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Chapter 3

LOGARITHMS
AND ANTILOGARITHMS

2 THE CONCEPT OF LOGARITHMS AND ANTILOGARITHMS

- Sample problems
  - $Antilog_2 \; 2 = ?$
  - $Antilog_2 \; 3 = ?$
  - $Antilog_{10} \; 2 = ?$
  - $Antilog_{10} \; 3 = ?$
  - $2^2 = ?$ $2^3 = ?$
  - $10^2 = ?$ $10^3 = ?$

3 Sample Problems

- Equivalent problems
  - $Antilog_2 \; 2 = 2^2 = 2 \times 2 = 4$
  - $Antilog_2 \; 3 = 2^3 = 2 \times 2 \times 2 = 8$
  - $Antilog_{10} \; 2 = 10^2 = 10 \times 10 = 100$
  - $Antilog_{10} \; 3 = 10^3 = 10 \times 10 \times 10 = 1{,}000$
- Thus, $antilog_x \; n = x^n$

4 Sample Problems

- $Antilog_{10} \; 1.7 = ?$
  - Same concept
  - $10^{1.7} = ?$
  - Simply need to learn computational procedure

5 SCALES OF MEASUREMENT

- Distinction between numerals and numbers
- Numerals
  - Symbols that label
  - e.g., X, II, IX, 6, 9, etc.
  - 6 and 9, as <u>numerals</u>, <u>cannot</u> be added or subtracted

6 SCALES OF MEASUREMENT

- Numbers
  - Symbols with fixed relation to other symbols
  - 6 and 9, as <u>numbers</u>, <u>can</u> be added or subtracted

7 ❑ **Four Scales of Measurement**

- Nominal
- Ordinal
- Interval
- Ratio

8 ❑ **(1) Nominal Scale**

- Objects are the same or different
- The letter A is different from the letter B
- The numeral 1 is different from the numeral 0
- Can sort into categories: Count the number of entries

9 ❑ **(2) Ordinal Scale**

- Two things are the same or different,

  and

- One object has more or less of some quantity (or the same as) than another
- Thus:
  - ♦ A > B> C > D > F
  - ♦ F < D< C < B < A
  - ♦ 4 > 3 > 2 > 1 > 0

10 ❑ **Ordinal Scale**

- Letters are not numbers: Cannot be added
- Numerals are not numbers either: Cannot be added

11 ❑ **Ordinal Scale**

- Even if letters are replaced by numerals:
  - ♦ A = 4, B = 3, C = 2, D = 1, F = 0
  - ♦ The numerals still cannot be added

12 ❑ **Example**

- Four men want to play a “competitive” golf match of doubles: Boomer (B), Draw (D), Slice (S), and Rake (R)
- Use driving range to establish rank order
  - ♦B > D > S > R
- Assign numerals
  - ♦B = 4, D = 3, S = 2, R = 1
  - ♦4 + 1 = 5; 3 + 2 = 5; So
  - ♦B + R vs. D + S
  - ♦D + S defeat B + R handily

13 ❑ **Example**

- **What is the fallacy?**
  - **Numerals were treated as numbers**
  - **B, D, and S were players of tournament quality**
  - **Rake tended the sand traps; had never swung a club before**
- **The size of the interval between adjacent <u>numerals</u> was not known and was not a constant**

14 (3) Interval (Linear) Scale

- **Size of interval between adjacent <u>numbers</u> is known and is a constant**
- **Size of interval is called the BASE**
- **Successive units formed by adding (or subtracting) base to each number**

15 Interval Scale

- **Because BASE is known, we can say that one object is <u>a certain number of intervals more or less than another</u>**
  - **Base = 1 5 is 1 interval > 4**
  - **Base = 2 6 is 1 interval < 8**
  - **Base = 10 20 is 3 intervals < 50**

16 (4) Ratio (Exponential, or Logarithmic) Scale

- **One unit on scale is so many <u>times</u> greater or less than another**
- **Successive units formed by multiplying (or dividing) each number by the BASE**
- **Successive units differ by a <u>constant ratio</u>, which is the BASE**

17 Ratio Scale

- **Thus,**
  - **Base = 2: 8 ÷ 4 = 2; 16 ÷ 8 = 2; etc.**
  - **Base = 10: 100 ÷ 10 = 10; 1,000 ÷ 100 = 10; etc.**
  - **Base = 1.5: 2.25 ÷ 1.5 = 1.5; 3.575 ÷ 2.25 = 1.5; etc.**
  - **Base = .1: .01 ÷ .1 = .1; .001 ÷ .01 = .1; etc.**
- **If scale values are: 1, 3, 9, 27, and 81,**
  What is the base?
  - ☑ **9 ÷ 3 = 3; 27 ÷ 9 = 3; 81 ÷ 27 = 3**
  - ☑ **Base = 3**

