

ABSTRACT

Physical Testing of Jack-Up Footings on Sand Subjected to Torsion

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With approximately 100 terra cubic feet (TCF) of gas estimated to be available on the North-West Shelf and the large quantity of oil in that region, the use of self-elevating mobile units such as jack-ups will be in great demand. Because of their economic importance in developing these resources, their safe use and reliability is critical. Jack-ups perform drilling in water depths up to 120 m. At such depths, the magnitude of loads acting on the three legs is substantial and becomes a combined load on the foundation. These include vertical (V), horizontal (2 orthogonal H_1 and H_2), moment (2 orthogonal M_1 and M_2), and torsional (T) loads. To incorporate this, an adequate foundation model with six degrees of freedom needs to be investigated. The main aim of this thesis is to examine the vertical and combined torsion load displacement behaviour. The thesis concentrates on footings on sand in an attempt to verify an existing three-dimensional plane strain formulation (known as 'Model C' for six degrees of freedom) in the Vertical load V – Torsion T space.

This thesis presents the experimental results of various size circular footings (20, 30, 40, and 50 mm radius) when subjected to vertical and torsional loads on dense and loose silica sand. Further, other common test procedures performed to determine the sand properties, such as sieve analysis, minimum and maximum density tests, and direct shear box tests are described. The test results include pure vertical-loading tests, vertical-torsion sideswipe tests (V - T space), and a radial displacement test in the V : T plane. Based on the experimental test results, Model C for six degrees of freedom in the V - T space was experimentally verified. Various model parameters, such as yield surface size in the V : T plane, have been defined and modifications to the existing model suggested. Based on these suggestions, successful retrospective numerical simulation of the experiments was performed.

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Dear Sir

It is with great pleasure that I submit this thesis entitled “Physical Testing of Jack-Up Footings on Sand Subjected to Torsion” as a partial requirement for the double degree of Bachelor of Science and Bachelor of Engineering (Civil).

Yours sincerely

Joseph Cheong

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"For God so loved the world that he gave his one and only Son, that whoever believes in him shall not perish but have eternal life." (John 3:16)

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NOMENCLATURE

Notation used in thesis. Symbols have been placed under chapter headings.

Chapter 1:

H	horizontal load
H_1, H_2	orthogonal horizontal loads
M	moment load
M_1, M_2	orthogonal moment loads
R	radius of a circular footing
T	torsional load
u	horizontal footing displacement
u_1, u_2	orthogonal horizontal footing displacements
V	vertical load
w	vertical footing displacement
θ	rotational footing displacement
θ_1, θ_2	orthogonal rotational footing displacements
θ_t	torsional (rotational) footing displacement

Chapter 2:

a	eccentricity of yield surface
A	surface area of the footing
b_c, b_γ, b_q	base inclination factors
B	footing width or diameter
c	cohesion (also shear strength, s_u)
d	(as prefix) increment in value
d_c, d_γ, d_q	depth factors
f	yield function
f_p	factor determining limiting magnitude of vertical load as $w_p \rightarrow \infty$
g	plastic potential function
g	dimensionless shear modulus factor
g_c, g_γ, g_q	ground inclination factors
G	representative elastic shear modulus
h_0	dimension of yield surface (horizontal)
$h_{0\ peak}$	horizontal dimension for the yield surface at peak bearing capacity
H	horizontal load
H_0	maximum horizontal load
H_1, H_2	orthogonal horizontal loads
i_c, i_γ, i_q	load inclination factors
k	initial plastic stiffness
k^e	elastic vertical stiffness
k'	rate of change in association factors
k_c	dimensionless elastic stiffness factor (horizontal/moment coupling)

k_h	dimensionless elastic stiffness factor (horizontal)
k_m	dimensionless elastic stiffness factor (moment)
k_t	dimensionless elastic stiffness factor (torsion)
k_v	dimensionless elastic stiffness factor (vertical)
m_0	dimension of yield surface (moment)
M	moment load
M_1, M_2	orthogonal moment loads
N_c, N_γ, N_q	bearing capacity factors
p_a	atmospheric pressure
q	overburden (surcharge pressure = γz)
q_{ult}	ultimate bearing capacity
Q	general deviator force
r	radius at the surface of a partially penetrated conical footing
R	radius of a circular footing
s_c, s_γ, s_q	shape factors
s_{fp}	dimensionless scaling factor in the “combined” strain hardening law
t_0	dimensionless variable that determines the size of the yield surface in the $T/2R$ plane and is the value of $T/(2RV_0)$ at the maximum T
$t_{0\ peak}$	torsional dimension for the yield surface at peak bearing capacity
T	torsional load (torque)
u	horizontal footing displacement
u_1, u_2	orthogonal horizontal footing displacements
V	vertical load
V_0	maximum vertical load capacity when all other loads are not present (eg $H = 0, M = 0, T = 0$)
V_0'	maximum vertical load for the current plastic potential shape
V_{peak}	maximum vertical load bearing capacity, i.e. peak value of V_0 in strain hardening law
V_{ult}	ultimate vertical load ($= q_{ult}A$)
w	vertical footing displacement
w_p	plastic component of the vertical penetration (also w^p)
w_{pm}	value of the plastic component of the vertical penetration at the peak value of V_0 (i.e. at V_{peak})
z	depth of a foundation below the lowest adjacent surface
α	footing roughness
α_h, α_m	horizontal and moment association factor respectively
$\alpha_{h\ \infty}, \alpha_{m\ \infty}$	association factor as horizontal displacement or rotation tend to infinity respectively
β	cone apex angle
β_1	curvature factor for yield surface (low stress)
β_2	curvature factor for yield surface (high stress)
β_3	curvature factor for plastic potential (low stress)
β_4	curvature factor for plastic potential (high stress)
ϕ	friction angle
γ	effective unit weight
λ	multiplication factor determining the magnitude of plastic displacement increments
ν	Poisson ratio

θ	rotational footing displacement
θ_1, θ_2	orthogonal rotational footing displacements
θ_t	torsional (rotational) footing displacement
σ_f	failure stress
σ_g	yield stress
σ_y	first yield stress

Superscripts:

e	elastic
p	plastic

Chapter 3:

d_{50}	maximum particle size of the smallest 50 per cent of the sample
e	void ratio
e_{max}	maximum void ratio
e_{min}	minimum void ratio
G_s	specific gravity
I_d	density index (also called relative density)
I_R	relative dilatancy index
p'	mean effective stress

ϕ	friction angle
ϕ'_{cv}	critical state friction angle
ϕ'_{peak}	peak friction angle
ρ_d	dry density
σ'_n	constant normal stress
σ'_v	effective vertical stress
τ	shear stress (= horizontal load / area)
τ / σ'_n	shear stress ratio

Chapter 4:

Y	horizontal displacement
Z	vertical displacement

Chapter 5:

d	(as prefix) increment in value
d_{50}	maximum particle size of the smallest 50 per cent of the sample
f	yield function
f_p	factor determining limiting magnitude of vertical load as $w_p \rightarrow \infty$
g	plastic potential function
G	representative elastic shear modulus

h_0	dimension of yield surface (horizontal)
H	horizontal load
I_d	density index (also called relative density)
k	initial plastic stiffness
k^e	elastic vertical stiffness
k_v	dimensionless elastic stiffness factor (vertical)
M	moment load
N_γ	bearing capacity factor (self-weight)
q	overburden (surcharge pressure = γz)
Q	general deviator force
r	auxiliary parameter
R	radius of a circular footing
t_0	dimensionless variable that determines the size of the yield surface in the $T/2R$ plane and is the value of $T/(2RV_0)$ at the maximum T
$t_{0\ peak}$	torsional dimension for the yield surface at peak bearing capacity
T	torsional load
T_m	maximum value of T
V	vertical load
V_0	maximum vertical load capacity when all other loads are not present
V_0'	maximum vertical load for the current plastic potential shape
V_{peak}	maximum vertical load bearing capacity, i.e. peak value of V_0 in strain hardening law
w	vertical footing displacement
w_p	plastic component of the vertical penetration (also w^p)
w_{pm}	value of the plastic component of the vertical penetration at the peak value of V_0 (i.e. at V_{peak})
z	depth of a foundation below the lowest adjacent surface
α	footing roughness
α	angle of “theoretical” plastic displacement direction vector
α_h, α_m	horizontal and moment association factor respectively
α_t	torsion association factor
β_1	curvature factor for yield surface (low stress)
β_2	curvature factor for yield surface (high stress)
β_3	curvature factor for plastic potential (low stress)
β_4	curvature factor for plastic potential (high stress)
ϕ	internal friction angle
ϕ'_{cv}	critical state friction angle
ϕ'_{peak}	peak friction angle
γ	effective unit weight
η	angle of “experimental” plastic displacement ratio (Eqn 5.21)
λ	multiplication factor determining the magnitude of plastic displacement increments
θ_t	torsional (rotational) footing displacement
θ_t^p	plastic component of torsional (rotational) displacement
σ	vertical stress
σ_{max}	maximum vertical stress
τ	shear stress

Superscripts:

e	elastic
p	plastic
t	total

Chapter 6:

f_p	factor determining limiting magnitude of vertical load as $w_p \rightarrow \infty$
g	dimensionless shear modulus factor
G	representative elastic shear modulus
k	initial plastic stiffness
k'_t	rate of change of association factor α_t
k_t	dimensionless elastic stiffness factor (torsion)
k_v	dimensionless elastic stiffness factor (vertical)
N_γ	bearing capacity factor (self-weight)
p_a	atmospheric pressure
R	radius of a circular footing
t_0	dimensionless variable that determines the size of the yield surface in the $T/2R$ plane and is the value of $T/(2RV_0)$ at the maximum T
T	torsional load
V	vertical load
V_0	maximum vertical load capacity when all other loads are not present
V_{peak}	maximum vertical load bearing capacity, i.e. peak value of V_0 in strain hardening law
w	vertical footing displacement
w_p	plastic component of the vertical penetration (also w^p)
w_{pm}	value of the plastic component of the vertical penetration at the peak value of V_0 (i.e. at V_{peak})
α_m	moment association factor
α_t	torsion association factor
$\alpha_{t\infty}$	association factor as torsional (rotational) displacement tend to infinity
β_1	curvature factor for yield surface (low stress)
β_2	curvature factor for yield surface (high stress)
β_3	curvature factor for plastic potential (low stress)
β_4	curvature factor for plastic potential (high stress)
γ	effective unit weight
η	angle of “experimental” plastic displacement ratio (Eqn 5.21)
θ_t	torsional (rotational) footing displacement
θ_t^p	plastic component of torsional (rotational) displacement

Chapter 7:

H	horizontal load
M	moment load

R	radius of a circular footing
t_0	dimensionless variable that determines the size of the yield surface in the $T/2R$ plane and is the value of $T/(2RV_0)$ at the maximum T
T	torsional load
V	vertical load
V_0	maximum vertical load capacity when all other loads are not present
α_t	torsion association factor
β_3	curvature factor for plastic potential (low stress)
β_4	curvature factor for plastic potential (high stress)

1 INTRODUCTION

1.1 Background of Jack-Up Drilling Rigs and Sign Conventions Used

Jack-up platforms are widely used in the oil and gas industry because they are quick and easy to install, are re-usable (transportable), and a cost effective means of drilling. Jack-ups are self-elevating mobile units that comprise a hull (triangular deck) supported by a number of legs, typically three, and attached to the base of each leg is a circular footing or a large inverted cone commonly known as a spudcan; a typical jack-up and some typical spudcans are shown in Figs. 1.1 and 1.2 respectively. Spudcans are roughly circular in plan with a sharp protruding spigot to provide extra horizontal stability by helping to minimise lateral movement in the soil. For larger units operating in the North Sea, spudcan diameters in excess of 20 m have become frequent. Jack-ups perform drilling in water depths of up to 120 m. At such depths, the magnitude of loads acting on the three legs is substantial and becomes a combined load on the foundation. In addition to vertical forces mainly by its own self weight, these loads include environmental forces such as horizontal loading imposed by current, wind and waves, moment loading induced by these loads acting at a certain lever arm above the footing, and torsional loading generated by twisting effects from skewed environmental loading. In the past, with only vertical (V), horizontal (H), and moment (M) loads considered, a two-dimensional plane strain formulation, three degrees of freedom model represented by Fig. 1.3 was adequate. However with vertical (V), horizontal (2 orthogonal H_1 and H_2), moment (2 orthogonal M_1 and M_2), and torsion (T) loads considered, a three-dimensional plane strain formulation, six degrees of freedom model represented by Fig. 1.4 is required.

Fig. 1.5 shows the positive directions of (V, M, H) loads and the corresponding vertical, rotational, and horizontal displacements (w, θ, u) for the three degrees of freedom model. For dimensional consistency, the model is formulated with moment and rotation described as $M/2R$ and $2R\theta$, respectively. Similarly, the (V, M_1, H_1, M_2, H_2, T) loads and corresponding ($w, \theta_1, u_1, \theta_2, u_2, \theta_t$) displacements for the six degrees of freedom model are shown in Fig. 1.4. Again, for dimensional consistency, similar formulations as for the three degrees of freedom model for the orthogonal moment and rotation is seen, i.e. described as $M_1/2R$ and $2R\theta_1$, and $M_2/2R$ and $2R\theta_2$, respectively. Further, the model is

also formulated with torsion load and torsional rotational displacement described as $T/2R$ and $2R\theta_t$, respectively. This will be the sign convention used throughout this thesis.

1.1.1 Jack-Up Installation

During the installation of jack-ups, they are pre-loaded to approximately twice their service vertical load, essentially “proof testing” the jack-ups before they are subjected to environmental forces. After floating the jack-up on its hull to its required destination, the “proof testing” is conducted by first lowering the legs to the seabed followed by the jacking down of the legs into the soil (consequently allowing the hull to be raised from the water). The footing continues to penetrate the soil while the applied bearing pressure (pre-load) exceeds the bearing capacity (strength) of the soil. The pre-load is applied firstly by the self-weight of the jack-up rig followed by the flooding of the ballast tank usually located within the hull. After the pre-load is applied for a minimum of 2 - 4 hours (Bienen, 2001), the water is then emptied from the tank resulting in a safer, more stable state. The reasoning is that by exposing the foundations to a larger vertical load, an interaction diagram (for the combined loading conditions) has been established and is large enough that for any environmental loading conditions the foundations will not have a bearing failure. The more pre-load applied, the better the “proof testing” since a greater safety margin exists with a higher pre-load. Obviously there are physical constraints governing the amount of pre-load that can be applied simply because pre-load tanks have a limit to the amount of water they hold and the dead weight of the jack-up is fixed. Once the water is emptied, the rig is jacked up to a specified air gap ready for drilling. These three stages of the installation procedure are well depicted in Fig. 1.6.

1.2 The Need for Further Research and Project Objectives

From around 1950 to 1990, the evaluation of bearing capacity of a shallow foundation subjected to combined loading is primarily reliant on classical bearing capacity theory. Jack-up footings are regarded as shallow foundations because the depth of penetration is relatively small compared to the footing’s large diameter. Classical bearing capacity

theory is often used to produce an “ultimate load failure surface”, the size of the surface is related to the vertical pre-load applied to the footing. Essentially, this surface defines the ultimate load combinations that can be applied to the footing during the analysis stage and “failure” is conservatively assumed once the force point of the currently applied load combination touches this surface. The prediction of the load combinations is essential for the stability of the jack-up platform to ensure that the loads applied by environmental forces such as horizontal force, moment and torsion, in addition to vertical forces such as its own self weight, is less than the ultimate load combinations. However, classical bearing capacity theory only relates to “strength” and does not take into account the effects of footing displacements related to the pre-failure plastic behaviour.

With more drilling performed in deeper and harsher environments, there is a need to better understand the behaviour of jack-ups and to develop more accurate predictions. Hence, since about 1990 there has been a move towards plasticity theory, which is capable of accounting for the effects of further footing penetration, and is believed to give more accurate and less conservative predictions. Essentially, plasticity models allow for the increase in capacity due to further penetration of the jack-up footing into soil by accounting for the work hardening of the soil. Previously, with work hardening not accounted for by classical bearing capacity theory, stiffness was conservatively assumed to reduce to zero at the yield surface boundary.

Furthermore, with a series of *ad hoc* procedures for describing foundation behaviour under combined loading and the many semi-empirical factors associated with classical bearing capacity theory, they are not suitable for numerical analysis. However, strain-hardening plasticity theory with the response of the foundation expressed purely in terms of force resultants is amenable to numerical analysis.

