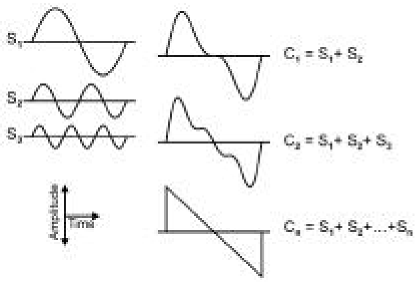
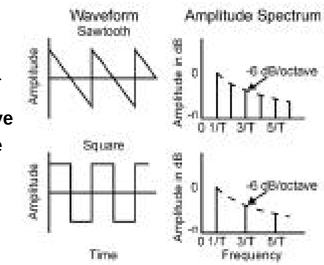
Summation of Sine Waves



1. A Sawtooth Wave

- A complex periodic wave with energy at <u>odd</u> and <u>even</u> harmonics that has a <u>spectral envelope slope</u> of - 6 dB / octave
- Amplitudes decrease as the inverse of the harmonic #
- dB = 20 $\log_{10} (1/h_i)$, where h_i is the harmonic #
- dB = - 20 $\log_{10} h_i$



Sawtooth Wave

- <u>Absolute</u> voltage of any harmonic depends on voltage of f_0!
- dB = - 20 $\log_{10} h_i$
- <u>Relative level</u>, in dB, is independent of voltage of f_0!
  - 2nd = - 20 $\log_{10}$ 2 = - 6 dB
  - 3rd = - 20 $\log_{10}$ 3 = - 9.5 dB
  - 4th = ?
    - ☑ - 20 $\log_{10}$ 4 = - 12 dB

Summary of Sawtooth Wave

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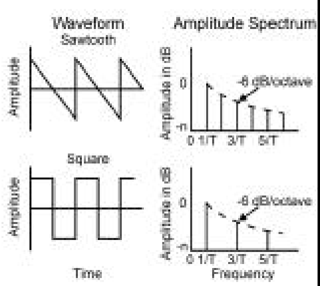
- A complex periodic wave
- Energy at odd & even integer multiples of f_0
- Spectral envelope slope of - 6 dB / octave
- dB = 20 $\log_{10} 1/h_i$ = - 20 $\log_{10} h_i$

2. Square Wave

- A complex periodic wave with energy only at <u>odd</u> integer multiples of f_0 that has a spectral envelope slope of - 6 dB / octave
- Amplitudes decrease as the inverse of the harmonic #
- dB = 20 $\log_{10} (1/h_i)$ = - 20 $\log_{10} h_i$

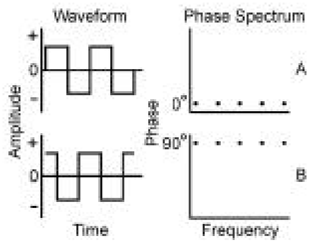


Summary of Square Wave

- Complex periodic wave
- Energy only at <u>odd</u> integer multiples of f_0
- Spectral envelope slope of - 6 dB / octave
- dB = 20 $\log_{10} 1/h_i$ = - 20 $\log_{10} h_i$

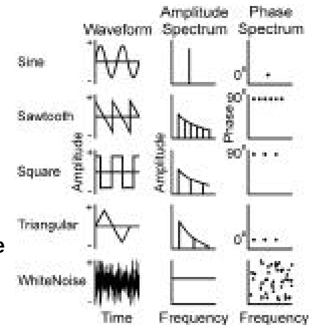
Summary of Square Wave

- What about the phase spectrum?
- Confusion among textbooks
  - Some will show the starting phase to be 0°; others as 90°
  - Each is correct, but all harmonics must have the same starting phase



3. Triangular Wave

- A complex periodic wave with energy only at <u>odd</u> harmonics
- What distinguishes the triangular wave from the square wave?
  - ☑ Slope of envelope is steeper for triangular wave



Triangular Wave

- Amplitudes decrease as the <u>reciprocal of the square</u> of the harmonic #
- dB = $20 \log_{10} 1/h_i^2$
  $= -20 \log_{10} h_i^2$
  $= -40 \log_{10} h_i$
- Why - 40?
  - ☑ Log Law 3

Table 5-4. Amplitudes (in voltage) of sinusoidal components of a **triangular** wave in which the amplitude of the fundamental frequency is 2 V