18 Ratio Scale

- **Why can a ratio scale be called an exponential scale?**
- **The scale of numbers can be represented as <u>the BASE raised to some power</u>, <u>or exponent</u>**

19 Examples

- **Base = 2**
  - $2^0 = 1$
  - $2^1 = 2$
  - $2^2 = 4$
  - $2^3 = 8$
  - $2^4 = 16$

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Ratio Scale

- The numbers on the scale differ by a constant ratio, and
- They can be expressed as the base, 2 in this case, raised to progressively higher powers
  - 2^0; 2^1; 2^2; 2^3; 2^4

21
Ratio Scale

- Write a parallel scale where base = 3
  - ☑ $3^0 = 1$
  - ☑ $3^1 = 3$
  - ☑ $3^2 = 9$
  - ☑ $3^3 = 27$
  - ☑ $3^4 = 81$

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Ratio Scale

- Defining equation for exponential series:
  - $x^n = \#$
- That means the BASE x is to be used n times in multiplication

23

- With either base, 2 or 10, successive entries on the scale that results differ by a constant ratio

24
Two Important Facts

- Any BASE raised to the zero power (X^0) equals 1
  - $X^0 = 1$
  - Thus, $2^0 = 1$, $10^0 = 1$, $3^0 = 1$, etc
- Any BASE raised to the first power (X^1) equals the BASE
  - $X^1 = X$
  - Thus, $2^1 = 2$, $10^1 = 10$, $3^1 = 3$, etc

25
MORE ON EXPONENTS

- Exponents specify how many times the BASE x is used in multiplication or division
- Multiplication
  - $x^n = \#$
    - >> $5^4 = 5 \times 5 \times 5 \times 5 = 625$
    - >> $5^5 = 5 \times 5 \times 5 \times 5 \times 5 = 3125$
    - >> $2^4 = ?$
      - ☑ $= 2 \times 2 \times 2 \times 2 = 16$

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MORE ON EXPONENTS

- Division

- $x^{-n} = \#$
- $x^{-n} = 1/x^n$
  - >> $2^{-2} = 1/2^2 = 1/4$
  - >> $10^{-4} = ?$
  - ☑ $1/10^4 = 1/(10 \times 10 \times 10 \times 10) =$ $1/10{,}000 = .0001$

27 ❑ **Three Facts Restated**

- $x^0 = 1$
- $x^1 = x$
- $x^{-n} = 1/x^n$
  - $5^{-4} = ?$
    - ☑ $1/(5 \times 5 \times 5 \times 5) = 1/625$

28 ❑ **Sample Problems**

- $2^6 = ?$
  - ☑ $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
  - ☑ Base of 2 used 6 times in multiplication
- $10^6 = ?$
  - ☑ $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000$
  - ☑ Base of 10 used 6 times in multiplication
- $2^{-6} = ?$
  - ☑ $2^{-6} = 1/2^6 = 1/64 = .015625$

29 ❑ **Sample Problems**

- $10^{-6} = ?$
  - ☑ $10^{-6} = 1/10^6 = 1/1{,}000{,}000 = .000001$
- $7.16^1 = ?$
  - ☑ $7.16^1 = 7.16$
- $7.16^0 = ?$
  - ☑ $7.16^0 = 1$

30 ❑ **Laws of Exponents**

- 1) Law 1: $(x^a) \bullet (x^b) = x^{a+b}$
  - $2^4 \times 2^2 = 2^{4+2} = 2^6 = 64$
  - $2^{1.4} \times 2^{3.6} = 2^{1.4+3.6} = 2^5 = 32$
- The product of some base x raised to some power, and the same base x raised to the same or different power, equals the base raised to the sum of the two powers

31 **Laws of Exponents**

- 2) Law 2: $x^a / x^b = x^{a-b}$
  - $2^6 / 2^4 = 2^{6-4} = 2^2 = 4$
  - $2^{5.5} / 2^{3.5} = 2^{5.5-3.5} = 2^2 = 4$
- The <u>ratio</u> of some base x raised to some power, to the same base raised to the same or different power, equals the base raised to the <u>difference</u> between the powers

32 **Law 2**

- More examples
  - $10^{-8} / 10^{-13} = 10^5$
  - $10^{-13} / 10^{-8} = 10^{-5}$
  - $10^{-7} / 10^{-7} = 10^0 = 1$
- Any number divided by itself must equal 1: Hence $10^0 = 1$ and, more generally, $x^0 = 1$