A theoretical strain-hardening plasticity model for sand for six degrees of freedom has been proposed by Cassidy and Bienen (2002). It is called ‘Model C’ and is an extension of the three degrees of freedom model described in Cassidy (1999) and Houlsby and Cassidy (2001). In order to examine the applicability of this proposed model, tests will be performed on circular footings on commercially available super fine silica sand using an experimental apparatus developed to load the footing with vertical, horizontal,

moment, and torsion loads. A main component of the experimental apparatus developed is the loading leg, consisting of axial, bending, and torsion strain gauges, calibrated to convert the strain read in the gauges to actual forces in the member, at the position of the gauge. A circular footing (of various sizes) can be attached at the bottom of the loading leg.

The main aim of this project is to examine the vertical load plastic displacement behaviour as well as the effects of torsion generated by the footings on sand in an attempt to verify the six degrees of freedom plane strain formulation in the V - T space. Pure vertical loading tests, which involve pushing the circular footing vertically into the sample are to be conducted in order to examine the vertical load plastic displacement behaviour, which forms the vertical strain hardening law for Model C. Vertical-torsion sideswipe tests are to be performed to allow the direct investigation of the shape of the yield surface at a given penetration, which involves pushing the footing vertically to a prescribed level (resulting in a specified vertical load), keeping the vertical displacement constant and rotating (twisting) the footing. Finally radial displacement tests in the V : T plane, which involve straight paths of vertical and torsional (rotational) displacements (i.e. constant gradient), are also to be performed to provide information about the hardening law and the flow rule.

From these tests, parameters for the numerical plasticity model can be evaluated. Retrospective numerical simulation of the tests will be made to investigate the capabilities of the theory to model footing and jack-up behaviour.

It is hoped that as a result of the experimental verification, a realistic model of the behaviour of jack-ups in three dimensions under environmental loads can be confidently employed in the Oil and Gas Industry. Since jack-up platforms are quick and easy to install, are re-usable (transportable), and a cost effective means of drilling, there will be more and more demand for them, which can soon be used with greater confidence when the realistic three-dimensional plasticity model is experimentally verified.

1.3 Thesis Structure

Chapter 2 provides a literature survey on the background of classical bearing capacity and plasticity theories, and past research into the analysis of jack-up footings on sand. This includes a review of the strain-hardening plasticity model for sand, Model C. Both the original three degrees of freedom model and the proposed six degrees of freedom model will be discussed. The theoretical formulation for the V - T load space in the six-degree model will also be examined.

Chapter 3 describes the sample preparations and various common testing procedures used in soil mechanics to determine the properties of the commercially available super fine silica sand used (such as sieve analysis, minimum and maximum density tests, and direct shear box tests).

Chapter 4 summarises the methods and procedures used for the experiments conducted in this research. This includes a description of the testing equipment, the computer set-up, and the calibration procedures. An outline of the testing programme is also described.

Chapter 5 presents the results of the tests conducted, a discussion of the results, and outlines some modifications of Model C for the V - T space based on the outcomes of the experiments.

Chapter 6 presents the numerical simulations performed for a number of representative experiments in the V - T space to investigate the capabilities of Model C. Comparisons are drawn on the experiments presented in Chapter 5.

Chapter 7 concludes and highlights the key findings of the research project, and proposes recommendations for future research into the physical testing of jack-up footings on sand subjected to torsion.

2 LITERATURE SURVEY

2.1 Background Theory

As discussed in chapter 1, classical bearing capacity theory is a well-known design method to predict the ultimate load combinations that can be applied to a shallow foundation. It has therefore been covered extensively in soil mechanics textbooks and research papers. Therefore, only a brief outline of bearing capacity theory of a foundation will be given here.

2.1.1 Classical Bearing Capacity Theory – Pure Vertical Load

The bearing capacity of a soil is essentially the ‘maximum’ strength of the soil in resisting the applied vertical pressure imposed by the foundation on the soil, on which the structure rests. This ‘maximum’ strength is commonly classified as the ultimate bearing capacity, q_{ult} defined as “the least pressure that will cause shear failure of the supporting soil immediately below and adjacent to the foundation” (Dufty, 2001). This definition correctly points out the failure of the soil due to shear, since bearing capacity failure usually occurs as a shear failure. The three modes of shear failure commonly identified, namely, general shear, local shear, and punching shear are well depicted by Fig. 2.1.

The general shear failure as shown in Fig. 2.1(a) has a well-defined failure pattern with heaving of the ground on both sides of the footing characterised by the low compression applied to the soil immediately beneath the footing. The punching shear failure shown in Fig. 2.1(c) on the other hand is characterised by the high compression applied to the soil immediately beneath the footing and has a poorly defined shearing pattern. As the name suggests, the high compression applied tends to punch the footing into the soil causing this type of failure. The local shear failure as shown in Fig. 2.1(b) is characterised by both general shear and punching shear failure, i.e. a transition mode of failure, exhibiting some heaving of the ground on both sides of the footing. However since the compression applied to the soil immediately beneath the footing is still quite significant, the footing penetrates a significant amount vertically, i.e. relatively large settlements are expected but not as much as that associated with punching shear failure.

Typically, shallow foundations, being the area of interest in this research, are most likely to follow the general shear mode of failure while deeper foundations tend to exhibit local shear failure.

The full formula by Hansen (1970) for determining the ultimate bearing capacity is an extension of the simple formula originally developed by Terzaghi for an infinitely long strip foundation and can be written as

$$q_{ult} = V_{ult}/A = cN_c s_c d_c i_c b_c g_c + qN_q s_q d_q i_q b_q g_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma i_\gamma b_\gamma g_\gamma \quad (2.1)$$

(cohesion) (overburden) (self-weight)

$$\text{where, } N_q = e^{\pi \tan \phi} \tan^2 \left(45^\circ + \frac{\phi}{2} \right) \quad (2.1 \text{ a})$$

$$N_c = (N_q - 1) \cot \phi \quad (2.1 \text{ b})$$

$$N_\gamma = 1.5(N_q - 1) \tan \phi \quad (\text{Hansen, 1970}) \quad (2.1 \text{ c})$$

This extension includes the following modifying factors (not originally in Terzaghi's simple formula) to account for footing shape (s), depth (d), load inclination (i), base inclination (b), and ground inclination (g). N_q and N_c are exact formulas developed by Prandtl (cited by Hansen, 1970) using plasticity methods. N_γ is an empirical formula developed by Lundgren-Mortensen, and later by Odgaard and N.H. Christensen (cited by Hansen, 1970). The three terms of Eqn 2.1, cohesion, overburden, and self-weight are the contributions to the total bearing capacity of the soil. The first term represents the contribution due to the cohesiveness or shear strength of the soil. The second term is due to the surcharge pressure applied, while the third term takes into account the self-weight of the soil.

This formula can be simplified for shallow footings on sands, for application in this research. The vertical penetration of the circular footing is expected to be small, and the only significant contribution to the bearing capacity is due to self-weight. This is because sand is usually considered to have no cohesion, and hence c can be assumed to be zero, and as the footing is not embedded (at least for dense sand where the vertical penetration is small before a peak load is reached), $q = \gamma z = 0$. Further, for pure vertical loading, the footing is not inclined nor is it placed under any overlying layer of sand. Therefore Eqn 2.1 can be reduced to

$$q_{ult} = 0.5\gamma BN_{\gamma} s_{\gamma} \quad (2.2)$$

and for a circular footing,

$$V_{ult} = q_{ult} A = s_{\gamma} N_{\gamma} \gamma \pi R^3 \quad (2.3)$$

where V_{ult} is ultimate load, A the surface area of the footing and s_{γ} is the shape factor. For circular footings, the shape factor is based on empirical correlations and assumed to be $s_{\gamma} = 0.6$ (Vesic, 1975). However the need for scaling the values calculated for strip footings by this empirical shape factor can be eliminated by effectively incorporating it into the dimensionless bearing capacity factor N_{γ} hence Eqn 2.3 can be further reduced to

$$V = N_{\gamma} \gamma \pi R^3 \quad (2.4)$$

A set of N_{γ} values for various conical shapes (cone apex angle β), roughness ($\alpha = 0$, smooth to $\alpha = 1$, rough), and soil conditions (friction angle ϕ) have been calculated theoretically by Cassidy (1999) based on a lower bound approach, and provided in Appendix D1. The notation for the conical shape footing is shown in Fig. 2.2. Full details of the procedure used to evaluate this set of N_{γ} values will not be given here but can be found in Cassidy (1999) and Cassidy and Houlsby (2002). Hence, the ultimate load bearing capacity for a flat circular footing ($\beta = 180^{\circ}$) can be estimated by Eqn 2.4 by reading off the appropriate N_{γ} value (in Appendix D1) that corresponds to the peak friction angle of the soil.

2.1.2 Classical Bearing Capacity Theory – Combined Loads

When concurrent horizontal and/or moment loads are applied to a footing, the combined bearing capacity reduces. Although bearing capacity theory for circular foundations under combined (V , H , M) loading will not be given here, the main aspects and applicability associated with this theory will be summarised nevertheless.

As can be seen earlier, the bearing capacity formulation of Eqn 2.1 reduced to Eqn 2.3 to calculate the ultimate vertical load V_{ult} , is based on two empirical terms s_{γ} and N_{γ} and thus has its drawbacks due to conservativeness associated with empirical formulations. Note that N_{γ} is a function of the internal friction angle, ϕ as shown in Eqn 2.1(c). Thus,

the results of the classical bearing capacity formulation are highly sensitive to the value of the internal friction angle of the soil and tend to be very conservative.

Due to the conservative nature demonstrated in the classical bearing capacity theory in the vertical load space for sand, there is a need to move away from this highly empirical method to plasticity methods. (The general bearing capacity formula in Eqn 2.1 is also intended primarily for conventional onshore use). This is because more accurate predictions (which can be obtained from plasticity methods) are needed for jack-ups moving into deeper and harsher waters offshore. It is hoped that this more accurate prediction would lead to a substantial reduction in the size of the jack-up footings and the length of its legs for example, whereas larger jack-up footings and length of its legs would be needed for the same magnitude of loads applied if more conservative methods are employed. Thus there is a potential saving in cost for the same magnitude of loads applied.

2.1.3 Plasticity Theory – Combined Loads

Plasticity theory is used in this thesis as the framework for interpreting the load-displacement behaviour of spudcans on sand. As will be described in section 4.3, the experiments have been conducted to allow interpretation of the footing behaviour within this framework. Therefore, a review of plasticity theory and its application to shallow foundations will be given here.

Though it will be applied to soils here, the basis of plasticity theory can be easily explained by the stress-strain behaviour of metals. There are four essential features associated with plasticity theory. They are

- (1) Elasticity
- (2) Yielding
- (3) Hardening
- (4) Flow

Yielding and *hardening* features will be illustrated by examining the idealised stress-strain behaviour of metal shown in Fig. 2.3. Initially, before any axial stress is applied

the axial strain is zero. When the metal is loaded to a stress less than the first yield stress, σ_y , the behaviour of the metal still lies in the elastic (recoverable) state since if the metal is unloaded, the axial strain returns to its original value of zero strain, as illustrated in Fig. 2.3. However when the metal is loaded to some point G (in Fig. 2.3) beyond the first yield stress, σ_y , plastic deformation occurs so that even when all the axial stress is now removed (i.e. metal is unloaded), a plastic (irrecoverable) strain is sustained at point B. This is shown in Fig. 2.3. If the metal is again reloaded to a stress before point G, i.e. with stress less than σ_g , the deformation here is again linearly elastic. (σ_g is assumed to be a stress less than the failure stress, σ_f at point F in Fig. 2.3).

So, *yielding* occurs when the metal becomes plastic, i.e. at all stresses greater than the first yield stress (σ_y). The magnitude of the plastic strain from points Y to G is given by the axial strain from zero to point B in Fig. 2.3. During this state of plastic straining from points Y to G, the yield stress is raised accordingly from σ_y to σ_g , whose effect is known as *strain hardening* or *work hardening*. In short, whenever plastic strain increases (i.e. this corresponds to the increase in yield stress), *strain hardening* or *work hardening* occurs. The relationship between the change ($\sigma_y - \sigma_g$) of yield stress and the plastic strain from points Y to G is known as a *hardening law* (Atkinson and Bransby, 1978). Essentially, this is a one-dimensional work-hardening model that defines the expansion of the yield point. The same concepts can be applied to soils and this can be thought of as a base case model from which plasticity models for soils are developed.

Therefore the four features of an elastic-plastic model have been shown here. Firstly, elastic behaviour is described between O to Y. The yield criterion is established when the stress exceeds the yield stress (initially at σ_y). Hardening is seen by expanding these yield stress from σ_y to σ_g to σ_f . And the flow rule describes the magnitude of the plastic displacement.

2.2 Past Research into the Analysis of Jack-Up Footings

The concept of plasticity models was first applied to the soil-structure interaction problem by Roscoe and Schofield (1956). They were interested in the design of short pier foundations in sand in terms of the plastic moment resistance of the footing in $M:V$

space. However, the move towards plasticity theory with the use of work-hardening plasticity models to account for the effects of footing penetration for jack-up spudcan footings was first spearheaded by Tan (1990) then later works by Martin (1994), Gottardi, Houlsby and Butterfield (1999), and Cassidy (1999).

Tan (1990) carried out his physical model tests (of various conical and spudcan footings on saturated sand) in a geotechnical centrifuge at Cambridge University. “Though Tan modelled experimental data with an incremental plasticity method, it was in two dimensional $H:V$ space with no moment component” (Cassidy, 1999).

Martin (1994) developed the plasticity theory for clay called ‘Model B’ based on applying horizontal and rotational displacements to a model spudcan while at different fixed initial vertical load. ‘Model B’ was developed from 33 footing tests conducted by Martin and Houlsby (1994) on soft Speswhite Kaolin clay. It was found that the three-dimensional ($V: M/2R: H$) yield surface shape (R being the radius of a circular footing) remained nearly constant, only the size expanded significantly with spudcan penetration. Similar findings for circular footings on dense Leighton-Buzzard sand were discovered by Gottardi and Houlsby (1995) and Gottardi *et al.* (1999). This led to the development of a plasticity model for sand called ‘Model C’ (for three degrees of freedom) by Cassidy (1999).

The extension to the complete three-dimensional plane strain formulation (Model C for six degrees of freedom) has been proposed by Cassidy and Bienen (2002). The experimental verification of the vertical-torsional component is the main aim of this project. Model C will be described in more detail in this chapter.

Further experiments in the centrifuge have been conducted to investigate the performance of spudcan footings and suction caissons on soft clay, subjected to $V:M:H$ loads (Byrne and Cassidy, 2002). Though performed to make comparative performance between spudcan footings and suction caissons, the tests have also helped to validate the existing three degrees of freedom plasticity model for clay. So far, most tests have been performed with single spudcan footings. Therefore, experimental tests (at 1g level) on a scaled three-legged jack-up unit model, with a spudcan footing attached to each leg, have been performed on soft clay by Vlahos *et al.* (2001). By applying combined $V:H$

loads at the hull level (resulting in combined $V:M:H$ loads at the footings), a more realistic, overall jack-up behaviour is simulated. Details of the soil-structure interaction and load sharing among the spudcan footings based on these tests can be found in Vlahos, Martin and Cassidy (2001). Nevertheless, this test also helps with the validation of the three degrees of freedom plasticity model for clay.

2.2.1 Model C for Three Degrees of Freedom

Model C is a plasticity model developed by Cassidy (1999) intended to describe the drained loading of a circular foundation on dense sand based on a series of experimental tests by Gottardi and Houlsby (1995) and Gottardi *et al.* (1999). Further reference to Model C can be found in Houlsby and Cassidy (2001).

The main features of the two-dimensional plane strain formulation, three degrees of freedom Model C in the $V:M:H$ load space, represented earlier in Fig. 1.3 will be described here. However, for a detailed description and formulation of Model C, reference can be made to Cassidy (1999).

The four major components discussed in section 2.1.3 are also associated with Model C. They are:

- (1) An empirical expression for the *yield surface* (see Fig. 2.4 for the yield surface in $V-M-H$ space),
- (2) A model for *elastic* load-displacement behaviour within the yield surface (discussed in section 2.2.1.2),
- (3) An empirical *strain-hardening expression* to define the expansion and contraction of the yield surface with the plastic component of vertical displacement,
- (4) A suitable *flow rule* to allow prediction of the footing plastic displacements during yield.

