1 # CHAPTER 5

COMPLEX WAVES

2 ## Preamble

- **The sine wave is the fundamental component of all other sound waves**
- **All waves that are not sinusoidal are complex waves**

3 ## FOURIER'S THEOREM

- **A complex wave is any sound wave that is not sinusoidal**
- **Complex waves consist of a series of simple sinusoids that can differ in amplitude, frequency, & phase**
- **This is called a Fourier series**
- **A Fourier series can be derived by a process that is called Fourier analysis**

4 ## Fourier Analysis

- **Any complex wave can be decomposed to determine the amplitudes, frequencies, and phases of the sinusoidal components**
- **All sound waves can be classified by reference to:**
  1. **Is periodicity present?**
  2. **How complex is the wave?**

5 ## PERIODIC WAVES

- **A wave that repeats itself over time**
- **Also called a periodic time function**
- **A complex periodic wave is periodic, but not sinusoidal**
  - **Panels A, C, and D are periodic waves, though panel A is not complex periodic**

6 ## Components of a Complex Periodic Wave

- **Sinusoidal components cannot be selected at random--They must satisfy an harmonic relation**
- **With an harmonic relation, each sinusoid in the series must be an integer multiple of the lowest in the series**
  - **e.g., lowest = 100 Hz: Components are?**
    - ☑ **100, 200, 300, 400, 500, etc.**
  - **e.g., lowest = 215 Hz: Components are?**
    - ☑ **215, 430, 645, 860, etc.**

7 ## Harmonic Series

- **If harmonic relation is present, the series of components is called an harmonic series**
- **Each of the components is called an harmonic**

- ♦ 1st (and f_0), 2nd, 3rd, 4th, 5th, etc.

8 ## Harmonic Series

- T = 8 ms; f_0 = 125 Hz
- What are the frequencies of the first five harmonics?
  - ☑ 1st (f_0) = 125 × 1 = 125
  - ☑ 2nd = 125 × 2 = 250
  - ☑ 3rd = 125 × 3 = 375
  - ☑ 4th = 125 × 4 = 500
  - ☑ 5th = 125 × 5 = 625

9 ## Harmonics, Partials, and Overtones

- If all components are exact integer multiples of f_0, harmonic # = partial #
- 2nd harmonic = 1st overtone

10 ## Summation of Sine Waves

- Progressive summation of 4 components with identical starting phases
- Each component is an odd integer multiple of f_0
- Compare S_1, C_1, C_2, C_3, & C_∞
- A square wave at ∞

11 ## Summation of Sine Waves

- Progressive summation of 3 components with identical starting phases
- The components are odd & even integer multiples of f_0
- Compare S_1, C_1, C_2, & C_∞
- A sawtooth wave at ∞

12 ## Summation of Sine Waves

- Summation of two components, S_1 and S_2
  - ♦ Starting phase of S_1 is held constant
  - ♦ Starting phase of S_2 varies

13 ## Summation of Sine Waves

- Note how shape of C changes with changes in starting phase of S_2
  - ♦ Panel A: 0^o
  - ♦ Panel B: 90^o
  - ♦ Panel C: 180^o
  - ♦ Panel D: 270^o

14 ## APERIODIC WAVES

- A wave that lacks periodicity
- Vibratory motion is random, & it sometimes is called a random time function
- Not necessarily "noise"

15 ## WAVEFORM AND SPECTRUM

- 1. Waveform
  - ♦ Plot of changes in some variable as a function of time
  - ♦ e.g., displacement, velocity, acceleration, pressure, etc. as a function of time

16 ## Waveform and Spectrum

- Observe the fundamental period: T = 8 ms
- Can calculate f_0 (125 Hz), but cannot easily see all frequency components, or their amplitudes or starting phases

17 ## Waveform and Spectrum

- 2. Amplitude spectrum
  - A graphic alternative to the waveform
  - Also called the amplitude spectrum in the frequency domain

18 ## Waveform and Spectrum

- Shows amplitude as a function of frequency
- The spectral envelope is given by connecting the peaks of the vertical lines: in this case the slope of the envelope is - 6 dB / octave

19 ## The Octave

- A doubling of frequency
- A frequency ratio of 2:1 or 1:2
  - 200:100
  - 500:250
  - 4000:2000
  - 100:200
  - 250:500
  - 2000:4000

20 ## Line Spectra

- In these cases, the amplitude spectrum is a line spectrum
  - Energy only at frequencies identified by vertical lines
  - Height of vertical line reflects amplitude