33 **Laws of Exponents**

- 3) Law 3: $(x^a)^b = x^{ab}$
  - $(10^2)^3 = (10)^{2 \cdot 3} = 10^6$

34 **LOGS AND ANTILOGS: Antilogs**

- Form two exponential series: Base = 2 and Base = 10
- We ask: $X^n = ?$

35 **Antilogs**

- What is 2 raised to the 0, 1, 2, 3, or 4 power?
  or
- What is 10 raised to the 0, 1, 2, 3, or 4 power?
- GENERALLY: What is the value of the base X raised to the nth power? $X^n = ?$

36 **Antilogs**

- That is the same as <u>determining the antilog of a number</u>
- Thus,
  - $X^n = ?$ is the same as $\text{antilog}_x\, n = ?$
  - $2^5 = 32$ and $\text{antilog}_2\, 5 = 32$
  - $10^3 = 1{,}000$ and $\text{antilog}_{10}\, 3 = 1{,}000$

37 **Sample Problems**

- $\text{Antilog}_2\, 6 = ?$
  - ☑ 64
- $\text{Antilog}_{10}\, 5 = ?$
  - ☑ 100,000
- $\text{Antilog}_3\, 2 = ?$
  - ☑ 9

38 **Sample Problems**

- **Antilog$_{10}$ -2 = ?**
  - $x^{-n} = 1/x^n$
  - antilog$_x$ -n = 1/antilog$_x$ n
  - $1/10^2 = .01$
- **Antilog$_{10}$ 0 = ?**
  - ☑ 1 ($x^0 = 1$)
- **Antilog$_{10}$ 1 = ?**
  - ☑ 10 ($x^1 = x$)

39 **LOGS AND ANTILOGS: Logs**

- The exponent is not given
- We ask: To what power must the BASE be raised to equal some number?
  - $x^? = \#$
  - $2^? = 8$
  - $10^? = 10{,}000$
- Solving for an exponent

40 **Taking the Log of a Number**

- $\log_2 8 = 3$ $2^? = 8$
- $\log_{10} 10{,}000 = 4$ $10^? = 10{,}000$
- $\log_{10} 1{,}000{,}000 = 6$ $10^? = 1{,}000{,}000$

41 **Taking the Log of a Number**

	Ratio Scale	Linear Scale
● $\log_{10}$ 10,000 =		4
	10:1	
● $\log_{10}$ 1,000 =		3
	10:1	
● $\log_{10}$ 100 =		2
	10:1	
● $\log_{10}$ 10 =		1
	10:1	
● $\log_{10}$ 1 =		0

42 **Taking the Log of a Number**

	Between	Thus
● $\log_{10}$ 200 =	2 & 3	2.xx
● $\log_{10}$ 700 =	2 & 3	2.xx
● $\log_{10}$ 3,000 =	3 & 4	3.xx
● $\log_{10}$ 27 =	1 & 2	1.xx
● $\log_{10}$ 5 =	0 & 1	0.xx
● $\log_{10}$ 0.5=	0 & -1	?

- We will see answer later

43 Bases for Logs and Antilogs

- The BASE <u>must be specified</u>
- Any number (other than 1) can be the BASE
- Three BASES are ordinarily encountered
  - 2
  - 10 (Common or Briggsian Log)
  - e (2.718: Natural or Naperian Log)

44 Summary

- When you take the <u>log</u> of a number, you are solving for an <u>exponent</u>
  - An <u>exponent</u> is a <u>log</u>
  - A <u>log</u> is an <u>exponent</u>

45 PROCEDURES FOR SOLVING LOG AND ANTILOG PROBLEMS: Logs

- Log_{10} 100 = ? Obvious
  - ☑ 2
- Log_{10} 1,000 = ? Obvious
  - ☑ 3
- Log_{10} 150 = ? Not so obvious
  - ☑ Between 2 and 3: 2.xxxx
  - ☑ 2.1761

46 Components (Anatomy) of a Log

Integer Decimal Values

- Log_{10} 100 = 2.000000000
- Log_{10} 1,000 = 3.000000000
- Log_{10} 150 = 2.176091259

47 Components of a Log

- Integer: the <u>characteristic</u>
- Decimals: the <u>mantissa</u>
  - Log_{10} 150 = 2.176091259
  - Characteristic = 2.
  - Mantissa = .176091259