2.2.1.1 Yield Surface

The yield surface for Model C has been defined experimentally by performing sideswipe tests. This allows direct investigation of the shape of the yield surface at a

given vertical penetration. It is performed by keeping the vertical displacement degree of freedom constant while allowing another degree of freedom to move. For instance, in a vertical-horizontal sideswipe test, after the footing is penetrated vertically to a resulting specified vertical load, the vertical displacement is kept constant while the footing is displaced horizontally. Similarly, in a vertical-torsion sideswipe test, after the footing is penetrated vertically to a resulting specified vertical load, the vertical displacement is kept constant while the footing is rotated. From experimental evidence of previous researchers mentioned previously, a three-dimensional yield surface in (V, M, H) space (termed by Houlsby and Cassidy, 2001 as “rugby ball” shaped and shown in Fig. 2.4) will be established at any penetration of a foundation into the soil. Any slices on the planes of constant V (i.e. in the H and $M/2R$ planes), its shape is a rotated ellipse while all other slices including the V axis, its shape is roughly parabolic.

From the experimental work performed (of spudcan footing on clay) by Martin (1994), it was found that the general shape of the $(V: M/2R: H)$ yield surface remained constant while expanding with increased penetration. Therefore, the shape of the yield surface is assumed to remain constant but the size of the yield surface expands as the footing is pushed further into the soil. This expression can therefore be explained through a vertical hardening law. This yield surface defines the boundary between elastic and elastic-plastic behaviour. Any footing behaviour within this surface is assumed to be elastic while elastic-plastic behaviour occurs once the load touches and expands the yield surface. This is analogous to the stress-strain behaviour of metal illustrated previously in section 2.1.3.

With the shape of the yield surface assumed constant, the yield surface can be normalised by V_0 , the pure vertical load capacity at any depth, i.e. it defines the size of the yield surface and is governed by the vertical plastic penetration in the strain hardening law. The three-dimensional $(V, M/2R, H)$ Model C yield surface (shown in Fig. 2.4) can be defined by the following equation:

$$f(V, M/2R, H) = \left(\frac{H}{h_0 V_0} \right)^2 + \left(\frac{M/2R}{m_0 V_0} \right)^2 - \frac{2aHM/2R}{h_0 m_0 V_0^2} - \left[\frac{(\beta_1 + \beta_2)^{(\beta_1 + \beta_2)}}{\beta_1 \beta_1 \beta_2 \beta_2} \right]^2 \left(\frac{V}{V_0} \right)^{2\beta_1} \left(1 - \frac{V}{V_0} \right)^{2\beta_2} = 0 \quad (2.5)$$

where h_0 and m_0 defines the dimensions of the yield surface in the horizontal and moment directions respectively (h_0 is the maximum value of H/V_0 when $M = 0$ and m_0 is the maximum value of $M/2RV_0$ when $H = 0$). The surface is an ellipse in the H - $M/2R$ plane with its eccentricities defined by a . The parameters β_1 and β_2 are chosen empirically to define the ‘degree of curvature’ (i.e. rounding) on the left and right of $(V/V_0)_{peak}$ respectively along the V/V_0 axis. They also affect the location of the peak of the curve in V/V_0 space. If β_1 and $\beta_2 < 1$, the peak will be located at a position of $V/V_0 < 0.5$. These β parameters will be adjusted later in the V - T plane to adjust the peak location.

Eqn 2.5 defines the yield surface of Model C (for three degrees of freedom). Parametric values recommended by Cassidy (1999) are

$$h_0 = 0.116; \quad m_0 = 0.086; \quad a = -0.2; \quad \beta_1 = 0.9; \quad \beta_2 = 0.99 \text{ or } 1.0.$$

In addition to the explanation for each of these parameters detailed above, the negative value of ‘ a ’ indicates an anticlockwise rotation of the yield surface with respect to the H and $M/2R$ axes. Also, the parameter β_2 is adopted as 0.99 if the normal vector to the surface is required to be uniquely defined and differentiable at $V/V_0 = 1$ whereas if this property is not necessary then $\beta_2 = 1.0$ allows for a mathematically less complex yield surface expression. Whether β_2 is 0.99 or 1.0, there is little difference to the overall yield surface shape.

2.2.1.1.1 Further Yield Surface Modification

A recent study by Byrne and Houlsby (2001) suggested that the general assumption of a constant yield surface shape expanding in size as the vertical plastic penetration increases (Martin, 1994; Gottardi *et al.*, 1999) could be improved. In a number of experiments performed on dense frictional materials, a slightly different observation was noted. Byrne and Houlsby (2001) proposed that the non-dimensional parameter $h_0 = H_0/V_0$ does not remain constant but changes according to stress level. They hypothesised that a relationship could be established between the size of the yield surface (based on sideswipe evidence) and the ratio of V_0 and peak bearing capacity of the footing (defined as V_{peak}). V_{peak} could be established for the fully penetrated spudcan

and Eqn 2.4. The relationship suggested by Byrne and Houlsby (2001) for the horizontal size is:

$$h_0 = h_{0peak} \left[1 - 0.36 \ln \left(\frac{V_0}{V_{peak}} \right) \right] \quad (2.6)$$

where V_0 is the vertical load where horizontal swipe occurs, V_{peak} is the maximum vertical load capacity, and $h_{0peak} = 0.11$ is the horizontal dimension for the yield surface at peak bearing capacity. Eqn 2.6 was validated for $0.025 < V_0 / V_{peak} < 1$ and is plotted in Fig. 2.5. For loose sands there is no observed peak bearing capacity, however, Byrne and Houlsby (2001) recommended using h_0 value of h_{0peak} . The original value of 0.116 is shown in Fig. 2.5 and represents V_0 / V_{peak} of $1600/2050 = 0.78$. There is also the same expression for m_0 .

In summary, the yield surface of Model C (for three degrees of freedom) is still accurately defined by Eqn 2.5, however, for dense sands, h_0 (in Eqn 2.5) will vary according to the vertical load V_0 at which the horizontal swipe is conducted and can either be estimated by Eqn 2.6 (which was empirically formulated) or obtained directly from the experiment. For loose sands, a constant h_0 of h_{0peak} can be taken for all V_0 .

2.2.1.1.2 Brief Discussion of the Development of the Yield Surface

In the development of this yield surface, Cassidy (1999) had to adjust the experimental values from Gottardi's and Houlsby's (1995) swipe tests when calculating V_0 to take soil elasticity and rig stiffness into account. Due to elasticity in the soil, but more importantly, due to the testing rig not being infinitely stiff, this results in some 'compression' of the soil and rig, hence resulting in the additional plastic penetration during the course of the swipe. This means that the swipe tests do not follow a yield surface of constant V_0 since a slight expansion of the yield surface occurs with the additional increase in the vertical plastic penetration due to the soil and rig. Therefore, appropriate adjustments were made to take soil elasticity and rig stiffness into account. Further details to the actual procedure can be found in Cassidy (1999).

2.2.1.2 Elastic Behaviour

With the yield surface of Model C established by Eqn 2.5, elastic behaviour is expected as long as the loading (or displacement) increment is fully contained within this yield surface and these were defined using finite element methods by Bell (1991) and Ngo Tran (1996). Interestingly they noted that cross coupling exists between the horizontal and rotational footing displacements (Cassidy and Bienen, 2002). As recommended by Cassidy (1999) elastic relationship between the increments of load (dV , dM , dH) and the corresponding elastic displacements (dw^e , $d\theta^e$, du^e) for a flat circular footing could be:

$$\begin{pmatrix} dV \\ dM / 2R \\ dH \end{pmatrix} = 2GR \begin{bmatrix} k_v & 0 & 0 \\ 0 & k_m & k_c \\ 0 & k_c & k_h \end{bmatrix} \begin{pmatrix} dw^e \\ 2Rd\theta^e \\ du^e \end{pmatrix} \quad (2.7)$$

where G is a representative elastic shear modulus and k_v , k_m , k_h , k_c are dimensionless elastic constants. For elastic behaviour within the yield surface in Model C, the parameters (k_v , k_m , k_h , k_c) of Eqn 2.7 is based on the assumption of Bell's results for Poisson ratio, $\nu = 0.2$ (Bell, 1991 as cited by Cassidy, 1999). Appropriate values, as listed in Cassidy (1999) and Cassidy and Bienen (2002) are:

$$k_v = 2.65; \quad k_m = 0.46; \quad k_h = 2.3; \quad k_c = -0.14.$$

The value for G is however one of the most difficult parameters to ascertain. This is because of the following two properties associated with G :

- The mobilised shear stiffness is strongly dependent on the shear strain
- The shear modulus also depends on stress level, which is approximately proportional to the square root of the mean effective stress.

Based on recent work, the following formula has been suggested by Houlsby and Cassidy (2001) to conveniently give an estimate of the shear modulus:

$$\frac{G}{p_a} = g \sqrt{\frac{V}{Ap_a}} \quad (2.8)$$

where p_a is atmospheric pressure, V is a representative vertical load on the foundation, A is the plan area of the foundation and g is a dimensionless constant. Typical values of g suggested by Houlsby and Cassidy (2001) are approximately 400.

2.2.1.3 Strain Hardening

An empirical *strain-hardening expression* is used to define the expansion and contraction of the yield surface with the plastic component of vertical displacement. This expression of the vertical load plastic vertical penetration curve for dense sand was formulated by Cassidy (1999) based on experimental data of Gottardi and Houlsby (1995). A vertical load penetration curve is obtained simply by performing a pure vertical loading test (since for pure vertical loading $V_0 = V$) and adjusting for plasticity in the soil. Similar load penetration curves were found in this research and are shown in chapter 5. For dense sand the experimental data could be described using the following *strain-hardening expression* formulated by Cassidy (1999):

$$V_0 = \frac{k w_p}{1 + \left(\frac{k w_{pm}}{V_{peak}} - 2 \right) \left(\frac{w_p}{w_{pm}} \right) + \left(\frac{w_p}{w_{pm}} \right)^2} \quad (2.9)$$

where k is the initial plastic stiffness, w_p is the plastic component of the vertical penetration, V_{peak} is the peak value of V_0 (as defined earlier in section 2.2.1.1.1, also see Eqn 2.4), and w_{pm} the value of the plastic component of vertical penetration at this peak. As correctly highlighted by Houlsby and Cassidy (2001), no special significance is attached to this particular form of the fit to the vertical load-penetration response, and alternative expressions could also be appropriate. Though more discussion of the experimental results obtained in this research will be given in section 5.1.1, the expression of Eqn 2.9 can be seen in Fig. 5.7.

For jack-ups in dense sand, loading post peak would not be expected and therefore the area of interest is really the pre-peak response (Cassidy, 1999). Although Eqn 2.9 unrealistically implies $V_0 \rightarrow 0$ as $w_p \rightarrow \infty$, it should still be adequate for pre-peak response of jack-ups. However, for a complete foundation model, a formula that models post-peak work softening as well as pre-peak performance was formulated by Cassidy (1999), with Eqn 2.9 altered to

$$V_0 = \frac{k w_p + \left(\frac{f_p}{1 - f_p} \right) \left(\frac{w_p}{w_{pm}} \right)^2 V_{peak}}{1 + \left(\frac{k w_{pm}}{V_{peak}} - 2 \right) \left(\frac{w_p}{w_{pm}} \right) + \left(\frac{1}{1 - f_p} \right) \left(\frac{w_p}{w_{pm}} \right)^2} \quad (2.10)$$

where f_p is a dimensionless constant that describes the limiting magnitude of vertical load as a proportion of V_{peak} (i.e. $V_0 \rightarrow f_p V_{peak}$ as $w_p \rightarrow \infty$) as illustrated in Fig. 2.6. It should be possible to use the same parametric values of k , V_{peak} , and w_{pm} . Eqn 2.10 is also shown (along with Eqn 2.9) in Fig. 5.7 as a best-fit curve to the experimental load-penetration curves obtained in this research. Again, further discussion will be detailed in section 5.1.1.

So far, the hardening law developed is for flat circular footings. However, since Model C was intended to reflect the soil-structure interaction of jack-up foundations, adaptation to the spudcan shape of jack-up footings had to be made. Cassidy (1999) derived this strain hardening law for spudcans by combining the experimental observations for flat circular footings and the theoretical predictions of conical footings. Full details of this “combined” strain hardening law together with the procedure used to evaluate bearing capacity factors N_γ for various conical shapes, roughness, and soil conditions will not be given here, but can be found in Cassidy (1999). The same set of N_γ values previously mentioned in section 2.1.1 are provided in Appendix D1 and the resulting hardening law (as shown in Fig. 2.7) is described by the following equation:

$$V_0 = s_{fp} \gamma^N \pi r^3 = s_{fp} \gamma^N \pi \left(w_p \tan\left(\frac{\beta}{2}\right) \right)^3 \quad (2.11)$$

where r is the radius at the surface of a partially penetrated conical footing, β the tip angle of the equivalent cone (shown in Fig. 2.8), w_p is the plastic component of the vertical displacement, and s_{fp} is a dimensionless scaling factor equal to Eqn 2.10 divided by V_{peak} , the experimental peak value of V_0 . Thus,

$$s_{fp} = \frac{V_0}{V_{peak}} = \frac{\frac{k w_p}{V_{peak}} + \left(\frac{f_p}{1-f_p} \right) \left(\frac{w_p}{w_{pm}} \right)^2}{1 + \left(\frac{k w_{pm}}{V_{peak}} - 2 \right) \left(\frac{w_p}{w_{pm}} \right) + \left(\frac{1}{1-f_p} \right) \left(\frac{w_p}{w_{pm}} \right)^2} \quad (2.12)$$

with all the parameters as defined previously for Eqns 2.9 and 2.10.

2.2.1.4 Flow Rule

The prediction of footing displacements during yield can be made from a suitable *flow rule*. If the plastic flow vectors were in the direction perpendicular to the yield surface, associated flow would be assumed. It was shown in the experiments of Gottardi and

Houlsby (1995) that the associated flow rule is found to model the ratios between plastic displacements well in the $(M/2R, H)$ plane but not in the $(V, M/2R)$ or (V, H) planes, where an associated flow rule under predicts the level of vertical displacements. Therefore, a different function is required and this is called the plastic potential function. In Model C the plastic potential (g) is defined as

$$g(V, M/2R, H) = \left(\frac{H}{\alpha_h h_0 V_0'} \right)^2 + \left(\frac{M/2R}{\alpha_m m_0 V_0'} \right)^2 - 2a \frac{H}{\alpha_h h_0 V_0'} \frac{M/2R}{\alpha_m m_0 V_0'} - \left[\frac{(\beta_3 + \beta_4)(\beta_3 + \beta_4)}{\beta_3 \beta_3 \beta_4 \beta_4} \right]^2 \left(\frac{V}{V_0'} \right)^{2\beta_3} \left(1 - \frac{V}{V_0'} \right)^{2\beta_4} = 0 \quad (2.13)$$

where α_h and α_m are non-dimensional horizontal and moment “association” factors, and V_0' is a dummy parameter giving the intersection of the plastic potential with the V -axis (and adjusted so that the potential passes through the current load point). The parameters β_3 and β_4 are chosen empirically (just like β_1 and β_2 previously) to allow for different variations in curvature.

The plastic potential described by Eqn 2.13 can be clearly illustrated by the graph in Fig. 2.9 (after Cassidy, 1999) with the horizontal axis being the vertical load V and the vertical axis being a general deviator force Q defined as:

$$Q^2 = \left(\frac{H}{h_0} \right)^2 + \left(\frac{M/2R}{m_0} \right)^2 - \frac{2aHM/2R}{h_0 m_0} \quad (2.14)$$

Essentially, Eqn 2.14 defines the combination of moment and horizontal loads in the $(M/2R, H)$ plane.

If $\alpha_h, \alpha_m = 1$ (in Eqn 2.13), the plastic potential coincides with the yield surface and associated flow is implied (as shown in Fig. 2.9). If $\alpha_h, \alpha_m < 1$, a ‘flattened plastic potential’ is seen with $V_0' > V_0$ (plastic potential ₍₁₎ in Fig. 2.9). If however, $\alpha_h, \alpha_m > 1$, an ‘elongated plastic potential’ is seen with $V_0' < V_0$ (plastic potential ₍₂₎ in Fig. 2.9).

