1 THE CONCEPT OF LOGARITHMS AND ANTILOGARITHMS

- **Equivalent problems**
  - **$\text{Antilog}_2\ 2 = 2^2 = 2 \times 2 = 4$**
  - **$\text{Antilog}_2\ 3 = 2^3 = 2 \times 2 \times 2 = 8$**
  - **$\text{Antilog}_{10}\ 2 = 10^2 = 10 \times 10 = 100$**
  - **$\text{Antilog}_{10}\ 3 = 10^3 = 10 \times 10 \times 10 = 1{,}000$**
- **Thus, $\text{antilog}_x\ n = x^n$**

2 Interval (Linear) and Ratio (Exponential, or Logarithmic) Scales

- **[Linear] Size of interval between adjacent <u>numbers</u> is known and is a constant**
- **[Linear] Size of interval is called the BASE**
- **[Linear] Successive units formed by adding (or subtracting) base to each number**
- **[Log] One unit on scale is so many <u>times</u> greater or less than another**
- **[Log] Successive units formed by multiplying (or dividing) each number by the BASE**
- **[Log] Successive units differ by a <u>constant ratio</u>, which is the BASE**
- **OR, The scale of numbers can be represented as <u>the BASE raised to some power</u>, <u>or exponent</u>**

3 Examples

- **Base = 2**
  - $2^0 = 1$
  - $2^1 = 2$
  - $2^2 = 4$
  - $2^3 = 8$
- **The numbers on the scale differ by a constant ratio, and**
- **They can be expressed as the base, 2 in this case, raised to progressively higher powers**
  - 2^0; 2^1; 2^2; 2^3; 2^4

4 Ratio Scale

- **Defining equation for exponential series:**
  - $x^n = \#$
- **That means the BASE <u>x</u> is to be used <u>n</u> times in multiplication**

5 Important Facts

- **Any BASE to the zero power (X^0) equals 1**
  - $X^0 = 1$ ie. $2^0 = 1$, $10^0 = 1$, $3^0 = 1$, etc
- **Any BASE to the first power (X^1) equals the BASE**
  - $X^1 = X$ ie. $2^1 = 2$, $10^1 = 10$, $3^1 = 3$, etc
- **Multiplication**
  - $x^n = \#$ ie. $5^4 = 5 \times 5 \times 5 \times 5 = 625$
- **Division**
  - $x^{-n} = 1 / x^n$ ie. $2^{-2} = 1 / 2^2 = 1 / 4$

6 Laws of Exponents

- **1) Law 1: $(x^a) \bullet (x^b) = x^{a+b}$**
  - $2^4 \times 2^2 = 2^{4+2} = 2^6 = 64$
  - $2 \quad \times 2^{3.6} = 2^{1.4+3.6} \quad {}^{5} = 32$
- **2) Law 2: $x^a / x^b = x^{a-b}$**
  - $2^6 / 2^4 = 2^{6-4} = 2^2 = 4$

- $2^{5.5} / 2^{3.5} = 2^{5.5 - 3.5} = 2^2 = 4$

7 Laws of Exponents

- 3) Law 3: $(x^a)^b = x^{ab}$
  - $(10^2)^3 = (10)^{2 \cdot 3} = 10^6$

- 1) Law 1: $(x^a) \bullet (x^b) = x^{a+b}$
- 2) Law 2: $x^a / x^b = x^{a-b}$
- 3) Law 3: $(x^a)^b = x^{ab}$

8 Antilogs

- Form two exponential series: Base = 2 and Base = 10
- We ask: X^n = ?
- Thus,
  - X^n = ? is the same as $\text{antilog}_x\ n$ = ?
  - $2^5 = 32$ and $\text{antilog}_2\ 5 = 32$
  - 10^3 = 1,000 and $\text{antilog}_{10}\ 3$ = 1,000

9 Sample Problems

- $\text{Antilog}_2\ 6$ = ?
  - ☑ 64
- $\text{Antilog}_{10}\ 5$ = ?
  - ☑ 100,000
- $\text{Antilog}_3\ 2$ = ?
  - ☑ 9

10 Sample Problems

- $\text{Antilog}_{10}\ -2$ = ?
  - $x^{-n} = 1 / x^n$
  - $\text{antilog}_x\ -n = 1 / \text{antilog}_x\ n$
  - $1 / 10^2 = .01$
- $\text{Antilog}_{10}\ 0$ = ?
  - ☑ 1 ($x^0 = 1$)
- $\text{Antilog}_{10}\ 1$ = ?
  - ☑ 10 ($x^1 = x$)