Harmonic Number	rms voltage ($1/h_i^2 \times 2$)	$-40 \log_{10} h_i$
1 (f_0)	$1/1^2 \times 2 = 2$	0
3	$1/3^2 \times 2 = .22$	-19.1
5	$1/5^2 \times 2 = .08$	-28
7	$1/7^2 \times 2 = .04$	-33.8
9	$1/9^2 \times 2 = .025$	-38.2

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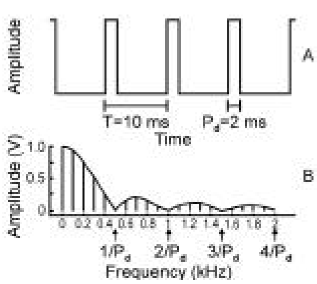
Summary of Triangular Wave

- Complex periodic wave
- Energy only at <u>odd</u> integer multiples of f_0
- Spectral envelope slope of -12 dB / octave
- dB = $20 \log_{10} 1/h_i^2 = -40 \log_{10} h_i$

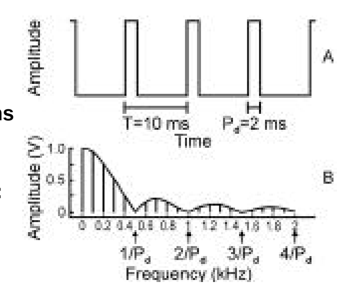
4. Pulse Train

- A repetitious series of rectangularly shaped pulses
- Each pulse has some width or duration (P_d)
- Is it periodic or aperiodic?
  - ☑ A line spectrum; it must be periodic



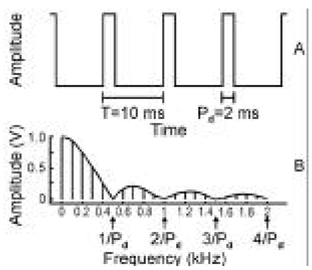
Pulse Train

- In this example, P_d = 2 ms
- The period (T) of the pulse is 10 ms
- 1/T defines the <u>pulse repetition frequency</u> (PRF): PRF = ?
  - ☑ PRF = 100 Hz



Pulse Train

- A complex periodic wave with harmonics at odd and even integer multiples of the pulse repetition frequency: 100, 200, 300, etc.
- Amplitude spectrum shows lobes and valleys (nulls)
- Nulls occur at integer multiples of reciprocal of P_d



Pulse Train

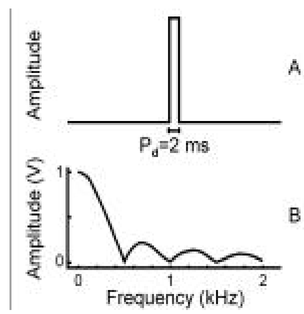
- Thus, nulls occur at $1/P_d$, $2/P_d$, $3/P_d$, etc.
  - 500 Hz, 1000 Hz, 1500 Hz
- Starting phases?
  - Below 1st null: 0^o
  - Between 1st and 2nd null: 180^o
  - Between 2nd and 3rd null: 0^o
  - and so forth

Why is White Noise Called Gaussian Noise?

- For white noise, the probability density function is a normal curve, or Gaussian distribution
- Spectral envelope slope of 0 dB/octave
- Starting phases in random array

6. A Single Pulse

- P_d = 2 ms
- Is the waveform periodic?
  - ☑ It cannot be!
  - ☑ It is aperiodic, & the amplitude spectrum must, therefore, be continuous



MEASURES OF SOUND PRESSURE FOR COMPLEX WAVES

- Different equations required for different signals
- True rms meter vs. average-responding meter
- Only the true rms meter will correctly read the rms voltage of signals other than sinusoids

Table 5-5. Measures of sound pressure for sine, square, and random waveforms. **A** refers to the peak or **maximum amplitude** as defined in Chapter 2

Metrics	Sine	Square	Random
rms	$A/\sqrt{2}$	A	~ 0.3 A
mean square	$A^2/2$	A^2	~ 0.1 A
FW_{avg}	$\frac{2}{2A}$	A	~ .25 A
peak	A	A	A

SIGNAL-TO-NOISE RATIO IN dB (dB S/N)

- It is the ratio of signal level to noise level
- dB S/N = $10 \log_{10} (I_S/I_N)$
- If S = 70 dB and N = 66 dB
  - dB S/N = 4 dB
- Why?
  - ☑ dB S/N = $10 \log_{10} (10^{-5} / 4 \times 10^{-6})$ = +4 dB
  - ☑ dB S/N = 70 - 66 = +4 dB
  - ☑ (Log Law 2)