The association factors α_h and α_m relates the shape of the plastic potential to the size of the yield surface. These parameters are formulated as hyperbolic functions in terms of plastic displacement histories as:

$$\alpha_h = \frac{k' + \alpha_{h\infty}(u^P / w^P)}{k' + (u^P / w^P)} \quad (2.15)$$

$$\alpha_m = \frac{k' + \alpha_{m\infty}(2R\theta^P / w^P)}{k' + (2R\theta^P / w^P)} \quad (2.16)$$

where k' determines the rate of change of the association factors. With the plastic potential defined by Eqn 2.13, the following values were evaluated as given in Houlsby and Cassidy (2001):

$$\beta_3 = 0.55; \quad \beta_4 = 0.65; \quad \alpha_{h\infty} = 2.5; \quad \alpha_{m\infty} = 2.15; \quad k' = 0.125.$$

A compromise solution of $\alpha_h = \alpha_m = 2.05$ as found in Cassidy (1999). Further details of the development of the plastic potential in Eqn 2.13 can also be found in Cassidy (1999).

Finally, plastic displacements occur when the force point is located on the yield surface and in the direction normal to the plastic potential. For Model C in three degrees of freedom the increments of the plastic component of the footing displacements are related to the plastic potential function g , expressed as:

$$\begin{pmatrix} dw^P \\ 2Rd\theta^P \\ du^P \end{pmatrix} = \lambda \begin{bmatrix} \frac{\partial g}{\partial V} \\ \frac{\partial g}{\partial M / 2R} \\ \frac{\partial g}{\partial H} \end{bmatrix} \quad (2.17)$$

where λ is a multiplier which can be derived from the condition of continuity with the strain hardening law.

2.2.2 Model C for Six Degrees of Freedom Incorporating Torsion

The main features of the three-dimensional plane strain formulation, six degrees of freedom Model C in the $V:H_1:M_1:H_2:M_2:T$ load space, is represented earlier in Fig. 1.4. This is the extension of the three degrees of freedom Model C (Cassidy, 1999) proposed by Cassidy and Bienen (2002) for jack-up footings on sand. The model was extended in analogy to the six degrees of freedom model for spudcans on clay outlined by Martin (1994). However, the proposed extension has not been experimentally validated. With torsion being an essential newly introduced loading parameter to the six-degree of

freedom model, a good starting point is the validation of this model in the V - T space. Prior to this research, only a limited number of tests of flat circular footings on sand have been conducted in the V - T space. These were performed at the University of Oxford by Platt (1998); further details are described in section 5.2.3. Following the recommendation by Cassidy and Bienen (2002), further experimental validation in the T : V plane has been adopted as the main aim of the research.

The extension to six degrees of freedom of the four major components of Model C will now be discussed.

2.2.2.1 Yield Surface

The three degrees of freedom Model C yield surface given by Eqn 2.5, has been extended to the six degrees of freedom yield surface under a circular footing (Cassidy and Bienen, 2002). This follows Martin's suggestion (Martin, 1994) and has a form of:

$$\begin{aligned}
 f\left(V, \frac{M_1}{2R}, H_1, \frac{M_2}{2R}, H_2, \frac{T}{2R}\right) = & \left(\frac{H_1}{h_0 V_0}\right)^2 + \left(\frac{M_1/2R}{m_0 V_0}\right)^2 - \frac{2aH_1 M_1/2R}{h_0 m_0 V_0^2} \\
 & + \left(\frac{H_2}{h_0 V_0}\right)^2 + \left(\frac{M_2/2R}{m_0 V_0}\right)^2 - \frac{2aH_2 M_2/2R}{h_0 m_0 V_0^2} \\
 & + \left(\frac{T/2R}{t_0 V_0}\right)^2 - \left[\frac{(\beta_1 + \beta_2)^{(\beta_1 + \beta_2)}}{\beta_1 \beta_1 \beta_2 \beta_2}\right]^2 \left(\frac{V}{V_0}\right)^{2\beta_1} \left(1 - \frac{V}{V_0}\right)^{2\beta_2} = 0 \quad (2.18)
 \end{aligned}$$

where H_1 and H_2 are orthogonal horizontal loads, M_1 and M_2 are orthogonal moment loads, T is the torque developed (or torsion generated), and t_0 is a dimensionless variable that determines the maximum size of the yield surface in the $T/2R$ plane and is the value of $T/(2RV_0)$ at the maximum T . The other parameters are as defined for Model C in three degrees of freedom. Due to symmetry, the formulation of Eqn 2.18 is possible since there can be no cross product terms of $H_1 H_2$, $M_1 M_2$, $H_1 M_2$, $H_2 M_1$ or any involving T .

The aim of this thesis will be to define the value of t_0 . For the purely V - T loading case ($H_1 = H_2 = 0$ and $M_1/2R = M_2/2R = 0$), Eqn 2.18 can be simplified to

$$f(V, T/2R) = \left(\frac{T/2R}{t_0 V_0} \right)^2 - \left[\frac{(\beta_1 + \beta_2)(\beta_1 + \beta_2)}{\beta_1 \beta_1 \beta_2 \beta_2} \right]^2 \left(\frac{V}{V_0} \right)^{2\beta_1} \left(1 - \frac{V}{V_0} \right)^{2\beta_2} = 0 \quad (2.19)$$

Swipe test will be conducted to establish whether Eqn 2.19 is an appropriate shape. Further, if it is, will the value of t_0 be a constant? Or, as suggested by Byrne and Houlsby (2001) for h_0 and m_0 , will t_0 also vary according to the vertical stress ratio of V_0/V_{peak} ?

2.2.2.2 Elastic Behaviour

The elastic response of the soil can be defined for any increments within the six degrees of freedom ($V: M_1/2R: H_1: M_2/2R: H_2: T$) yield surface by extending Eqn 2.7 to:

$$\begin{pmatrix} dV \\ dM_1/2R \\ dH_1 \\ dM_2/2R \\ dH_2 \\ dT/2R \end{pmatrix} = 2GR \begin{bmatrix} k_v & 0 & 0 & 0 & 0 & 0 \\ 0 & k_m & k_c & 0 & 0 & 0 \\ 0 & k_c & k_h & 0 & 0 & 0 \\ 0 & 0 & 0 & k_m & k_c & 0 \\ 0 & 0 & 0 & k_c & k_h & 0 \\ 0 & 0 & 0 & 0 & 0 & k_t \end{bmatrix} \begin{pmatrix} dw^e \\ 2Rd\theta_1^e \\ du_1^e \\ 2Rd\theta_2^e \\ du_2^e \\ 2Rd\theta_t^e \end{pmatrix} \quad (2.20)$$

with the values of k_v , k_m , k_h , and k_c as previously defined.

As cited by Bienen (2001) and Cassidy and Bienen (2002), the torsional displacement of a flat rigid circular footing on the surface of a homogeneous elastic half space (referenced to Poulos and Davis, 1974) is given by

$$\theta_t^e = \frac{3}{16GR^3} T \quad (2.21)$$

Therefore substituting Eqn 2.21 into Eqn 2.20, the value of k_t is non-dimensionalised as

$$k_t = \frac{T/2R}{(2GR)(2R\theta_t^e)} = \frac{2}{3} \quad (2.22)$$

For elastic behaviour within the yield surface in Model C, the parameters (k_v , k_m , k_h , k_c , and k_t) of Eqn 2.20 is also based on the assumption of Bell's results for Poisson ratio, $\nu = 0.2$ just as for Eqn 2.7 previously.

2.2.2.3 Strain Hardening

The empirical strain hardening law for six degrees of freedom is the same as the three-degree of freedom Model C as defined in Eqns 2.9 and 2.10, and V_{peak} can be estimated by Eqn 2.4.

2.2.2.4 Flow Rule

A possible plastic potential for Model C six degrees of freedom (based on the six degrees of freedom yield surface) has been proposed by Cassidy and Bienen (2002). The proposal was made based on using the same assumptions for the flow in the moment and horizontal directions as well as associated behaviour in the $T:V$ plane. From this, the plastic potential extension is formulated as

$$\begin{aligned}
 g(V, \frac{M_1}{2R}, H_1, \frac{M_2}{2R}, H_2, \frac{T}{2R}) = & \left(\frac{H_1}{\alpha_h h_0 V_0'} \right)^2 + \left(\frac{M_1 / 2R}{\alpha_m m_0 V_0'} \right)^2 - 2a \frac{H_1}{\alpha_h h_0 V_0'} \frac{M_1 / 2R}{\alpha_m m_0 V_0'} \\
 & + \left(\frac{H_2}{\alpha_h h_0 V_0'} \right)^2 + \left(\frac{M_2 / 2R}{\alpha_m m_0 V_0'} \right)^2 - 2a \frac{H_2}{\alpha_h h_0 V_0'} \frac{M_2 / 2R}{\alpha_m m_0 V_0'} \\
 & + \left(\frac{T / 2R}{\alpha_t t_0 V_0'} \right)^2 - \left[\frac{(\beta_3 + \beta_4)^{(\beta_3 + \beta_4)}}{\beta_3 \beta_3 \beta_4 \beta_4} \right]^2 \left(\frac{V}{V_0'} \right)^{2\beta_3} \left(1 - \frac{V}{V_0'} \right)^{2\beta_4} = 0 \quad (2.23)
 \end{aligned}$$

For association in the V - T plane the value of the new association factor α_t would be 1.0.

As with the three degrees of freedom model, plastic displacements occur when the force point is located on the yield surface and in the direction normal to the plastic potential. For Model C in six degrees of freedom the increments of the plastic component of the footing displacements are related to the plastic potential function g in Eqn 2.23 and can be expressed as:

$$\begin{pmatrix} dw^P \\ 2Rd\theta_1^P \\ du_1^P \\ 2Rd\theta_2^P \\ du_2^P \\ 2Rd\theta_t^P \end{pmatrix} = \lambda \begin{bmatrix} \frac{\partial g}{\partial V} \\ \frac{\partial M_1 / 2R}{\partial g} \\ \frac{\partial H_1}{\partial g} \\ \frac{\partial M_2 / 2R}{\partial g} \\ \frac{\partial H_2}{\partial g} \\ \frac{\partial T / 2R}{\partial g} \end{bmatrix} \quad (2.24)$$

with the multiplier λ as previously defined.

Another of the aims of the experimental research will be to investigate the validity of Eqn 2.23. If it is appropriate, the experiments performed in the $T:V$ plane in this research to investigate this are described in chapters 4 and 5 to define values for β_3 , β_4 , and α_t .

2.3 Application of Model C to Structural Analysis of Jack-Ups

The main advantages of plasticity theory can be summarised below:

- Can express in stiffness matrix (terminology of structural engineers)
- Includes work hardening – accounts for non-linear behaviour when yielding, hence allows for the increase in capacity due to further penetration of the jack-up spudcans into soil
- Amenable to numerical analysis since response of the foundation expressed purely in terms of force resultants.

2.4 Summary

Classical bearing capacity for pure vertical loads, plasticity theories, and their application to the analysis of jack-up footings on sand have been reviewed here. This included a review of the strain-hardening plasticity model for sand called ‘Model C’. At the start of this chapter, the conservative nature demonstrated in classical bearing capacity theory has been shown, hence a need to move away from this highly empirical method to plasticity methods. This is because more accurate predictions (which can be

obtained from plasticity methods) are needed for jack-ups moving into deeper and harsher waters offshore. It is hoped that this more accurate prediction would lead to a substantial reduction in the size of the jack-up footings and the length of its legs for example whereas larger jack-up footings and length of its legs would be needed for the same magnitude of loads applied if more conservative methods are employed. Thus there is a potential saving in cost for the same magnitude of loads applied.

The general concept of plasticity theory was first illustrated by the stress-strain behaviour of metal from which the theories of elasticity and plasticity can be applied to the stress-strain behaviour of soils. Leading on from this, the plasticity model, entitled Model C, has been introduced first in three degrees of freedom (Cassidy, 1999) then to six degrees of freedom (Cassidy and Bienen, 2002) with the four major components associated with Model C explained. Incorporated in the six degrees of freedom model is the vertical-torsional load space and its corresponding vertical and torsional (rotational) displacements respectively. Experiments on circular footings in sand will be performed in the V - T space to validate the applicability of Model C six degrees of freedom in this load space.

3 SOIL CHARACTERISATION TESTS

This chapter describes the sample preparations and various common test procedures used to determine the properties of the sand. These included sieve analysis, minimum and maximum density tests, and direct shear box tests.

3.1 Soil Type – Super Fine (SF) Silica Sand

Commercially available super fine (SF) silica sand with a specific gravity (G_s) of 2.65 was used as the soil sample in all tests conducted. Minimum and maximum density tests were conducted according to the Australian Standards (AS 1289.5.5.1 – 1998) yielding minimum and maximum dry densities of 1.5167 t/m^3 and 1.8296 t/m^3 respectively. This corresponds to maximum and minimum voids ratio of 0.7472 and 0.4485 respectively (more details will be outlined in section 3.2). Sieve analysis was used to determine the particle size distribution, with the sand sample found to have an average grain size (d_{50}) of 0.190 mm, as shown in the grading curve obtained in Fig. 3.1 (Cheong, 2002). The critical state friction angle of 34.9° and a peak friction angle of 39.4° were determined though a direct shear box test (more details in section 3.4). These soil properties are listed in Table 3.1 along with other grain sizes. It was found that some values were comparable to values quoted by Blinco (2000) but others, such as e_{min} (found to be a critical parameter in determining relative density) differed quite substantially. Hopefully with the reanalysis of these soil properties, a greater confidence of the results of this research can be gained.

3.2 Minimum and Maximum Density Tests

Two methods were employed to determine the minimum and maximum dry density of a cohesionless material, using loose pouring to obtain the minimum dry density and vibratory compaction to obtain the maximum dry density.

3.2.1 Minimum Dry Density Test (to obtain maximum void ratio, e_{max})

The minimum dry density obtained (of the silica sand used) of 1.5167 g/cm^3 (equivalent unit, t/m^3) gives a maximum void ratio, e_{max} of 0.7472. A dry placement method

according to the Australian Standards (AS 1289.5.5.1 – 1998) was employed. This involved gently placing a sand sample into a cylindrical metal mould to the prescribed height so that the volume of sand in the mould can be accurately obtained (by measuring the internal diameter of the mould and the prescribed height). When placing the sand sample, a pouring funnel is used to allow the material to flow in a steady stream so that the free fall is not more than 20 mm. The pouring funnel is moved in a spiral motion from the outside towards the centre to form a layer of uniform thickness with the minimum of segregation (AS 1289.5.5.1 – 1998). When the sand is filled to the prescribed height of the mould, a straight edge is used to level it off from which the mass of sand in the mould is obtained. With the mass and volume known, the dry density can be obtained. This procedure was repeated three times and an average value for the minimum dry density obtained. The dimensions of the cylindrical metal mould, the mass of sand, the calculations, and the results of the three tests can be found in Appendix B1.

Another method to determine the minimum dry density of the sand was used as a check. This method is not according to the Australian Standards because a measuring cylinder was used instead of the cylindrical metal mould. The drawback with this method is the rough estimation of the volume simply read from the measuring cylinder after a sand sample is carefully placed by holding the measuring cylinder initially at a near horizontal position and tilted gradually (to a vertical position eventually) once the sand is scooped in (to ensure minimal free fall). Nevertheless the minimum dry density of 1.5259 g/cm^3 and the corresponding maximum void ratio, e_{max} of 0.7368 is quite close to the results obtained earlier from using the cylindrical metal mould according to the Australian Standards.

3.2.2 Maximum Dry Density Test (to obtain minimum void ratio, e_{min})

The maximum dry density obtained (of the silica sand used) of 1.8296 g/cm^3 gives a minimum void ratio, e_{min} of 0.4485. A wet placement method according to the Australian Standards (AS 1289.5.5.1 – 1998) was employed. This involves saturating the sand in water (left for half an hour to ensure full saturation). The saturated sand sample is placed into the same cylindrical metal mould used for the minimum density test. The cylindrical metal mould is then placed onto a vibrating table and vibrated for 5

min (at minimum amplitude) to remove the excess water from the saturated sand and to 'densify' the sand. A surcharge is then placed on top to further compact the sand, while vibrating for 10 min (at maximum amplitude). The sand is levelled off with a straight edge and transferred into a metal tin to be placed in an oven (at 105 °C) for 24 hours to dry from which the mass of the dry density is then obtained. This procedure was also repeated three times and an average value for the maximum dry density obtained. The dimensions of the cylindrical metal mould, the mass of sand, the calculations, and the results of the three tests can also be found in Appendix B1.

