

Guía 3

Desigualdades, Simplificación, Racionalización, Ecuaciones

1. Demostrar las siguientes proposiciones

a. $(\forall a, b, c, d \in \mathbb{R})(a > b \wedge c > d \Rightarrow a + c > b + d)$

b. $(\forall x, y, z \in \mathbb{R}^+)[(x + y)(y + z)(z + x) \geq 8xyz]$

c. $(\forall a \in \mathbb{R}^+)(a + \frac{1}{a} \geq 2)$

d. $(\forall x, y \in \mathbb{R}^+)[(x + y)(x^{-1} + y^{-1}) \geq 4]$

e. $(\forall x, y \in \mathbb{R})(4x(x + y) \leq 8x^2 + y^2)$

f. $(\forall x, y \in \mathbb{R})(x < y \Rightarrow x < \frac{x + y}{2} < y)$

g. $(\forall x, y, z \in \mathbb{R})(x^2 + y^2 + z^2 \geq xy + xz + yz)$

h. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq \frac{9}{a + b + c}$

i. $(\forall x, y \in \mathbb{R}^+)(x < y \Rightarrow x < \sqrt{xy} < \frac{x + y}{2} < y)$

j. ¿Es cierto que: $(\forall x, y \in \mathbb{R})(x < y \Rightarrow x^2 < y^2)$

2. Simplifique las siguientes expresiones, indicando restricciones

a. $\frac{21x^5y^6 + 28x^4y^3 - 35x^5y^4}{7x^2y^3}$, Sol: $x^2(3xy^3 + 4 - 5xy)$

b. $\frac{x^2 + 4x}{x^2 - 16}$, Sol: $\frac{x}{x - 4}$

c. $\frac{4}{(a + 5b)(a - 4b)} + \frac{1}{(a - 4b)(a - b)} + \frac{2}{(a + 5b)(a - b)}$

d. $\frac{x^3 + y^3}{x^2 + xy + 2x + 2y}$, Sol: $\frac{x^2 - xy + y^2}{x + 2}$

e. $\frac{\frac{1}{a} + \frac{1}{b} - \frac{c}{ab}}{\left(\frac{a^2 + b^2 - c^2}{a^2 b^2} + \frac{2}{ab}\right) \frac{1}{a + b + c}}$

f. $\frac{\frac{3}{x} + \frac{2}{y}}{\frac{5}{x} - \frac{6}{y^2}}$, Sol: $\frac{y(3y + 2x)}{5y^2 - 6x}$

g. $\frac{x + y}{x^{-1} + y^{-1}}$, Sol: xy

h. $\frac{x + y + \frac{x - 2y}{2}}{x + y - \frac{x + 3y}{3}}$

i. $\frac{\frac{-1}{y - 2} + \frac{5}{x}}{\frac{3}{x} - \frac{4}{xy - 2x}}$, Sol: $-\frac{x - 5y + 10}{3y - 10}$

j. $\frac{\frac{2}{a - 3} - \frac{3}{a + 3}}{\frac{5}{a^2 - 9} - \frac{2}{a - 3}}$, Sol: $\frac{a - 15}{2a + 1}$

k. $\frac{2a - 3}{3a} + \frac{3x + 2}{10x} + \frac{x - a}{5ax}$, Sol: $\frac{29a - 24}{30a}$

l. $\frac{2x - 1}{4 - x} + \frac{x + 2}{3x - 12}$; Sol: $\frac{5(x - 1)}{3(4 - x)}$

m. $\frac{a - b}{b} + \frac{2a}{a - b} - \frac{a^3 + a^2 b}{a^2 b - b^3}$, Sol: $\frac{b}{a - b}$

n. $\left(\frac{3}{2x-4} + \frac{x-6}{2x^2-8} - \frac{1}{x+2}\right)(x^3-8)$; Sol: $x^2 + 2x + 4$

o. $\frac{\left(\frac{1}{x} + \frac{x^2}{y^3}\right) : \left(\frac{1}{x} - \frac{1}{y} + \frac{x}{y^2}\right)}{\left(\frac{x+2y}{x+y} + \frac{x}{y}\right) : \left(\frac{x+2y}{y} - \frac{x}{x+y}\right)}$; Sol: $\frac{x+y}{y}$