21 ## Continuous Spectra

- Here the amplitude spectrum is a continuous spectrum
  - Energy present at all frequencies between certain frequency limits
  - What is slope of envelope?
    - ☑ 0 dB / octave
- A slope of 0 dB / octave is not a requirement for continuous spectra

22 ## Waveform and Spectrum

- 3. Phase Spectrum
  - The phase spectrum in the frequency domain defines the starting phase as a function of frequency
  - The combination of the amplitude spectrum & the phase

spectrum defines the waveform completely in the frequency domain

23 **EXAMPLES OF COMPLEX WAVES**

- Waveforms, amplitude spectra, & phase spectra of four complex waves
- Compare each with the reference sine wave at the top

24 **EXAMPLES OF COMPLEX WAVES**

- Can you describe amplitude & phase spectra of the sine wave?
  - ☑ Amplitude spectrum: A line spectrum with energy at a single frequency
  - ☑ Phase spectrum: Starting phase is 0^o (in this example)

25 **1. A Sawtooth Wave**

- A complex periodic wave with energy at <u>odd</u> and <u>even</u> harmonics that has a <u>spectral envelope slope</u> of - 6 dB / octave
- Amplitudes decrease as the inverse of the harmonic #

26 **Sawtooth Wave**

- dB = 20 $\log_{10} (1 / h_i)$, where h_i is the harmonic #
- dB = - 20 $\log_{10} h_i$
- Why?
  - ☑ Log Law 4

27 **Sawtooth Wave**

- Voltage of f_0 = 2v
- Voltage of other harmonics: $1/h_i \times 2$
  - ♦ 2nd harmonic: $1/2 \times 2 = 1$ v
  - ♦ 3rd harmonic = ?
    - ☑ $1/3 \times 2 = .67$ v
  - ♦ 4th harmonic = ?
    - ☑ $1/4 \times 2 = .5$ v

28 **Sawtooth Wave**

- What if voltage of f_0 is 4v?
- What are voltages of 2nd, 3rd, and 4th harmonics?
  - ☑ 2nd = $1/2 \times 4 =$ 2v
  - ☑ 3rd = $1/3 \times 4 =$ 1.33v
  - ☑ 4th = $1/4 \times 4 =$ 1.0v

29 **Sawtooth Wave**

- <u>Absolute</u> voltage of any harmonic depends on voltage of f_0!
- <u>Relative level</u>, in dB, is independent of voltage of f_0!
  - ♦ 2nd = - 20 $\log_{10} 2$ = - 6 dB
  - ♦ 3rd = - 20 $\log_{10} 3$ = - 9.5 dB
  - ♦ 4th = ?
    - ☑ - 20 $\log_{10} 4$ = - 12 dB

30 **Sawtooth Wave**

- Comparison among harmonics

- ♦ 2nd re: 1st (f_0) : - 6 dB
- ♦ 4th re: 2nd : - 6 dB
- ♦ 6th re: 3rd : - 6 dB

- The spectral envelope slope is - 6 dB / octave

31 Sawtooth Wave

- What does amplitude spectrum of a sawtooth wave “look like?”
  - ♦ Depends on choice of linear vs. log scales

32 Summary of Sawtooth Wave

- A complex periodic wave
- Energy at odd & even integer multiples of f_0
- Spectral envelope slope of - 6 dB / octave
- $dB = 20 \log_{10} 1/h_i = -20 \log_{10} h_i$

33 2. Square Wave

- A complex periodic wave with energy only at <u>odd</u> integer multiples of f_0 that has a spectral envelope slope of - 6 dB / octave

34 Square Wave

- Amplitudes decrease as the inverse of the harmonic #
- $dB = 20 \log_{10} (1/h_i) = -20 \log_{10} h_i$

35 Summary of Square Wave

- Complex periodic wave
- Energy only at <u>odd</u> integer multiples of f_0
- Spectral envelope slope of - 6 dB / octave
- $dB = 20 \log_{10} 1/h_i = -20 \log_{10} h_i$

36 Summary of Square Wave

- What about the phase spectrum?
- Confusion among textbooks
  - ♦ Some will show the starting phase to be 0º; others as 90º
  - ♦ Each is correct, but all harmonics must have the same starting phase

37 3. Triangular Wave

- A complex periodic wave with energy only at <u>odd</u> harmonics
- What distinguishes the triangular wave from the square wave?
  - ☑ Slope of envelope is steeper for triangular wave