Another method to determine the maximum dry density of the sand was again used as a check. This method is not according to the Australian Standards because dry sand is used instead of wet (saturated) sand. The drawback with this method is the insufficient compaction of dry sand as opposed to the Australian Standards (AS 1289.5.5.1 – 1998) of using wet (saturated) sand. The procedure is the same as the wet placement method above, except dry sand is used and hence, an oven was not required. The maximum dry density of 1.7437 g/cm³ and the corresponding maximum void ratio, e_{max} of 0.5198 is obtained. The results from this method differ substantially from the wet placement method, nevertheless within 'ball park' figures. However, since this method is not according to the Australian Standards (AS 1289.5.5.1 – 1998), the results obtained here are discarded while the previous results obtained by the wet placement method are kept (see short discussion in section 3.5 as to the choice of keeping the e_{min} value obtained by the wet placement method according to the Australian Standards).

3.3 Sample Preparation – Dense and Loose Sand

3.3.1 Dense Sand Preparation

The dense sand sample is prepared by raining sand into the strongbox using an automatic sand rainer device constructed at the University of Western Australia, and shown in Fig. 3.2. The purpose of the sand rainer is to create a homogeneous, uniform dense sand sample. This is the best method available of obtaining homogeneous, uniform sand. Other methods such as vibration and compaction by a pneumatic drill have been considered but have their setbacks. The vibratory method which involves a side-to-side movement of the strongbox filled with sand causes segregation of the

particles, with the bigger particles falling to the bottom and the finer particles left at the top, hence creating a non-homogeneous, non-uniform soil. The pneumatic hammer procedure on the under hand causes crushing of the particles and layers of varying degrees of compaction hence also resulting in a non-homogeneous, non-uniform sand. Thus, a homogeneous, uniform dense sand sample is obtained via a sand rainer with the dry density of the sand determined from the internal dimensions of the strongbox and the mass of sand in the strongbox, from which the density index, I_d (also called relative density) of the sand can be determined. (A page of calculations is shown in Appendix B2 for Sand Sample # 18 as an example). After the strongbox is filled with sand, the top surface of the sand is vacuumed off to ensure a reasonably flat surface before the mass of the strongbox with sand is 'weighed'. The sand rainer consists of a sand hopper (enlargement shown in Fig. 3.3), which was set to traverse horizontally at 105 mm/s while raining down sand at a gap of 1.2 mm (measured using slip gauges) at a height of about 1 to 1.1 m. Essentially the width of this gap determines the degree of density of the sand. The finer the gap, the denser the sand. At a gap of 1.2 mm, the average density index of the dense samples used throughout this thesis is 86.4% with a difference of -6.1% , $+5.9\%$ as shown in Table 3.2a. Most of the error here comes from physical limitations in measuring the height of sand in the strongbox and the slight unevenness of the sand surface since a variation of just 2 mm in height measurement affects the density index by about 6%. Another source of error is the precision of the scales used to 'weigh' the mass of sand. Nevertheless the average density index of 86.4% lies within the 'state of compaction' classification (Whitlow, 1995) of 'very dense' sands (85%-100%). A sand sample with a relative index that lies within 65%-85% has a 'dense' 'state of compaction classification' (Whitlow, 1995).

3.3.2 Loose Sand Preparation

The loose sand sample is prepared by allowing the sand to fall slightly from a scoop into the strongbox. As with the preparation of the dense sand sample, after the strongbox is filled with sand, the top surface of the sand is vacuumed off to ensure a reasonably flat surface before the mass of the strongbox with sand is 'weighed'. The density index (I_d) of the sand is then determined in the same manner as with the preparation of the dense sand sample. The average density index of the loose samples

(shown in Table 3.2b) used throughout this thesis is 26.1%, which lies within the ‘state of compaction’ classification (Whitlow, 1995) of ‘loose’ sands (15%-35%).

3.4 Friction Angle via Direct Shear Box Test

The critical state and peak friction angles of the silica sand used were obtained through a direct shear box test using an automatic direct shear apparatus. The direct shear test involves placing a sample of soil under some constant normal stress (σ_n' of about 95 kPa) and driving a shearing action by a motor (the soil is sheared by applying a horizontal force at a constant rate of strain of 0.2 mm/min horizontally). The automatic direct shear apparatus consists of a computer, which automatically logs the vertical load, vertical displacement, horizontal load, and horizontal displacement throughout the entire test, which takes about 60 minutes.

A square metal box of approximately 100 x 100 mm that is split into two halves horizontally is used to house the sample of soil, in this case, dense silica sand. The dense sample is levelled, put under a pneumatic drill to further reduce the height of the sample and then the mass and volume is determined. From these values, the dry density, void ratio, and density index are calculated and tabulated in Table 3.3.

Three direct shear box tests with different top and bottom half interfaces (sand against sand, sand against aluminium, and sand against sand glued on aluminium) were performed on very dense sands in order to obtain the critical state and peak friction angles under these three interfaces. The direct shear box test with the ‘sand against sand glued on aluminium’ interface is performed to mimic the same test conditions conducted on the footing (glued with sand) in the strongbox. Hence, the critical state and peak friction angles obtained from the ‘sand against sand glued on aluminium’ interface will be used (i.e. Shear Box 3 sand sample in Table 3.3 with a density index of 87.4%). The friction angle (ϕ) can be calculated via the following formula found in Whitlow (1995):

$$\tan \phi = \tau / \sigma_n' \text{ (shear stress ratio).} \quad (3.1)$$

where τ is the shear stress (=horizontal load / area) and σ_n' is the constant normal stress. The critical state friction angle is computed at the residual τ / σ_n' point while the peak friction angle is computed at the maximum value of τ / σ_n' .

A plot of the shear stress ratio (τ/σ_n') versus shear displacement (mm) for each of these three interfaces is shown in Fig. 3.4, which follows the expected theoretical direct shear box test behaviour depicted in Fig. 3.5. The dry density (ρ_d), void ratio (e), density index (I_d), constant normal stress (σ_n'), critical state friction angle (ϕ'_{cv}), and peak friction angle (ϕ'_{peak}) for each of these three interfaces are shown in Table 3.3.

3.4.1 Peak Friction Angle Discussion

It has been observed that the peak friction angle (ϕ'_{peak}) of a soil is a function of both density index (I_d) and the mean effective stress (p') applied to the soil for p' greater than 150 kPa (Bolton, 1987) where p' can be approximated by the effective vertical stress (σ_v') since p' is usually less than σ_v' , hence this is a conservative assumption. However for any value of p' smaller than 150 kPa, the peak friction angle is only a function of the density index (I_d), since p' should be substituted by 150 kPa (Bolton, 1987). Therefore, the empirical formulation was given by Bolton (1987) as:

$$I_R = I_D \left[5 - \ln \left(\frac{p'}{150} \right) \right] - 1 \quad \text{for } p' > 150 \text{ kPa} \quad (3.2)$$

and can then be reduced to

$$I_R = 5I_D - 1 \quad \text{for } p' < 150 \text{ kPa} \quad (3.3)$$

where I_R is the relative dilatancy index, I_D is the density index (or relative density), and p' is the mean effective stress. The peak friction angle (ϕ'_{peak}) is dependent on I_R as given in the following equation by Bolton (1986):

$$\phi'_{peak} = 3I_R + \phi'_{cv} \quad (3.4)$$

Thus, although the direct shear box test is conducted at a relatively high constant vertical stress of 95 kPa compared to the actual stress applied by the footing on the soil in the sand sample strongbox (at least the initial loading stage), both vertical stresses are still less than 150 kPa. Hence the peak friction angle is only a function of density index. Thus the peak friction angle computed earlier from the direct shear box test's peak stress ratio is acceptable and should be adopted. The peak friction angle computed via Bolton's empirical formulation in Eqn 3.4 should only be used when a peak friction angle cannot be obtained directly from the direct shear box test [for instance if the

constant normal stress (σ_n) applied in the direct shear box test is well greater than 150 kPa (given that the actual stress applied by the footing on the soil in the sand sample strongbox is less than 150 kPa)].

3.5 Short Discussion

Detailed below is an added rationale (to that already given in section 3.2.2) as to why the minimum voids ratio of 0.4485 obtained via the Australian Standards (AS 1289.5.5.1 – 1998) is acceptable and hence, adopted.

For the direct shear box test detailed in section 3.4, the density index of sand (in the 100 by 100 mm square box) tabulated in Table 3.3 is calculated using the maximum and minimum voids ratio of 0.7472 and 0.4485 respectively (obtained in section 3.2) by following the Australian Standards (AS 1289.5.5.1 – 1998) procedure. Specifically, note that the minimum voids ratio (0.4485) obtained by following the Australian Standards (AS 1289.5.5.1 – 1998) procedure differed substantially from the value (0.5198) obtained by another method also described in section 3.2. The properties of sand in Shear Box 1 in Table 3.3 shows that using a minimum voids ratio of 0.4485, a very dense density index of 99.8% is obtained, whereas if the other minimum voids ratio of 0.5198 was used, a density index of 131.1% is calculated, which is ridiculously high (and greater than 100%), indicating a likely error in the latter minimum voids ratio value (being too high). This comparison (with calculations) is shown in Appendix B3.

4 EXPERIMENTAL EQUIPMENT AND PROCEDURES

This chapter describes the testing equipment, the computer set-up, and the calibration procedures. A series of ten pure vertical loading tests (seven on dense sand, three on loose sand), twenty-six vertical-torsion sideswipe tests (twenty-two on dense sand, four on loose sand) and one radial displacement test on dense sand were performed.

4.1 Combined Loading Apparatus and Computer Set-Up

A general overview of the testing equipment is shown in Fig. 4.1 – it is located in the Civil Engineering Laboratory at the University of Western Australia. The main components are a loading leg, a cone actuator, an Acer Pentium III computer, two 486 computers, and a rectangular sample strongbox with internal dimensions of 390 mm by 650 mm in plan and 325 mm high. The tests were carried out at 1g level (i.e. at earth's gravity).

An experimental apparatus has been developed to load a footing with vertical, horizontal, moment, and torsion loads. A main component of the experimental apparatus developed is the loading leg (shown in Figs. 4.2 and 4.3), consisting of axial, bending, and torsion strain gauges, which needs to be calibrated to convert the strain read in the gauges to actual forces in the member, at the position of the gauge. A circular footing (of various sizes, such as 20, 30, 40, and 50 mm radius – the latter three shown in Fig. 4.4) can be attached at the bottom of the loading leg. Since the circular footing is not permanently fixed, but removable, its proper attachment is crucial especially in tests involving torsional (rotational) movement such as vertical-torsional sideswipe tests (see Appendix A1 for a brief note on this).

The loading leg is placed in a loading-rig frame with three separate cone actuators (shown in Figs. 4.5a and 4.5b) that allow the leg to be driven by three motors that can apply rotational (in the torsional plane), horizontal, and vertical displacements independently (as shown in Fig. 4.2). The outer shaft of the loading leg is fixed to the motor-head piece and prevented from twisting, while the inner shaft, which extends down to where the footing is attached, is allowed to twist. The motors are controlled by two 486 computers (which operates in bits), which will be conveniently named Comp 2

and Comp 1 for easy reference shown from left to right in the photograph, Fig. 4.6. Comp 1 controls the motors (via Comp 2) with a control program that issues commands, which details the vertical and horizontal velocity, and also the torsional (rotational) velocity to drive the motor. Comp 2 uses the new Data Acquisition System (DAS-8), with Digital to Analogue Converter (DAC-02) cards, to read information from the 8 distribution channels located on a central electronic board labelled 'Geo Cal Trolley', which is connected to an electronic box labelled # 32, shown in Fig. 4.7. Box # 32 has bottom axial load, torsional load, torsional (rotational) displacement, torsional (rotational) velocity, overshoot, *Y* displacement, and *Z* displacement cables attached while another electronic box labelled # 8 has bottom bending, top bending, and top axial load cables attached in order to read some of these parameters generated by the gauges on the loading leg. The parameters from both electronic boxes are sent to the Acer Pentium III shown in Fig. 4.8. The Pentium III computer operates in Volts (V) and is used to log data using a data acquisition program called LabView (snapshots shown in Figs. 4.9a, 4.9b, 4.9c, and 4.9d) from both electronic boxes (LabView can log at a frequency of up to 100 Hz). Hence, calibration factors are needed to convert the Volts to the relevant units – for example, axial force is converted from Volts to N, bending from Volts to N mm, *Y* and *Z* displacements from Volts to mm, etc. (The procedure for determining these calibration factors will be explained in section 4.2). A schematic diagram of the entire interaction of the three computers is shown in Fig. 4.10.

4.2 Calibration of Loading Leg

Calibration factors are required to convert the strain read in the gauges to actual forces in the member, at the position of the gauge. Before the axial, moment, and torsion calibration factors were determined, a reference load cell (which will be called the J load, shown in Figs. 4.11a and 4.11b) was first calibrated.

4.2.1 J Load Calibration

The J load was calibrated against a known vertical force obtained by applying known weights onto the J load. The J load was taped onto a forklift arm and a spherical ball was placed on top of it. A hanger was then placed on top of the ball from which weights were placed onto it as shown in Fig. 4.12. The ball was used to make sure the vertical

load was transferred directly down onto the J load, without any eccentricity. The loading on the J load was displayed as voltage readings by LabView on the PIII computer and recorded each time an additional weight was placed on the J load, starting by recording the initial voltage reading before anything was placed onto the J load. The vertical load was plotted against its voltage reading as shown in Fig. 4.13 and the gradient of the line of best fit through this data was obtained. This gradient of 893.89 is the calibration factor for the J load with units of N/V, which is placed in a calibration file loaded by LabView. So subsequently, any data logged by LabView from the J load channel, the J load reading originally in V will be automatically multiplied by the J load calibration factor giving the resultant unit of N.

4.2.2 Axial Gauge Calibration

The calibration factors for the top and bottom axial gauges can now be obtained by calibrating against the J load. This was performed by first placing the J load onto the strongbox, with the ball resting on it. The loading leg in the loading-rig frame was carefully positioned so that the base of the loading leg just touches the ball, which sat on the J load. The loading leg was driven vertically downward (against the J load) to generate an axial force, which in turn affects the top and bottom axial strain gauges, whose readings were continuously logged by LabView. The axial reading on the J load was also logged. The top and bottom axial was logged in V while the calibrated J load was logged in N. The J load was plotted against the top and bottom axial loads as shown in Fig. 4.14. The gradient of the two lines of best fit gives the calibration factors for the top and bottom axial gauges of 351.27 N/V and 351.23 N/V respectively.

4.2.3 Moment Gauge Calibration

Calibration factors for top and bottom bending gauges were determined as follows. The J load was glued sideways onto an aluminium block, which rested against the side of the strongbox. This is shown in Fig. 4.15. The loading leg was carefully positioned against the J load, with the ball in between. The loading leg was then driven horizontally against the J load to generate bending moments in the top and bottom bending gauges, whose readings were continuously logged by LabView. The axial reading on the J load was also logged. The top and bottom bending was logged in V while the calibrated J

load was logged in N. The eccentricity from the point of application of the horizontal force to the top and bottom bending gauges were measured to be 232 mm and 172 mm respectively. The calibrated axial reading from the J load gave the horizontal force applied. Hence, the calibrated axial reading was multiplied by the eccentricity of 232 mm and 172 mm to give the actual applied bending moments on the top and bottom bending gauges respectively. The actual applied bending moments (N m) were plotted against the voltage readings from both top and bottom gauges as shown in Fig. 4.16. The gradient of the two lines of best fit gives the calibration factors for the top and bottom bending gauges of 10.263 Nm/V and 16.911 Nm/V respectively.

4.2.4 Torsion Gauge Calibration

4.2.4.1 Pure Torsion Calibration

The determination of the calibration factor for the torsion gauge was needed for most of the experiments conducted throughout this thesis. This involved setting up the loading leg from the edge of the strongbox with the centre core propped by a horizontal bar which extended across the sides of the strongbox as shown in Fig. 4.17a. The idea is to apply torsion loads (N mm) by hanging known weights (as shown in Figs. 4.17b and 4.17c) and calibrating against the torsion gauge readings (V). Equal weights had to be applied on either side of the rectangular bar at the same eccentricity to create pure torsion. The pulley system was needed to apply the upward force on the right-hand side (generated by hanging weights) as shown in Figs. 4.17a, 4.17b, and 4.17c. By plotting the applied torsion load (N mm) with its torsion gauge reading (V), the torsion calibration can be obtained. This was repeated at various eccentricities of 20, 40, 60, and 80 mm (Figs. 4.18a, 4.18b, 4.18c, 4.18d, 4.18e, 4.18f, and 4.18g) and an average torsion calibration was taken to be 1448.4 Nmm/V with an acceptable percentage difference of +3.6, -3.8 % as shown in Table 4.1a.