11 LOGS AND ANTILOGS: Logs

- The exponent is not given
- What power must the BASE be raised to equal some number?
  - $x^?$ = #
  - $2^? = 8$
  - $10^?$ = 10,000
- Solving exponent

- $\text{Log}_2\ 8 = 3$
- Log_{10} 10,000 = 4

12 Bases for Logs and Antilogs

- The BASE **must be specified**
- Any number (other than 1) can be the BASE
- Three BASES are ordinarily encountered
  - 2
  - 10 (Common or Briggsian Log)
  - e (2.718: Natural or Naperian Log)

13 ## Summary

- When you take the log of a number, you are solving for an exponent
  - An exponent is a log
  - A log is an exponent

14 ## Scientific Notation

- A number written in scientific notation is expressed as the product of a coefficient and the base 10 raised to some power
- The coefficient
  - A number equal to or greater than 1.00, but
  - less than 10 (e.g., 1.0000; 3.14159; 9.9999)

15 ## Procedure

- Move decimal point leftward (successive division by 10) or rightward (successive multiplication by 10) until requirements for coefficient are met
- The number of places moved specifies the value of the exponent
  - Successive division: + exponent
  - Successive multiplication: - exponent

16 ## Examples

- $100 = 1.00 \times 10^2$
- $200 = 2.00 \times 10^2$
- $300 = 3.00 \times 10^2$
- $315 = 3.15 \times 10^2$

17 ## Examples

- $100 = 10^2$ (or) 1×10^2
- $1{,}000 = 10^3$ (or) 1×10^3
- 200 = ?
  - ☑ 2×10^2
- 2,000 = ?
  - ☑ 2×10^3

18 ## Logs With Bases Other Than 10

- $\log_Y X = \log_{10} X / \log_{10} Y$

- $\log_2 8 = \log_{10} 8 / \log_{10} 2$
- = .9031 / .3010
- = 3

19 ## Laws of Logarithms

- 1. Law 1: Log ab = Log a + Log b
  - Log (10 × 10) = ?
  - Log 10 + Log 10 = 2
- 1. Law 1: Log a / b = Log a - Log b
  - Log (10 / 10) = ?
  - Log 10 - Log 10 = 0

20 ## Laws of Logarithms

- 1. Law 1: Log 145 = ?
  - Log (1.45×10^2) =
  - Log 1.45 + Log 10^2 =
  - 2 + .1614 = 2.1614
- 2. Law 2:
  - Log (100 / 10) = ?
  - Log 100 - Log 10 =
  - 2 - 1 = 1
- Law 2 will be encountered repeatedly in solution of decibel problems

21 ## Law 2

- Log (2.16 / 1.58) = ?
  - Log 2.16 - Log 1.58 =
  - 0.3345 - 0.1987 =
  - 0.1358
- Log $(1 \times 10^4) / (2 \times 10^2)$ = ?
  - Log 10^4 - Log (2×10^2) =
  - 4 - 2.3 = 1.7
    - Note: in this problem we applied both Law 1 and Law 2

22 ## Laws of Logarithms

- 3. Law 3: Log a^b = b Log a
  - Log 10^3 = 3 Log 10 = 3 × 1 = 3
  - Log $6^{3.5}$ = 3.5 Log 6 = 3.5 × .78 = 2.72
- This law will be used for decibels in Chapter 4
- 4. Law 4: Log 1 / a = - Log a
  - Log (1 / 10) = - Log 10 = - 1
  - Log (1 / 12) = - Log 12 = - 1.08
- This law will be encountered in solving problems in Chapters 5 and 8

23 ## Logs Without Log Tables or Calculators

1 - Log 1 = 0
- Log 2 = .3
- Log 3 = .48
- Log 4 =
  - ☑ Log 4 = Log (2 × 2) = .60 (.30 + .30)
- Log 5 =
  - ☑ Log 5 = Log (10 / 2) = .70 (1.0 - .30)
- Log 6 =
  - ☑ Log 6 = Log (2 × 3) = .78 (.30 + .48)

2 - MEMORIZE
- MEMORIZE
- MEMORIZE

24 ❑ Logs Without Log Tables or Calculators

1 - Log 7 = .85
- Log 8 =
  - ☑ Log 8 = Log (4 × 2) = .90 (.60 + .30)
- Log 9 =
  - ☑ Log 9 = Log (3 × 3) = .96 (.48 + .48)
- Log 10 =
  - ☑ Log 10 = 1

2 - MEMORIZE