁰ = 1)
- Antilog₁₀ 1 = ?
 10 (x¹ = x)

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LOGS AND ANTILOGS: Logs

- The exponent is not given
- What power must the BASE be raised to equal some number?
 ◆ x[?] = #
 ◆ 2[?] = 8
 ◆ 10[?] = 10,000
- Solving exponent
- Log₂ 8 = 3
- Log₁₀ 10,000 = 4

Table 3-6. Two exponential series for base = 2 and base = 10

Base = 2		Base = 10	
X ⁿ	Answer	X ⁿ	Answer
2 ⁰ = 1	0	10 ⁰ = 1	0
2 ¹ = 2	1	10 ¹ = 10	1
2 ² = 4	2	10 ² = 100	2
2 ³ = 8	3	10 ³ = 1,000	3
2 ⁴ = 16	4	10 ⁴ = 10,000	4

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Bases for Logs and Antilogs

- The BASE must be specified
- Any number (other than 1) can be the BASE
- Three BASES are ordinarily encountered
 - ◆ 2
 - ◆ 10 (Common or Briggsian Log)
 - ◆ e (2.718: Natural or Napierian Log)

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Summary

- When you take the log of a number, you are solving for an exponent
 - ◆ An exponent is a log
 - ◆ A log is an exponent

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Scientific Notation

- A number written in scientific notation is expressed as the product of a coefficient and the base 10 raised to some power
- The coefficient
 - ◆ A number equal to or greater than 1.00, but
 - ◆ less than 10 (e.g., 1.0000; 3.14159; 9.9999)

10	1.00×10^1
100	1.00×10^2
1,000	1.00×10^3
121	1.21×10^2
800	8.00×10^2
0.1	1.00×10^{-1}
0.0121	1.21×10^{-2}

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Procedure

- Move decimal point leftward (successive division by 10) or rightward (successive multiplication by 10) until requirements for coefficient are met
- The number of places moved specifies the value of the exponent
 - ◆ Successive division: + exponent
 - ◆ Successive multiplication: - exponent

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Examples

- $100 = 1.00 \times 10^2$
- $200 = 2.00 \times 10^2$
- $300 = 3.00 \times 10^2$
- $315 = 3.15 \times 10^2$
- $0.125 = ?$
 - ☑ 1.25×10^{-1}
- $0.0367 = ?$
 - ☑ 3.67×10^{-2}
- $0.000001 = ?$
 - ☑ $1 \times 10^{-6} = 10^{-6}$

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Examples

- $100 = 10^2$ (or) 1×10^2
- $1,000 = 10^3$ (or) 1×10^3
- $200 = ?$
 - ☑ 2×10^2
- $2,000 = ?$
 - ☑ 2×10^3

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Logs With Bases Other Than 10

- $\log_Y X = \log_{10} X / \log_{10} Y$
 - ◆ $\log_2 8 = \log_{10} 8 / \log_{10} 2$
 - ◆ $= .9031 / .3010$
 - ◆ $= 3$

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Laws of Logarithms

- 1. Law 1: $\text{Log } ab = \text{Log } a + \text{Log } b$
 - ◆ $\text{Log } (10 \times 10) = ?$
 - ◆ $\text{Log } 10 + \text{Log } 10 = 2$
- 1. Law 1: $\text{Log } a / b = \text{Log } a - \text{Log } b$
 - ◆ $\text{Log } (10 / 10) = ?$
 - ◆ $\text{Log } 10 - \text{Log } 10 = 0$

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Laws of Logarithms

- 1. Law 1: $\text{Log } 145 = ?$
 - ◆ $\text{Log } (1.45 \times 10^2) =$
 - ◆ $\text{Log } 1.45 + \text{Log } 10^2 =$
 - ◆ $2 + .1614 = 2.1614$
- 2. Law 2:
 - ◆ $\text{Log } (100 / 10) = ?$
 - ◆ $\text{Log } 100 - \text{Log } 10 =$
 - ◆ $2 - 1 = 1$
- Law 2 will be encountered repeatedly in solution of decibel problems

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Law 2

- $\text{Log } (2.16 / 1.58) = ?$
 - ◆ $\text{Log } 2.16 - \text{Log } 1.58 =$
 - ◆ $0.3345 - 0.1987 =$
 - ◆ 0.1358
- $\text{Log } (1 \times 10^4) / (2 \times 10^2) = ?$
 - ◆ $\text{Log } 10^4 - \text{Log } (2 \times 10^2) =$
 - ◆ $4 - 2.3 = 1.7$
 - ◆ Note: in this problem we applied both Law 1 and Law 2

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Laws of Logarithms

- 3. Law 3: $\text{Log } a^b = b \text{Log } a$
 - ◆ $\text{Log } 10^3 = 3 \text{Log } 10 = 3 \times 1 = 3$
 - ◆ $\text{Log } 6^{3.5} = 3.5 \text{Log } 6 = 3.5 \times .78 = 2.72$
- This law will be used for decibels in Chapter 4
- 4. Law 4: $\text{Log } 1 / a = - \text{Log } a$
 - ◆ $\text{Log } (1 / 10) = - \text{Log } 10 = - 1$
 - ◆ $\text{Log } (1 / 12) = - \text{Log } 12 = - 1.08$
- This law will be encountered in solving problems in Chapters 5 and 8

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Logs Without Log Tables or Calculators

- $\text{Log } 1 = 0$ ● MEMORIZE
- $\text{Log } 2 = .3$ ● MEMORIZE
- $\text{Log } 3 = .48$ ● MEMORIZE
- $\text{Log } 4 =$
 - ☑ $\text{Log } 4 = \text{Log } (2 \times 2) = .60$ $(.30 + .30)$
- $\text{Log } 5 =$
 - ☑ $\text{Log } 5 = \text{Log } (10 / 2) = .70$ $(1.0 - .30)$
- $\text{Log } 6 =$
 - ☑ $\text{Log } 6 = \text{Log } (2 \times 3) = .78$ $(.30 + .48)$

Ch3-23

Logs Without Log Tables or Calculators

- $\text{Log } 7 = .85$ ● MEMORIZE
- $\text{Log } 8 =$
 - ☑ $\text{Log } 8 = \text{Log } (4 \times 2) = .90$ $(.60 + .30)$
- $\text{Log } 9 =$
 - ☑ $\text{Log } 9 = \text{Log } (3 \times 3) = .96$ $(.48 + .48)$
- $\text{Log } 10 =$
 - ☑ $\text{Log } 10 = 1$

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