4.2.4.2 Combined Torsion Calibration with Axial Load

The calibration of the torsion strain gauge as described above was performed with no axial load applied. Another torsional calibration technique was later employed to check whether there was any interaction between the axial and torsion strain gauges (i.e. to

check whether the torsion calibration factor varied with various axial loads applied). The procedure was similar to that described above. The only modification was the attachment of the J load to a metal rod, which was attached along the central longitudinal shaft of the loading rig frame. The loading rig frame was then positioned on its side. The loading leg with a rectangular bar attached to its end (so as to apply torque by weights) was also oriented sideways. The set up of the loading rig frame (with the J load) and the loading leg (with the rectangular bar) with a ball in between them (to apply an accurate point load) is shown in Fig. 4.17d (obviously the loading leg and frame were clamped to prevent lateral movement before any axial load was applied). This enabled the loading leg to be compressed by driving the metal rod (in the loading rig frame) longitudinally against it, hence generating an axial (compressive) load, which was 'picked up' by the J load. With a specified axial load sustained (at a level that corresponds to axial loads reached from vertical loading tests), torque was then applied as shown in Figs. 4.17e and 4.17f (same procedure of hanging weights, using pulley system as described earlier). This was repeated with various specified axial loads of 0, 460, and 680 N (with calibration plots shown in Figs. 4.18h, 4.18i, and 4.18j) and an average torsion calibration was taken to be 1441.4 Nmm/V with an acceptable percentage difference of +2.4, -2.4 % as shown in Table 4.1b. There seemed to be an increase in the torsion calibration factor with increasing applied axial loads, but this is minimal. Moreover, the difference from the earlier torsion calibration obtained of 1448.4 Nmm/V is small. Hence, it is believed to be a reasonable assumption that there is no interaction between the axial and torsion strain gauges.

4.2.5 Torsional (Rotational) Displacement Calibration

The other task was to determine the calibration factor for rotational displacement (in the torsional plane). A rectangular piece was attached to the end of the load cell so that the torsional (rotational) displacement can be seen easily. The rectangular piece was rotated between 0 and 90 degrees several times, each time its voltage reading was recorded from LabView. By plotting the actual rotation in degrees versus the voltage reading from the PIII computer, the gradient of the line of best fit was found to be 37.274 deg/V as shown in Fig. 4.19. This is the calibration factor for torsional (rotational) displacement.

4.2.6 Horizontal and Vertical Displacement Calibration

The calibration for Y and Z displacements are described as follows. This was simply carried out by measuring the actual Y (horizontal) and Z (vertical) displacements and recording its voltage reading before and after it was displaced. This was repeated at various displacements. The actual Y and Z displacements were plotted up against its voltage reading from which the gradient of line of best fit was found to be -39.52 mm/V and -50.83 mm/V respectively as shown in Figs. 4.20 and 4.21.

4.2.7 Summary of Calibration Factors

The calibration factors are summarised in Table 4.2. These calibration factors are used to convert data obtained from volts to the relevant units.

4.3 Testing Programme

A total of thirty-seven successful tests (some tests had to be discarded, as discussed in Appendix A1) were carried out on circular footings subjected to pure vertical loads and combined vertical and torsion loads. Most of the tests conducted were on a 50 mm radius circular footing. However other footing radii of 20, 30, and 40 mm were also used. Since spudcan footings are circular in plan, circular footings were used in all tests performed in this thesis as similar behaviours are expected for the same size radius. The original three-degree of freedom Model C was also formulated from tests on circular flat footings (Gottardi *et al.*, 1999). A maximum of three tests of 50 mm radius circular footings can be conducted in the rectangular sample strongbox to avoid unwanted interactions between tests. The thirty-seven tests performed consisted of the following types of tests:

- Pure vertical loading
- Vertical-torsion sideswipe tests
- Radial displacement test

4.3.1 Pure Vertical Loading Tests

Pure vertical loading tests, which involved pushing the circular footing vertically into the sample were conducted in order to examine the vertical load plastic displacement behaviour, which forms the vertical strain hardening law for Model C. A series of ten of these tests were conducted (seven on dense sand, three on loose sand).

4.3.2 Vertical-Torsion Sideswipe Tests

Vertical-torsion sideswipe tests were performed to allow the direct investigation of the shape of the yield surface at a given penetration, which involved pushing the footing vertically to a prescribed level (resulting in a specified vertical load), keeping the vertical displacement constant and rotating (twisting) the footing. A series of twenty-six of these tests were conducted (twenty-two on dense sand, four on loose sand).

4.3.3 Radial Displacement Test in the Vertical – Torsion Loading Space

A radial displacement test in the vertical-torsion plane, which involved straight paths of vertical and torsional (rotational) displacements (i.e. constant gradient), was performed to provide information about the hardening law and the flow rule. One test on dense sand was conducted.

Experimental results will be discussed in the following chapter.

5 DISCUSSION OF RESULTS

This chapter presents and discusses the experimental results. A summary table of results both in the order of tests performed as well as rearranged according to test type (each separated into dense and loose sand) has been provided in Appendix C1 and Appendix C2 respectively. The results (rearranged according to test type) are presented in graphical format and can be found in Appendix C3 through to Appendix C7. A selection of the results obtained will be presented and discussed below according to test type.

5.1 Pure Vertical Loading Tests (Vertical Loading Space)

5.1.1 Pure Vertical Loading Tests on Dense Silica Sand

The aim of the pure vertical loading tests is to confirm the hardening law expression in Eqn 2.10. However, minimal tests of this type are needed since previous work has already been performed by Tan (1990) and Gottardi *et al.* (1999) on dense Leighton-Buzzard sand (a type of silica sand with $d_{50} = 0.8$ mm) from which this law is developed. However, new parametric values are required to fit the super fine dense silica sand ($d_{50} = 0.190$ mm) used here.

The vertical load increases as the footing is pushed vertically into the sample (during pre-peak behaviour) as shown in Fig. 5.1 for a 50 mm radius circular footing on dense sand. The vertical downward forces applied to the footing causes the vertical displacement of the footing, which must be resisted by an equal upward force by the bearing capacity (strength) of the sand; this is the vertical load that is sensed by the axial strain gauges. For all pure vertical-loading tests performed on dense sand (given in Appendix C3), the vertical axis shows the vertical load and the horizontal axis shows the plastic component of vertical displacement, which has been adjusted from the total (experimental, measured) vertical displacement by following the procedure given in the following section (5.1.1.1).

5.1.1.1 Procedure for the Evaluation of Vertical Plastic Displacement

The total footing (experimental, measured) displacement is equal to the sum of the elastic and plastic displacement components, i.e.

$$w^t = w^e + w^p \quad (5.1)$$

w^t is obtained from experiments. Before w^p can be obtained, w^e can be evaluated from the elastic stiffness matrix given previously in Eqn 2.7 involving V and w^e , i.e.

$$V = 2GRk_v w^e \quad (5.2)$$

where $k_v = 2.65$, a dimensionless elastic stiffness vertical factor given earlier. Hence G , the elastic shear modulus, is the only unknown needed.

Now, the general elastic response equation is

$$V = k^e w^e \quad (5.3)$$

where k^e is the elastic stiffness obtained from the experimental unloading lines of the unload-reload cycles (since elastic relaxation associated with unloading) performed in pure vertical loading tests.

Equating Eqns 5.2 and 5.3, the following expression is obtained:

$$k^e = 2GRk_v \quad (5.4)$$

With k^e now known from the unload-reload cycles, G can be evaluated from Eqn 5.4.

Hence at every data point, with V known and G (a constant) evaluated, the corresponding elastic component of vertical displacement, w^e can now be obtained from Eqn 5.2.

In the absence of unload-reload cycles, G can be estimated directly by Eqn 2.8 given earlier and then substituted into Eqn 5.2 to obtain w^e for each data point of load V .

With w^e evaluated, the plastic component of vertical displacement, w^p can be obtained from Eqn 5.1. A graphical representation of w^e , w^p , and w^t for soil is given in Fig. 5.2. Fig. 5.2 is analogous to the elastic and plastic strains for metal given in Fig. 2.3 earlier.

5.1.1.2 Adjusting for Vertical Plastic Displacements

With the unloading section of eight unload-reload lines taken from a number of pure vertical loading tests on dense sand (such as from Fig. 5.1), and shown in Fig. 5.3, the elastic vertical stiffness (defined by Eqn 5.3) can be estimated as $k^e = 2030 \text{ N/mm}$ by averaging the unloading section of the unload-reload lines. A large scatter in the elastic vertical stiffness is observed with a standard deviation of about 350 N/mm . This result is similar to the large scatter also observed for Gottardi and Houlsby's (1995) unload-reload lines as cited by Cassidy (1999). This reflects the difficulties in accurately measuring extreme stiff response and also very small displacements. Nevertheless, with the known k^e , the corresponding value of the shear modulus G from Eqn 5.4 is 7.66 N/mm^2 . This value is used to obtain the plastic component of vertical displacement by subtracting the elastic vertical displacement (evaluated from Eqn 5.2) calculated from the measured total displacement for each data point (i.e. applying Eqn 5.1). Similarly, Fig. 5.1 and all subsequent figures of this test type will show the adjusted plastic component of vertical displacement along the horizontal axis.

5.1.1.3 Elastic Behaviour Within the Yield Surface

Elastic behaviour can be best demonstrated by the unload-reload cycles performed throughout the vertical loading test, as shown by the linear unload-reload lines in Fig. 5.1. The vertical load drops during the unloading stage and stays within the yield surface but rises during the loading stage without much change in vertical displacement. To further scribe the yield surface, a greater vertical displacement is needed. The footing is pushed further downward vertically and usually passes through a peak for dense sand before levelling to a residual level, as can be seen in Fig. 5.1. This behaviour is typical for dense sand, corresponding to a dilative behaviour post-peak.

5.1.1.4 Verification of Strain-Hardening Law

All of the dense sand, pure vertical-loading data is shown in Fig. 5.4. This includes data for 20, 30, 40, and 50 mm footings. One test of three footing sizes have been selected and replotted in Fig. 5.5, with greater peak loads observed for increasing footing sizes. By normalising the load and displacements consistently, the hardening law can be

investigated. Figs. 5.4 and 5.5 are normalised as follows: the horizontal axis is normalised by dividing w^p by twice the footing radius $2R$ and the vertical axis is normalised by dividing V by $\gamma\pi R^3$ so that both axes are non-dimensional. The resulting normalised plots are shown in Figs. 5.6 and 5.7 respectively. The strain hardening law for Model C given by Eqn 2.9 (pre-peak behaviour) and Eqn 2.10 (both pre-peak and post-peak behaviour) are normalised accordingly and plotted as a best fit to the experimental load penetration curves in Fig. 5.7. The best-fit parameters are

$$V_{peak} = 1010 \text{ N}; \quad w_{pm} = 6.9 \text{ mm}; \quad k = 40 \text{ N/mm}; \quad f_p = 0.6.$$

Hence, the pure vertical loading tests performed confirms the hardening law expression in Eqn 2.10.

Numerical simulation of pure vertical loading tests can then be carried out based on the above parameters. It is shown earlier that V_{peak} could alternatively be estimated by applying Eqn 2.4, with reference to Appendix D1. A N_γ value can be obtained from the plot in Appendix D1 corresponding to $\beta = 180^\circ$ (since flat circular footing used in this thesis) and $\phi = \phi'_{peak} \approx 40^\circ$ (given earlier in Table 3.3). The peak friction angle of the sand is used since this corresponds to the peak bearing capacity (V_{peak}). V_{peak} for a 50 mm radius footing (i.e. the same radius used in the evaluation of the best-fit parameters above) can be evaluated as follows:

- $\alpha = 1$, rough (since sand is glued on the base of footing used)
- $N_\gamma \approx 130$ (estimated from Appendix D1)
- $\gamma = 17.5 \times 10^3 \text{ N/m}^3$ (from Table 3.2(a))
- $\therefore V_{peak} \approx 890 \text{ N}$

Indeed the value of V_{peak} calculated is reasonably close to that obtained from experiments (as shown by the V_{peak} best-fit parameter above).

5.1.1.5 Stress Levels for Different Size Footings

Finally, to observe the stress levels for different size footings, the vertical axis is normalised by dividing V by πR^2 and the horizontal axis is normalised as before. The resulting normalised plot (i.e. Fig. 5.5 normalised by area) is shown in Fig. 5.8. As expected, the larger size footings encountered larger vertical stresses as shown in Fig. 5.8. [Fig. 5.8 also shows that the stress levels obtained from the tests conducted is comparable to the constant normal stress of 95 kPa applied during shearing in the direct

shear box test performed (in section 3.4)]. It is also interesting to note that even for these small-scale model footings tested at 1g, this observation is seen. This suggests that the stresses encountered by actual size spudcans (being much larger) in the offshore field will also be much larger. The behaviour of actual size spudcans can be modelled in the centrifuge at 100g say (using a footing size of 100 mm diameter as a model to the prototype of 10 m diameter for example) to account for the larger stresses beneath the footing. It is expected that post-peak behaviour would not be seen for the prototype footing because with a much larger vertical stress encountered (which can be visualised as a confining stress) for the much larger footing, almost no dilative behaviour will be expected (i.e. particles of sand would not be able to ride up over each other).

5.1.2 Pure Vertical Loading Tests on Loose Silica Sand

Pure vertical loading tests on loose silica sand were also performed and shown in Appendix C4. An elastic vertical stiffness is estimated as $k^e = 1300$ N/mm for the loose sand tests. This is calculated by averaging the unloading section of four unload-reload lines shown in Fig. 5.9 with a similar large scatter in k^e observed (with a standard deviation of about 300 N/mm). However, the k^e obtained for loose sand is a substantial reduction from $k^e = 2030$ N/mm obtained earlier for dense sand, as would be expected since this reduction in stiffness is probably a function of the lower stress levels encountered. Hence a lower corresponding shear modulus G of 4.91 N/mm² is evaluated.

Just as for dense sand, the vertical load increases as the footing is pushed vertically into the sample as shown in Fig. 5.10 for various size footings (of radius 40 and 50 mm) on loose silica sand and also presented in a normalised plot shown in Fig. 5.11 for comparison. In Fig. 5.10, greater vertical loads are also observed with increasing footing size, as with dense sand. However, no observable peak loads are reached in all tests performed just as is envisaged by Byrne and Houlsby (2001) from their tests on loose carbonate sands. This is because an excessive overflow of sand onto the footing would be seen when it is driven down to a greater vertical penetration, eventually hitting the base of the metal strongbox (and hence reach a very large erroneous load). By overlaying the response of dense silica sand (Fig. 5.7) with loose silica sand (Fig. 5.11),

with both normalised plots shown in Fig. 5.12, some interesting comparisons can be drawn.

First, a similar behaviour is observed as shown in Byrne and Houlsby's (2001) plot of the response of loose carbonate sand with Gottardi *et al.*'s (1999) dense Leighton-Buzzard sand and shown in Fig. 5.13. The response observed in Figs. 5.12 and 5.13, to a degree, follows the typical stress-strain relationship obtained from a direct shear box test as depicted previously in Fig. 3.5 and also shown in Fig. 3.4 from a direct shear box test conducted. Dense sand is seen to undergo a dilative work-softening behaviour (post-peak) while loose sand (and dense sand pre-peak) is seen to undergo a contractive work-hardening behaviour. However, due to the overflow of sand (for tests in loose sand) as the depth of penetration increases, this causes an increasing overburden pressure $q = \gamma z$ (in the bearing capacity formulation, Eqn 2.1) with depth z . This accounts for the continual increase of vertical load with depth for loose carbonate sand response in Fig. 5.13, with the vertical load going past the expected residual load level (estimated from dense sand). Similar behaviour is observed in loose silica sand as shown in Fig. 5.12 with the vertical load continually increasing with depth, and if allowed to penetrate deeper, the vertical load will also go past the residual load level. These observations show that although an increase in bearing capacity with depth is obtained, the vertical load level in loose sand should not go past the residual load level for dense sand for surface footings; at most, should only converge at that level. This is consistent with the physical expectation for sands, with a higher load capacity expected for dense sand than for loose sand. However, although the bearing capacity for penetrated footings would be greater than that encountered by surface footings, the load capacity for loose sand should not keep increasing indefinitely with depth. Nevertheless, behaviour under 20% of the footing's diameter being penetrated is acceptable (as shown in Fig. 5.12 where only slight overflow is observed for dense sand at $w_p/2R = 0.2$).