38 Triangular Wave

- Amplitudes decrease as the <u>reciprocal of the square</u> of the harmonic #
- dB= $20 \log_{10} 1/h_i^2$
  $= -20 \log_{10} h_i^2$
  $= -40 \log_{10} h_i$
- Why - 40?
  - ☑ Log Law 3

39 **Triangular Wave**

- We saw earlier that the 5th harmonic of a square or sawtooth wave was -14 dB
- But, the 5th harmonic of triangular wave is -28 dB ($20 \log_{10} 1/5^2$)
- The spectral envelope slope is -12 dB / octave

40 **Summary of Triangular Wave**

- Complex periodic wave
- Energy only at <u>odd</u> integer multiples of f_0
- Spectral envelope slope of -12 dB / octave
- $dB = 20 \log_{10} 1/h_i^2 = -40 \log_{10} h_i$

41 **4. Pulse Train**

- A repetitious series of rectangularly shaped pulses
- Each pulse has some width or duration (P_d)
- Is it periodic or aperiodic?
  - ☑ A line spectrum; it must be periodic

42 **Pulse Train**

- In this example, P_d = 2 ms
- The period (T) of the pulse is 10 ms
- 1/ T defines the <u>pulse repetition</u> <u>frequency</u> (PRF): PRF = ?
  - ☑ PRF = 100 Hz

43 **Pulse Train**

- A complex periodic wave with <u>harmonics</u> at odd and even integer multiples of the pulse repetition frequency: 100, 200, 300, etc.
- Amplitude spectrum shows <u>lobes</u> and <u>valleys</u> <u>(nulls)</u>
- <u>Nulls</u> occur at integer multiples of reciprocal of P_d

44 **Pulse Train**

- Thus, nulls occur at $1/P_d$, $2/P_d$, $3/P_d$, etc.
  - ♦ 500 Hz, 1000 Hz, 1500 Hz
- Starting phases?
  - ♦ Below 1st null: 0^o
  - ♦ Between 1st and 2nd null: 180^o
  - ♦ Between 2nd and 3rd null: 0^o
  - ♦ and so forth

45 **5. White, or Gaussian, Noise**

- An aperiodic waveform with equal energy in every frequency band 1 Hz wide: from
  - ♦ f - 0.5 Hz to f + 0.5 Hz
- Why is this noise called “white noise?”
  - ♦ Analogous to white light -- equal energy in all light wavelengths

46 **Why is White Noise Called Gaussian Noise?**

- A random time function can be described by a <u>cumulative probability distribution</u> (left)
- A plot of the changing slope of a cumulative probability distribution is called a <u>probability density function</u> (right)

47 **Why is White Noise Called Gaussian Noise?**

- For white noise, the probability density function is a <u>normal curve</u>, or Gaussian distribution
- Spectral envelope slope of 0 dB / octave
- Starting phases in random array

48
6. A Single Pulse

- P_d = 2 ms
- Is the waveform periodic?
  - ☑ It cannot be!
  - ☑ It is <u>aperiodic</u>, & the amplitude spectrum must, therefore, be <u>continuous</u>

49
Pulse Train (Revisited)

1
- T (ms)
  - 10
  - 20
  - 40
  - 80
  - 160
  - 320
  - ∞

- At T = ∞, PRF = 0 and the spacing between harmonics = 0
- The result is a continuous spectrum
- Nulls still occur at $1/P_d$, $2/P_d$, $3/P_d$, etc.

2
- PRF (Hz)
  - 100
  - 50
  - 25
  - 12.5
  - 6.25
  - 3.125
  - 0

50 **MEASURES OF SOUND PRESSURE FOR COMPLEX WAVES**

- Different equations required for different signals
- True rms meter vs. average-responding meter
- Only the true rms meter will correctly read the rms voltage of signals other than sinusoids

51 **SIGNAL-TO-NOISE RATIO IN dB (dB S / N)**

- It is the ratio of signal level to noise level
- dB S / N = $10 \log_{10} (I_S / I_N)$

- If S = 70 dB and N = 66 dB
  - ♦dB S / N = 4 dB
- Why?
  - ☑ dB S / N = $10 \log_{10} (10^{-5} / 4 \times 10^{-6})$
    = + 4 dB
  - ☑ dB S / N = 70 - 66
    = + 4 dB
  - ☑ (Log Law 2)