p. $\left[\frac{x-y - \frac{(x+y)^2}{x-y}}{x+y - \frac{x^2+y^2}{x-y}}\right] : \left[\frac{\frac{x^2}{y^2} - \frac{1-x^2}{1-y^2}}{\frac{x-1}{y+y^2} + \frac{x+1}{y-y^2}}\right]$; Sol: $\frac{4x}{x-y}$

q. $\frac{\frac{xy}{x+y} + 2x}{2y - \frac{xy}{x+y}} - \frac{\frac{xy}{x+y} - 2x}{2y + \frac{xy}{x+y}} + \frac{4xy}{4y^2 - \frac{x^2y^2}{(x+y)^2}}$, Sol: $\frac{2x(3y+2x)}{y(x+2y)}$

r. $\left(\frac{\frac{a+b}{2} - a}{\frac{a+b}{2} - b}\right)^3 - \left(\frac{\frac{a+b}{2} - 2a + b}{\frac{a+b}{2} + a - 2b}\right)$, Sol: 0

3. Reduzca las siguientes expresiones numéricas

a. $3\sqrt{8} + 2\sqrt{18}$	b. $2\sqrt{28} - 3\sqrt{63} + 8\sqrt{7}$
c. $\sqrt[3]{2} + 2\sqrt[3]{16} - \sqrt[3]{54}$	d. $5\sqrt{12} + 4\sqrt{50} + 2\sqrt{3} - 9\sqrt{32}$
e. $\frac{5\sqrt{48}}{6} - \frac{3\sqrt{12}}{4}$	f. $\frac{3\sqrt[3]{54}}{2} - \frac{5\sqrt[3]{16}}{3}$

4. Multiplique y Reduzca

a. $(7\sqrt{6})(2\sqrt{18})$	b. $(3\sqrt{6})(2\sqrt{8} - 3\sqrt{12})$
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c. $(2\sqrt{3})(5\sqrt{2} - 4\sqrt{10})$	d. $(2\sqrt[3]{6})(5\sqrt[3]{4})$
e. $(\sqrt{3} + \sqrt{4} + \sqrt{5})(\sqrt{3} + \sqrt{4} + \sqrt{5})$	f. $\sqrt{3 - \sqrt{5}}\sqrt{3 + \sqrt{5}}$
g. $(3\sqrt{x} + 5\sqrt{y})(3\sqrt{x} - 5\sqrt{y}); x, y \in \mathbb{R}_0^+$	

5. Reduzca al máximo las siguientes expresiones

a. $2\sqrt[3]{8x^2} + 5\sqrt[3]{27x^2} - 3\sqrt[3]{x^2}$, Sol: $16\sqrt[3]{x^2}$

b. $\sqrt[3]{x^5} - \frac{2\sqrt[3]{x}}{\sqrt[3]{x^{-1}}} + \sqrt[3]{\frac{8}{x^{-5}}}$, Sol: $\sqrt[3]{x^2}(3x - 2)$

c. $\sqrt{9x^{-4}y^6} + \sqrt{16x^8y^{-2}} - \sqrt[3]{8x^6y^{-3}} + \sqrt[4]{81x^{16}y^{-4}} - \sqrt[5]{\frac{-243x^{-10}}{y^{-15}}} + \sqrt[3]{\frac{1}{x^{-6}y^3}}$

6. En cada uno de los siguientes casos determine las condiciones sobre x e y de modo que J sea un número real. Luego racionalice y simplifique J .