The overflow of sand can be subtracted from the response obtained but since the main focus of this research is to investigate the expansion of the yield surface through vertical-torsion sideswipe tests (detailed later in section 5.2) which were conducted at a penetration level under 7% of the footing's diameter for dense sand (i.e. $w_p/2R < 0.07$ in Fig. 5.12), this is not necessary. For loose sand, the maximum penetration before a torsion "swipe" is conducted (as shown later in Table 5.2) is 36% of the footing's

diameter, however after examining Fig. 5.12, this level of $w_p/2R = 0.36$ is still less than the expected residual level (estimated from dense sand at $w_p/2R = 0.2$ in Fig. 5.12). Hence overflow is not of any significant concern in this research.

Besides the overflow of sand pointed out previously, the other likely explanation for the experimental behaviours differing from theoretical (as demonstrated in both Figs. 5.12 and 5.13) is the inaccurate modelling of pure vertical loading tests at 1g level. Centrifuge modelling (at 100g for example) is not only capable of taking into account the much larger magnitude of stresses beneath the same sized footing at 100g than at 1g but can also adequately account for vertical strains. Hence, future work in the centrifuge for pure vertical loading tests on sand (especially loose sand) is recommended.

5.2 Vertical-Torsion Sideswipe Tests (Vertical – Torsion Loading Space)

5.2.1 Swipe Tests on Dense Silica Sand

A number of vertical-torsion sideswipe tests were performed (on dense silica sand with an average relative density of 86%, given earlier in section 3.3.1) for different size footings of radius 20, 30, and 50 mm. Results for these experiments can be found in Appendix C5. These torsion swipes were performed as part of the experimental verification of Model C six degrees of freedom in analogy to previous work by Gottardi *et al.* (1999) with horizontal and moment swipes performed to determine Model C in three degrees of freedom in the (V, H, M) space. The horizontal and moment swipes were performed on dense Leighton-Buzzard sand (larger grain diameter $d_{50} = 0.8$ mm). Swipe tests were conducted at $V = 1600$ N with $V_{peak} = 2050$ N obtained from pure vertical loading tests. For the experiments outlined here (with dense silica sand $d_{50} = 0.190$ mm and for the same size footing), V_{peak} was around 1150 N (as shown in Fig. 5.1). Swipe tests at $V = 1000$ N were commonly conducted.

Since all vertical-torsion sideswipe tests are performed on sand, only compression loading is applicable (i.e. no tension loading). In order for torque to be developed, there must be a compressive force between the footing and the sand beneath it. If the axial load applied is zero, no torque can be developed when twisting the footing.

5.2.1.1 Vertical-Torsion Swipes at $V_0 = 1000$ N

An example of a vertical-torsion sideswipe test is shown in Fig. 5.14(a). This is a typical curve traced out for a swipe test and was performed in this manner:

- A 50 mm radius circular footing was pushed to a prescribed vertical load level (as shown by the reasonably straight line along the horizontal axis from 0 N to V_0 of about 1000 N)
- The vertical displacement was then kept constant and the footing twisted.

As the footing is twisting, the bearing capacity of the soil reduces (i.e. the vertical load reduces), while the torsional load increases from zero to a peak then drops back to zero as the soil is sheared to failure with further twisting applied to the footing. The curve traced out in Fig. 5.14(a) can be assumed to be a yield surface. As can be seen from Fig. 5.14(d), which shows the plot of T versus the plastic component of torsional (rotational) displacement θ_t^p for the first few degrees of the test (i.e. a subset of Fig. 5.14(c)), a peak torsion of about 2500 Nmm is reached only after 2 degrees of twist. (The plastic component of torsional displacement is adjusted from measured torsional displacement in a similar manner to the adjustments for vertical plastic displacement previously described). This demonstrates why torsion is incorporated into the plasticity model (Model C) for jack-up spudcan footings on sand, since it only takes a slight twist of these footings to reach peak torsion, more critically, the peak bearing capacity V_0 (i.e. maximum vertical load reached) is halved as shown in Figs. 5.14(a) and 5.14(b). Fig. 5.14(b) is a normalised plot of Fig. 5.14(a) obtained by dividing V by V_0 and T by $2RV_0$ so that both axes are non-dimensional (R being the radius) and shows peak torsion occurring at V/V_0 of about 0.5 (i.e. halved). Two other sideswipe tests on 50 mm circular footings, also swiping at around 1000 N, were performed so that the behaviour observed in Fig. 5.14 was repeatable. All three tests are shown in Fig. 5.15, and indeed, the behaviour observed is consistent with that observed in Fig. 5.14.

So far the observation of swipe tests has been at one V_0 level of 1000 N. More tests were conducted to observe behaviours at various V_0 levels as discussed in the following section (5.2.1.2) in an attempt to answer the question posed in section 2.2.2.1 as to whether the value of t_0 is a constant or will it vary according to the vertical stress ratio of V_0/V_{peak} ?

5.2.1.2 Vertical-Torsion Swipes at Various V_0

From the vertical-torsion sideswipe tests performed, a number of observations can be made. Starting with Fig. 5.16(a), which shows the V - T plot of six yield surface curves obtained by swiping at various V_0 , it can be observed that greater peak torsional loads are obtained with swipes at increasing V_0 . This plot demonstrates the expansion of the yield surface in the V - T plane with increasing vertical load, which is the result of increasing vertical penetration. By normalising Fig. 5.16(a) as described previously, with the resulting plot shown in Fig. 5.16(b), it can be seen that the expansion of the yield surface is not constant as shown by the variation of $t_0 = T/2RV_0$ (by definition, $T = T_m$, the maximum value of T for this t_0 expression). In fact, with swipes at increasing V_0 , the value of t_0 reduces as shown in Figs. 5.16(a) and 5.16(b). Fig. 5.16(c) shows the normalised plot of $T / (2RV_0)$ versus $2R\theta_t^p$ (for dimensional consistency) where θ_t^p is the plastic component of torsional (rotational) displacement expressed in radians here. Again, lower t_0 values (for the same level of torsional (rotational) displacement) corresponding to swipes at larger V_0 are shown in Fig. 5.16(c).

This observation was discussed in section 2.2.1.1.1 and 2.2.2.1 of the literature survey, with reference to the Byrne and Houlsby (2001) proposal that the non-dimensional parameter h_0 also not being constant (in dense sand) but varies according to V_0/V_{peak} level (Eqn 2.6). A similar formulation can be derived for the variation of t_0 (also a non-dimensional parameter) by plotting t_0 versus V_0/V_{peak} for all tests of this type performed in this research. All of the experimental tests of this type can be found in Table 5.1 and are also shown graphically in Fig. 5.17. The formulation of the best-fit curve is

$$t_0 = t_{0peak} \left[1 - 0.3846 \ln \left(\frac{V_0}{V_{peak}} \right) \right] \quad (5.5)$$

where V_0 is the vertical load where torsional swipe occurs, V_{peak} is the maximum vertical load capacity, and $t_{0peak} = 0.0234$ is the torsional dimension for the yield surface at peak bearing capacity. Eqn 5.5 was validated for $0.1 < V_0/V_{peak} < 1$.

So far the observation of swipe tests has been for one footing size of 50 mm radius. More tests were conducted to observe behaviours for various size footings as discussed in the following section (5.2.1.3).

5.2.1.3 Vertical-Torsion Swipes for Various Size Footings

A similar observation can be made from sideswipe tests on different size circular footings. Fig. 5.18(a) shows that the larger the footing, the greater the torsional loads generated (with all tests swiping at the largest possible V_0 pertaining to the footing size). With the exception of the 20 mm radius footing, the value of t_0 is also seen to decrease with swipes at increasing V_0 , as shown in Figs. 5.18(a) and 5.18(b). However, doubt should be cast on the results of the 20 mm radius footing. This is because its size is too small and with maximum axial and torsion loads generated of only 58 N and 54 Nmm respectively, it is difficult to read the results. Also, the “jaggedness” along the horizontal axis, as shown in Fig. 5.18(b) associated with the slight drift in the torsion control system produced a significant amount of torsion fluctuation even before the “swipe” is started. Torsional drift is also seen in tests on larger size footings, however since the footings are larger, the percentage error is smaller. Ideally, with absolutely no torsional drift, a perfectly straight horizontal line along the horizontal axis should be seen as this corresponds to no torsion load generated before the swipe. As seen previously, it only takes a few degrees of torsional rotation to cause a peak torsion load; so even with the slightest of torsional rotation, a torsion load is generated.

Further swipe tests are conducted to establish whether the yield surface defined by Eqn 2.19 (discussed in section 2.2.2.1) is an appropriate shape. This is examined in the following section (5.2.1.4).

5.2.1.4 More Vertical-Torsion Swipes (together with those presented so far)

A few other sideswipe tests, together with those presented so far, can be placed together in a single plot as shown in Fig. 5.19(a). Since each test has a different t_0 value (which varies according to V_0 as shown earlier in Fig. 5.16(b)), therefore, in order to compare tests, Fig. 5.19(a) is normalised by dividing V by V_0 and T by $2Rt_0V_0$ where t_0 is obtained from each experiment (shown in Table 5.1), with the resulting plot shown in Fig. 5.19(b). (Alternatively t_0 could be obtained from Eqn 5.5, but since this is an empirical fit, t_0 obtained from each experiment is more accurate). Theoretically, a vertical-torsion sideswipe fit in the $T:V$ plane is expected; this is given by Eqn 2.19 previously.

The curves obtained in Fig. 5.19(b) at almost the same path indicates a consistent behaviour described by Eqn 2.19, which is also normalised accordingly and shown in Fig. 5.19(b), by experimental best-fit. The best-fit parameters are $\beta_1 = 0.45$, $\beta_2 = 0.55$, and $t_0 = 0.025$ (t_0 used here corresponds to the experimental value for a 50 mm radius footing at $V_0 = 1000$ N). Note that Eqn 2.19 is limited to describing the experimental swipes from $V/V_0 = 1$ to about $V/V_0 = 0.1$. Once the swipe reaches $V/V_0 = 0.1$, a straight line is obtained as shown in Fig. 5.19. This line is believed to be the critical state line and is proposed to correspond to the pure shearing mechanism at the base of the footing. Thus a piecewise function is required to describe the yield surface in the V - T space as a refinement to the current six-degree of freedom Model C proposed by Cassidy and Bienen (2002).

5.2.1.5 Proposed Yield Surface Equation in V - T Space

This function can be derived theoretically by assuming a triangular stress distribution beneath the circular footing (on sand), which is shown in Fig. 5.20 with the stress distribution given by

$$\sigma(r) = \sigma_{\max} \left(1 - \frac{r}{R} \right) \quad (5.6)$$

where $\sigma_{\max}$ is the maximum vertical stress, R the radius of the footing, and r an auxiliary parameter. For sand, a simple Coulomb failure criterion (for pure shearing) is assumed thus:

$$\tau(r) = \sigma(r) \tan \phi \quad (5.7)$$

where ϕ is the internal friction angle of the sand, $\sigma(r)$ is the vertical stress defined by the assumed stress distribution in Eqn 5.6, and $\tau(r)$ is the shear stress.

The infinitesimal change of loads (dV , dT) are represented and given in Fig. 5.21. Therefore, by integrating the stresses over the radius R , the total vertical load obtained is given by

$$V = \int_0^R \sigma(r) (2\pi r) dr \quad (5.8)$$

and the total torque generated is

$$T = \int_0^R \tau(r)(2\pi r)dr \quad (5.9)$$

where $\sigma(r)$ is again defined by the assumed stress distribution in Eqn 5.6.

Thus Eqn 5.8 is evaluated as

$$V = \frac{1}{3} \pi R^2 \sigma_{\max} \quad (5.10)$$

and Eqn 5.9 as

$$T = \frac{1}{6} \pi R^3 \sigma_{\max} \tan \phi \quad (5.11)$$

Therefore the following linear (straight line) equation to describe the critical state line behaviour is obtained:

$$\frac{T}{2RV} = \frac{1}{4} \tan \phi \quad (5.12)$$

where $\phi = \phi'_{cv} = 34.9^\circ$ (as found in section 3.4 and given in Table 3.3).

The complete derivation of Eqn 5.12 can be found in Appendix E1. With Eqn 5.12 normalised accordingly, it is shown in Fig. 5.19(b) in the range of $V/V_0 = 0$ to about $V/V_0 = 0.1$. It fits the experiments quite well. Theoretical derivations for other stress distributions, shown in Appendix E2, were considered before the triangular stress distribution was chosen to fit the experiments best. Thus the six-degree of freedom Model C in the vertical-torsion space has been successfully verified and refined by Eqn 5.12. When combined with Eqn 2.19 the full yield equation can be written as:

$$f(V, T/2R) = \frac{T}{2RV} - \frac{1}{4} \tan \phi = 0 \quad \text{for } 0 \leq V/V_0 \leq 0.1 \quad (a)$$

$$= \left(\frac{T/2R}{t_0 V_0} \right)^2 - \left[\frac{(\beta_1 + \beta_2)^{(\beta_1 + \beta_2)}}{\beta_1 \beta_1 \beta_2 \beta_2} \right]^2 \left(\frac{V}{V_0} \right)^{2\beta_1} \left(1 - \frac{V}{V_0} \right)^{2\beta_2} = 0 \quad \text{for } 0.1 \leq V/V_0 \leq 1.0 \quad (b)$$

(5.13)

The complete interaction for the vertical-torsion sideswipe curve can be illustrated in Fig. 5.22, noting the following:

- The swipe starts at the bearing failure line
- Ends with a critical state line (which corresponds to the pure shearing mechanism) and
- A transitional curve enclosed between the two lines.

The critical state line behaviour can be observed quite prominently for some tests, such as shown in Fig. 5.19, with a pointed peak (torsion) occurring at the top of the straight line. Interestingly this behaviour tends to occur with swipes at lower V_0 for the 50 mm radius footing. This is most likely due to the tendency for sand particles to dilate (for swipes at lower V_0) during the pure shearing phase. At lower V_0 , the sand particles beneath the footing are placed under a less-confined vertical pressure, hence when a twist is applied to the footing, the particles tend to ride up over each other, resulting in the pointed peak (torsion) occurring at the top of the straight line, i.e. at the start of the pure shearing phase.

The following tests are conducted to examine the “proof-testing” pre-loading procedure (as described in section 1.1.1) and are discussed below.

5.2.1.6 Vertical-Torsion Swipes at Various Fraction of V_0 (after unloading)

These vertical-torsion swipe tests involved loading the 50 mm radius footing to V_0 of around 1000 N, then unloading it to various fraction of V_0 before the “swipe” takes place as shown in Fig. 5.23. This type of test corresponds to the pre-loading, “proof testing” procedure. Here, $V_0 \approx 1000$ N in Fig. 5.23(a) corresponds to the pre-load level, and unloading to say, $0.65 V_0$ as shown in Fig. 5.23(b) before a sideswipe is performed. This sideswipe corresponds to a likely torsional loading generated by twisting effects from a potential cyclone passing by. The outer yield surface obtained by swiping at $V_0 \approx 1000$ N defines the boundary between elastic and elastic-plastic behaviour, corresponding to a “safety margin”. Any footing behaviour (such as swipes at lower fraction of V_0) within this surface is assumed to be elastic (i.e. safe) while the elastic-plastic behaviour occurs once the load reaches the yield surface. Therefore, by pre-loading to a greater V_0 possible, the better the “proof testing” since a greater safety margin exists with a higher pre-load.

5.2.1.7 Various Combinations of Vertical-Torsion Swipe Tests

Finally, a few of the tests (with swipes performed after unloading from a pre-load of 1000 N) shown previously in Fig. 5.23 are replotted in Fig. 5.24 together with similar test results obtained by purely swiping at a similar V_0 level but without pre-loading

before the swipe. It is interesting to note that the swipes performed after pre-loading has a steeper shape curve at the start of the swipe as can be seen in Fig. 5.24(a), meaning that torsional load picks up initially without much loss of vertical load capacity. The reason for this behaviour is due to the pre-loading beforehand exposing the footing to a larger vertical load as shown by the larger yield surface swiped at 1000 N, hence the swipe performed after unloading (from a pre-load level) is within the elastic region with the steep line corresponding to linear-elastic behaviour at the start of the swipe. The other tests (at about the same V_0 level) performed without pre-loading before the swipe, shows a less-steep, more rounded behaviour at the start of the swipe as shown in Fig. 5.24(a), corresponding to the elastic-plastic yield surface. Both sets of tests (at about the same V_0 level) reach a similar peak torsion load (again shown in Fig. 5.24(a)) although showing different behaviours at the start of the swipe.