48 How to Solve Log Problems

- Calculator
- Log table
  - If log table, it is helpful to convert <u>conventional number</u> into

scientific notation

- Scientific notation, however, is important in its own right

49 **Scientific Notation**

- A number written in scientific notation is expressed as the product of a coefficient and the base 10 raised to some power
- The coefficient
  - A number equal to or greater than 1.00, but
  - less than 10 (e.g., 1.0000; 3.14159; 9.9999)
- See Table 3-7

50 **Scientific Notation**

51 **Procedure**

- Move decimal point leftward (successive division by 10) or rightward (successive multiplication by 10) until requirements for <u>coefficient</u> are met
- The number of places moved specifies the <u>value</u> of the exponent
  - Successive division: + exponent
  - Successive multiplication: - exponent

52 **Examples**

- $100 = 1.00 \times 10^2$
- $200 = 2.00 \times 10^2$
- $300 = 3.00 \times 10^2$
- $315 = 3.15 \times 10^2$

53 **Examples**

- $100 = 10^2$ (or) 1×10^2
- $1{,}000 = 10^3$ (or) 1×10^3
- 200 = ?
  - ☑ 2×10^2
- 2,000 = ?
  - ☑ 2×10^3

54 **Scientific Notation**

- A number, in scientific notation, is the product of some simple number (the <u>coefficient</u>) and 10 raised to some <u>power</u>
- 12500 = ?
  - ☑ 1.25×10^4
- 12541 = ?
  - ☑ 1.2541×10^4, or just 1.25×10^4

55 ## Scientific Notation

- When the number, in conventional notation, is 10 or greater:
  - Count the number of places that the decimal point must be moved to the <u>left</u> to lie between 1 and 9 (successive division by 10)
- This specifies the <u>positive</u> exponent
  - $114 = 1.14 \times 10^{2}$

56 ## Scientific Notation

- When the number, in conventional notation, is less than 1:
  - Count the number of places that the decimal point must be moved to the <u>right</u> (successive multiplication by 10)
- This specifies the <u>negative</u> exponent
  - $.114 = 1.14 \times 10^{-1}$

57 ## Scientific Notation

- When the number, in conventional notation, equals 1:
  - The decimal point need not be moved
- The exponent is <u>zero</u>
  - $1.00 = 1.00 \times 10^{0}$
  - $9.00 = 9.00 \times 10^{0}$

58 ## Solution For Logs

- Three steps for solution of log problems: $\text{Log}_{10}\ 145 = ?$
- Step 1: Express number in scientific notation
  - $\text{Log}_{10}\ 145 = \log_{10} 1.45 \times 10^{2}$
- Step 2: The <u>exponent</u> in scientific notation is the <u>characteristic</u> of the log
  - $\text{Log}_{10}\ 1.45 \times 10^{2} = 2.\text{xxxx}$

59 ## Solution For Logs

- Step 3: Look up <u>mantissa</u> in a log table
  - The first two digits of coefficient indicate proper <u>row</u> in log table; the third digit specifies the correct <u>column</u>. Thus, 1.45 directs you to row 14 and column 5
  - $\text{Log}_{10}\ 1.45 \times 10^{2} = 2.1614$

60 ## Log Table

61 ## Sample Problems

- $\text{Log}_{10}\ 846 =$

☑ 2.9274

- **Log_{10} 923 =**
  - ☑ **2.9652**
- **Log_{10} 1,000 =**
  - ☑ **3.0000**

62 Sample Problems

- **Log_{10} 2,315 =**
  - ☑ **3.3646**
- **Log_{10} 1 =**
  - ☑ **0.0000**
- **NOTE: If x^0 = 1, and if an exponent is a log, then the Log_{10} 1 = 0 !**

63 Sample Problems

- **Log_{10} 2 =**
  - ☑ **0.3010 (Remember !)**
- **Log_{10} 0.0002 =**
  - ☑ **4.3**
- **NOTE: The characteristic can be positive <u>or</u> negative, but the mantissa can <u>only</u> be positive**

64 Explanation

1
- **Log 2000**
- **Log 1000**
- **Log 200**
- **Log 100**
- **Log 20**
- **Log 10**
- **Log 2**
- **Log 1**

- **Note: Each bracket spans one power of 10 and represents 1 log unit because log_{10} 10 = 1**

2
- **3.3**
- **3**
- **2.3**
- **2**
- **1.3**
- **1**
- **0.3**
- **0**

65 Explanation

1
- **Log .2**
- **Log .1**

- **Log .02**
- **Log .01**
- **Log .002**
- **Log .001**
- **Log .0002**
- **Log .0001**

- **.2 is one power of 10 <u>less</u> than 2 and therefore is 1 log unit <u>less</u> than the log of 2**
  - ♦ **0.3 - 1 = - 0.7**
- 2 **-1.3? NO! 1.3 = - 0.7**
- **1**
- **2.3**
- **2**
- **3.3**
- **3**
- **4.3**
- **4**