a. $J = \frac{y}{\sqrt{x+y} - \sqrt{x}}$, Sol: $J = \sqrt{x+y} - \sqrt{x}$

b. $J = \frac{16x^2 - 81y^2}{2\sqrt{x} + 3\sqrt{y}}$, Sol: $J = (4x + 9y)(2\sqrt{x} - 3\sqrt{y})$

c. $J = \frac{4(8x - y)^5}{4\sqrt{x} - \sqrt{2y}}$, Sol: $J = 2(4\sqrt{x} + \sqrt{2y})(8x - y)^4$

d. $J = \frac{5x^3 + 20x^2}{\sqrt{(x+3)(x+2)} - \sqrt{x^2 + 6}}$, Sol: $J = \left(\sqrt{(x+3)(x+2)} + \sqrt{x^2 + 6}\right)(x+4)x$

e. $J = \frac{4x^2 + 4xy + y^2}{\sqrt[3]{x} - \sqrt[3]{x+y}}$, Sol: $J = \frac{(2x+y)^2\left(\sqrt[3]{x^2} + \sqrt[3]{x(x+y)} + \sqrt[3]{(x+y)^2}\right)}{-y}$

f. $J = \frac{18x - 6}{\sqrt[3]{4x - 2} - \sqrt[3]{-2x}}$, Sol: $J = 3\left(\sqrt[3]{(4x - 2)^2} + \sqrt[3]{(4x - 2)(-2x)} + \sqrt[3]{4x^2}\right)$

7. Resuelva en \mathbb{R} las siguientes ecuaciones:

a. $\frac{6}{x + 3} + \frac{20}{(x + 3)(x - 2)} = \frac{5}{x - 2}$, Sol: $\{x = 7\}$

b. $\frac{4}{x - 2} + \frac{x}{x + 1} = \frac{x^2 - 2}{(x - 2)(x + 1)}$, Sol: $\{x = -3\}$

c. $\frac{1}{1 + \frac{1}{1 + \frac{1}{x}}} = \frac{x}{2} + 1$, Sol: $\{x = 0\}, \left\{x = \frac{-3}{2}\right\}$

d. $\frac{1}{1 + \frac{1}{x + \frac{1}{2}}} = \frac{1}{1 + \frac{1}{1 + \frac{1}{3}}}$, Sol: $\left\{x = \frac{5}{6}\right\}$

e. $\frac{\frac{1}{3}\left(x + \frac{1}{4}\right)}{x + \frac{1}{5}} + \frac{\frac{1}{6}\left(x + \frac{1}{5}\right)}{x + \frac{1}{4}} = \frac{1}{3} + \frac{1}{6}$, Sol: $\left\{x = \frac{-3}{10}\right\}$

f. $42x^2 = 2 + 5x$, Sol: $\frac{5 \pm \sqrt{57}}{8}$

g. $8x^2 + 30x + 27 = 0$, Sol: $\left\{x = \frac{-9}{4}\right\}, \left\{x = \frac{-3}{2}\right\}$

h. $2x(x - 1) = 3(1 - x^2)$, Sol: $\left\{x = \frac{-3}{5}\right\}, \{x = 1\}$

i. $x(x + 7) = x(3 - x) - 2$, Sol: $\{x = -1\}$

j. $\frac{6 - x}{6x} = \frac{1}{x + 1}$, Sol: $\{x = -3\}, \{x = 2\}$

k. $\frac{2x+11}{x+4} - \frac{x-2}{x-4} = \frac{12}{x^2-16} + \frac{7}{2}$, Sol: $\left\{x = -\frac{8}{5}\right\}, \{x = 2\}$

l. $\frac{x^2-7}{x^2+4} = -1$, Sol. $\left\{x = \frac{\pm\sqrt{6}}{2}\right\}$

m. $2x^4 - 49x^2 - 25 = 0$, Sol. $\{x = \pm 5\}$

n. $2x - 11\sqrt{x} = -5$, Sol: $\left\{x = \frac{1}{4}\right\}, \{x = 25\}$

o. $\left(\frac{7-x}{x-2}\right)^2 = 8\left(\frac{7-x}{x-2}\right) - 15$, Sol: $\left\{x = \frac{17}{6}\right\}, \left\{x = \frac{13}{4}\right\}$

p. $(1-\sqrt{3})x^4 + 1 = (2-\sqrt{3})x^2$, Sol $\{x = \pm 1\}$

q. $\sqrt[3]{x^2-7x} + 9\sqrt[6]{x^2-7x} = 10$, Sol: $\left\{x = \frac{7 \pm \sqrt{53}}{2}\right\}$

8. Encuentre “d” de tal modo que la ecuación $\frac{2x-1}{x+2} - \frac{x+d}{x+2} = 0$ no tenga solución.