66 Logs With Bases Other Than 10

- **$\log_Y X = \log_{10} X / \log_{10} Y$**
  - ♦ **$\log_2 8 = \log_{10} 8 / \log_{10} 2$**
  - ♦ **$= .9031 / .3010$**
  - ♦ **$= 3$**

67 PROCEDURES FOR SOLVING LOG AND ANTILOG PROBLEMS: Antilogs

- **Same three steps used for solving log problems, but in reverse order**
  - ♦ **Antilog_{10} 2.1614 (>100; <1,000)**
- **Step 1**

.1614 is the <u>mantissa</u>. Find 1614 as cell entry in log table. The two-digit row and one-digit column designators yield the <u>coefficient</u> in scientific notation: 1.45

- ♦ **$\text{Antilog}_{10}\ 2.1614 = 1.45 \times 10^{?}$**

68 Solutions For Antilogs

- **Step 2:**

The characteristic is 2. It yields the <u>exponent</u> in scientific notation.

- ♦ **Therefore: $\text{Antilog}_{10}\ 2.1614 = 1.45 \times 10^2 = 145$**

69 Logs and Antilogs with a Calculator

- **Logs: $\text{Log}_{10}\ 167 = ?$**
  - ♦ **Enter # in key pad: 167**
  - ♦ **Press log key**
  - ♦ **Display shows 2.2227**
- **Antilogs: $\text{Antilog}_{10}\ 2.2227$**

- Enter # in key pad: 2.2227
- Press 10^x key
- Display shows 167

70 ❑ **Laws of Logarithms**

- 1. Law 1: Log ab = Log a + Log b
  - Log (10×10) = ?
  - Log 10 + Log 10 = 2
- You have encountered this law before: when a conventional number is expressed in scientific notation, you find the log of the product of a coefficient and 10^x by summing the log of the coefficient and the log of 10^x

71 ❑ **Law 1**

- Log 145 = ?
  - Log (1.45×10^2) =
  - Log 1.45 + Log 10^2 =
  - 2 + .1614 = 2.1614

72 ❑ **Laws of Logarithms**

- 2. Law 2: Log a / b = Log a - Log b
  - Log (100 / 10) = ?
  - Log 100 - Log 10 =
  - 2 - 1 = 1
- Law 2 will be encountered repeatedly in solution of decibel problems

73 ❑ **Law 2**

- Log (2.16 / 1.58) = ?
  - Log 2.16 - Log 1.58 =
  - 0.3345 - 0.1987 =
  - 0.1358
- Log (1×10^4) / (2×10^2) = ?
  - Log 10^4 - Log (2×10^2) =
  - 4 - 2.3 = 1.7
    - Note: in this problem we applied both Law 1 and Law 2

74 ❑ **Laws of Logarithms**

- 3. Law 3: Log a^b = b Log a
  - Log 10^2 = 2 Log 10 = $2 \times 1 = 2$
  - Log 10^3 = 3 Log 10 = $3 \times 1 = 3$
  - Log $10^{2.75}$ = 2.75 Log 10 = 2.75
  - Log $6^{3.5}$ = 3.5 Log 6 = $3.5 \times .78$

= 2.72

- This law will be used to derive the equation for decibels for <u>pressure</u> from the equation for decibels for <u>intensity</u> in Chapter 4

75 Laws of Logarithms

- 4. Law 4: Log 1 / a = - Log a
  - Log (1 / 10) = - Log 10 = - 1
  - Log (1 / 12) = - Log 12 = - 1.08
- This law will be encountered in solving problems in Chapters 5 and 8

76 Logs Without Log Tables or Calculators

1
- Log 1 = 0
- Log 2 = .3
- Log 3 = .48
- Log 4 =
  - ☑ Log 4 = Log (2 × 2) = .60 (.30 + .30)
- Log 5 =
  - ☑ Log 5 = Log (10 / 2) = .70 (1.0 - .30)
- Log 6 =
  - ☑ Log 6 = Log (2 × 3) = .78 (.30 + .48)

2
- MEMORIZE
- MEMORIZE
- MEMORIZE

77 Logs Without Log Tables or Calculators

1
- Log 7 = .85
- Log 8 =
  - ☑ Log 8 = Log (4 × 2) = .90 (.60 + .30)
- Log 9 =
  - ☑ Log 9 = Log (3 × 3) = .96 (.48 + .48)
- Log 10 =
  - ☑ Log 10 = 1

2
- MEMORIZE