5.2.2 Swipe Tests on Loose Silica Sand

Four vertical-torsion sideswipe tests were conducted on loose silica sand. The average relative density was 26% as was discussed in section 3.3.2. The swipes were performed at various V_0 values. This is similar to the dense sand but at lower V_0 levels. The T - V plots are shown in Fig. 5.25(a). Again, the results are normalised as before to compare between tests of different V_0 and these results are shown in Fig. 5.25(b). Similar behaviour is seen with the expansion of the yield surfaces shown in Fig. 5.25(a) with greater torsional load obtained for swipes at increasing V_0 . Again, when the swipes (starting at $V/V_0 = 1$) reach a V/V_0 level of 0.1, a straight line is scribed as shown in Fig. 5.25(b) and is believed to correspond to the critical state line (just as seen for dense sand) obtained from pure shearing occurring between the footing and sand. However, peculiar to loose sand is the looping behaviour seen at the start of the critical state line at around $V/V_0 = 0.1$, again shown in Fig. 5.25. This could correspond to dilation occurring as the sand particles are sheared up over each other resulting in a higher torsion load encountered as shown by the looping behaviour. This is peculiar since dilative behaviour is not usually seen for loose sand; rather a contractive behaviour is usually expected. Nevertheless, the dilative behaviour shown here could indicate that the voids (air pockets) present in the sand are quite low (at low confining stress), i.e. at a 'denser than critical' state, hence the tendency for particles to dilate.

A slightly different trend could also be seen in Fig. 5.25(b) with only slight variation of t_0 with different levels of V_0 (in Fig. 5.25(a)) for loose sand. (For dense sand, t_0 was seen to decrease with increasing V_0 as shown previously in Fig. 5.16(b)). Therefore, for loose sand, t_0 is assumed constant for all V_0 . It is also similar in magnitude to the t_{0peak} value of the dense sand relationship outlined in Eqn 5.5. By plotting t_0 obtained for each test (data shown in Table 5.2) versus penetration depth (normalised by $2R$), with the resulting plot shown in Fig. 5.26, only a slight increase in t_0 can be seen with penetration depth, hence the assumption of t_0 being constant for all V_0 is reasonable. Constant t_0 implies a constant expansion of the yield surface (in the torsion plane) for loose sand (whereas for dense sand the expansion of the yield surface is not constant as shown by the variation of t_0 earlier).

The same piecewise function given earlier by Eqn 5.13 for dense sand is also valid for loose sand. Eqn 5.13 is normalised accordingly and plotted in Fig. 5.25(b) by experimental best fit (with a constant t_0 assumed, the vertical axis need not be normalised by t_0 as required for dense sand). The best-fit parameters are however slightly different for loose sand with $\beta_1 = 0.4$ and $\beta_2 = 0.45$ in order to fit the experiments best. Hence, the curvature (given by the curvature factors β_1 and β_2) also varied slightly (from dense sand). The theoretical curve (defined by these best-fit parameters) in Fig. 5.25(b) fits the experiments reasonably well except for the start of the swipe from $V/V_0 = 1$ to $V/V_0 = 0.9$ (approx) with the experimental curves seem to be over-predicted by the theoretical line (i.e. the experimental curves seen to lie under the theoretical line at the start of the swipe). This difference can be explained by a slight “relaxation” (i.e. reduction) of the vertical load capacity V_0 before the “swipe” is started. This is due to the manual operation of the “swipe” involving the stopping of vertical displacement (motor), followed by the starting of torsional (rotational) displacement (motor) being difficult to be performed instantaneously. Other than this, the rest of the experiments follow the theoretical curve quite well.

5.2.3 Summary of Vertical-Torsion Swipes on Dense and Loose Sands

In summary, similar behaviours are observed for vertical-torsion sideswipe tests on both dense and loose silica sands, however, there are some differences, as highlighted below:

Dense Silica Sand ($I_d = 86\%$, 22 tests)		Loose Silica Sand ($I_d = 26\%$, 4 tests)	
t_0	0.02 $\rightarrow$ 0.04 (Table 5.1 and Eqn 5.5) i.e. expansion of yield surface not constant	t_0	0.029 average (Table 5.2) i.e. t_0 is assumed constant (since small variation)
t_0	0.025 (for $R = 50$ mm, $V_0 = 1000$ N) For Eqn 5.13(b) ($0.1 \leq V/V_0 \leq 1.0$):		For Eqn 5.13(b) ($0.1 \leq V/V_0 \leq 1.0$):
β_1	0.45	β_1	0.40
β_2	0.55	β_2	0.45

For $0 \leq V/V_0 \leq 0.1$, Eqn 5.13(a) describes swipes of both dense and loose silica sands, with $\phi = \phi'_{cv} = 34.9^\circ$.

With a substantial number of torsion swipes performed in this research, the observations above can be confidently recorded. Prior to this research, only a limited number of torsion swipes (by Platt, 1998) of flat circular footings on sand were conducted with $t_0 = 0.053$ for Leighton Buzzard sand and $t_0 = 0.059$ for Goodwyn sand (as cited by Cassidy and Bienen, 2002), both values being considerably larger than the t_0 values obtained in this research for dense and loose silica sand. In Platt (1998), only two torsion swipes for each sand type were conducted, using a different type of testing apparatus at the University of Oxford. With only four tests, further discussion with the researchers at Oxford University is recommended before the results of Platt (1998) are fully considered.

5.2.4 Comparison of Vertical-Torsion Swipe With Vertical-Horizontal Swipe

A brief comparison will be made here between vertical-torsion loading (as part of the six degrees of freedom model) and vertical-horizontal loading (established in the three degrees of freedom model). To do this, comparisons will be made between non-dimensional parameters t_0 and h_0 . The average value of t_0 obtained experimentally is approximately 0.029 for loose sand and 0.03 for dense sand (i.e. mid-way between t_0 varying from 0.02 to 0.04). The value for h_0 obtained on the other hand is much larger,

with a value of 0.116 recommended by Cassidy (1999) (see section 2.2.1.1 for details). These are shown in Fig. 5.27 with a typical yield surface “swipe” corresponding to each of these non-dimensional parameters t_0 and h_0 and the relevant best-fit parameters (such as β_1 , β_2 , etc) given previously. It is clear from Fig. 5.27 that the non-dimensional loads generated from a torsional swipe are much smaller than the loads generated from a horizontal swipe with $t_0/h_0 = 0.029/0.116 = 0.25$.

This difference may be attributed to the way in which the sand particles beneath the footing are being “sheared to failure”. A circular shearing motion corresponding to torque applied and a horizontal sliding motion corresponding to horizontal load applied during the “swipe” are depicted in Fig. 5.28. Just as the theoretical formulation given by Eqn 5.12 was derived for torsion loading during the pure shearing phase (i.e. assuming pure friction), the theoretical formulation for horizontal loading (during the pure shearing phase) can also be derived as follows (again, by assuming a triangular stress distribution beneath the circular footing shown in Fig. 5.20 earlier):

The stress distribution is given by

$$\sigma(r) = \sigma_{\max} \left(1 - \frac{r}{R} \right) \quad (5.14)$$

where $\sigma_{\max}$ is the maximum vertical stress, R the radius of the footing, and r an auxiliary parameter. For sand, a simple Coulomb failure criterion (for pure shearing) is assumed thus:

$$\tau(r) = \sigma(r) \tan \phi \quad (5.15)$$

where ϕ is the internal friction angle of the sand, $\sigma(r)$ is the vertical stress defined by the assumed stress distribution in Eqn 5.14, and $\tau(r)$ is the shear stress.

The infinitesimal change of loads (dV , dH) are represented and given in Fig. 5.29. Therefore, by integrating the stresses over the radius R , the total vertical load obtained is given by

$$V = \int_0^R \sigma(r) (2\pi r) dr \quad (5.16)$$

and the total horizontal load obtained is

$$H = \int_0^R \tau(r) (2\pi r) dr \quad (5.17)$$

where $\sigma(r)$ is again defined by the assumed stress distribution in Eqn 5.14.

Thus Eqn 5.16 is evaluated as

$$V = \frac{1}{3} \pi R^2 \sigma_{\max} \quad (5.18)$$

and Eqn 5.17 as

$$H = \frac{1}{3} \pi R^2 \sigma_{\max} \tan \phi \quad (5.19)$$

Therefore

$$\frac{H}{V} = \tan \phi \quad (5.20)$$

where $\phi = \phi'_{cv} = 34.9^\circ$ (as found in section 3.4 and given in Table 3.3).

Comparing Eqn 5.12 with Eqn 5.20, it is obvious that the ratio $(T/2RV_0)$ is $\frac{1}{4}$ of the ratio (H/V) just as seen earlier with $t_0/h_0 = 0.029/0.116 = 0.25$ “experimentally”. The theoretical derivations therefore confirm the experimental observations that the non-dimensional loads generated from a torsional swipe are much smaller than the non-dimensional loads generated from a horizontal swipe. Interestingly the theoretically derived ratio of $\frac{1}{4}$ is the same as the experimental peak ratio. This implies that torsional loading does not have such a severe interaction as horizontal loading have with vertical load.

5.3 Radial Displacement Test (Vertical – Torsion Loading Space)

One radial displacement test in the V - T plane on dense sand was conducted to provide information about the hardening law and the flow rule. This test involves straight paths of vertical and torsional (rotational) displacements (i.e. constant gradient) as shown in Fig. 5.30(d). This test was for a very small amount of torsion displacement when compared to vertical displacement and therefore the experimental reading jumps between values (a best-fit curve has been shown in Fig. 5.30(d)). The corresponding V - T plot is shown in Fig. 5.30(a) showing the simultaneous increase of vertical and torsional loads at constant gradient as a result of the simultaneous vertical and torsional (rotational) displacements applied at a constant ratio.

The angle of “experimental” plastic displacement ratio is defined from the horizontal axis as shown in Fig. 5.31. The angle is given by

$$\eta = \tan^{-1} \left(\frac{d2R\theta_t^p}{dw^p} \right) \quad (5.21)$$

From Fig. 5.30(d), the angle of plastic displacement ratio can be evaluated showing the direction of plastic flow at an angle of $\eta = 12.2^\circ$. This incremental “experimental” plastic displacement vector (at the evaluated angle of 12.2°) can be plotted as arrows (marked ‘Exp’) as shown in Fig. 5.32, with selected $(V, T/2R)$ points from Fig. 5.30(b) (at intervals of $V = 250$ N approx) from which the corresponding yield points V_0 can be evaluated by solving Eqn 2.19. The following best-fit parameters obtained earlier (for dense sand) are used to solve Eqn 2.19, i.e. using $\beta_1 = 0.45$, $\beta_2 = 0.55$, and for simplicity, t_0 assumed to be 0.025 (t_0 has not been varied since this does not effect the overall general trend to be illustrated), from which the theoretical yield surfaces can be plotted as shown in Fig. 5.32.

Within plasticity theory, if associated flow is assumed in the V - T plane, the theoretical change, in vertical displacement and torsional (rotational) displacement (analogous to the theoretical change in horizontal displacement and rotation for the three degrees of freedom model given in Cassidy, 1999), can be derived for the yield surface equation of Eqn 2.19 as

$$\partial w^p = \lambda \frac{\partial f}{\partial V} = \lambda \left(-2 \left[\frac{(\beta_1 + \beta_2)^{(\beta_1 + \beta_2)}}{\beta_1^{\beta_1} \beta_2^{\beta_2}} \right]^2 \left(\frac{V}{V_0} \right)^{2\beta_1} \left(1 - \frac{V}{V_0} \right)^{2\beta_2} \left(\frac{\beta_1}{V} - \frac{\beta_2}{V_0} \left(1 - \frac{V}{V_0} \right)^{-1} \right) \right) \quad (5.22)$$

$$\partial \theta_t^p = \lambda \frac{\partial f}{\partial (T/2R)} = \lambda \left(2 \left(\frac{1}{t_0 V_0} \right)^2 \left(\frac{T}{2R} \right) \right) \quad (5.23)$$

where λ is a multiplier which can be derived from the condition of continuity with the strain hardening law (evaluation of partial directional derivatives, $\partial f / \partial V$ and $\partial f / \partial (T/2R)$ can be found in Appendix E3, showing full working).

The arrows marked ‘Theory’ (in Fig. 5.32) indicate the “theoretical” plastic flow direction vector if associated flow is assumed, whose direction is again defined by the angle from the horizontal axis evaluated by taking the $\tan^{-1}$ of Eqn 5.23 divided by Eqn 5.22 as follows (the same λ assumed):

$$\alpha = \tan^{-1} \left(\frac{\frac{\partial f}{\partial(T/2R)}}{\frac{\partial f}{\partial V}} \right) \quad (5.24)$$

By applying Eqn 5.24, the angle obtained at those four positions shown in Fig. 5.32 is evaluated to be approximately $\alpha = 80^\circ$.

Before progressing further, the concept of associated flow will be illustrated. Associated flow can be best demonstrated by pure vertical-loading tests or by the initial stages of vertical-torsion sideswipe tests prior to the “swipe”. This is best illustrated by drawing a few theoretical swipes (defined by Eqn 2.19) at various values of V_0 using the best-fit parameters evaluated earlier for dense sand (i.e. using $\beta_1 = 0.45$, $\beta_2 = 0.55$, etc). This is shown in Fig. 5.33. The arrows marked “Exp” along the vertical load axis in Fig. 5.33 represents the “experimental” incremental plastic displacement direction vector expected as a result of pure vertical displacement (since no torsional (rotational) displacement exists before the “swipe” nor for pure vertical loading tests). The arrow marked “Theory” corresponds to the plastic flow direction vector (perpendicular to the yield surface) at the start of the swipe. Since both arrows “Exp” and “Theory” are at a zero degree angle (i.e. the same direction), this means that the “experimental” incremental plastic displacement direction vector is perpendicular to the tangent line at each of those points in Fig. 5.33, therefore, associated flow is illustrated. Essentially the strain-hardening law established by pure vertical loading tests can also be regarded as a type of radial displacement test exhibiting associated flow.

5.3.1 Adjusting Plastic Potential by curvature factors (β_3 and β_4)

Non-associated flow was found in Cassidy (1999) for radial displacement tests conducted for the three-degree of freedom model as shown in the Q - V plot in Fig. 5.34 (where Q is a general deviator force in the $(M/2R, H)$ plane defined earlier by Eqn 2.14). Consistent with this is the strong non-association in the V - T plane and is shown by the angle of “experimental” plastic displacement direction vector of $\eta = 12.2^\circ$ (obtained earlier for the radial displacement test conducted in the $T/2R$ - V plot in Fig. 5.30) being much less than the “theoretical” angle of $\alpha = 80^\circ$ by Eqn 5.24. This is illustrated in Fig. 5.32. (For associated flow, the experimental angle should equal the “theoretical” angle so that the plastic displacement direction vector is perpendicular to the tangent to the

yield surface at each of the four points). The smaller angle is due to the vertical displacement component being much larger than the torsional (rotational) displacement component exhibited in the experiment (Fig. 5.30(d)). Therefore, since associated flow rule under predicts the level of vertical displacements, a different plastic potential from the yield surface is required. In section 2.2.2.4, a plastic potential function g is outlined. For the six degrees of freedom model which, for the T - V plane can be reduced to:

$$g(V, T/2R) = \left(\frac{T/2R}{\alpha_t t_0 V_0'} \right)^2 - \left[\frac{(\beta_3 + \beta_4)(\beta_3 + \beta_4)}{\beta_3 \beta_3 \beta_4 \beta_4} \right]^2 \left(\frac{V}{V_0'} \right)^{2\beta_3} \left(1 - \frac{V}{V_0'} \right)^{2\beta_4} = 0 \quad (5.25)$$

where V_0' is a dummy parameter giving the intersection of the plastic potential with the V -axis (and adjusted so that the potential passes through the current load point). The parameters β_3 and β_4 are chosen empirically (just like β_1 and β_2 previously) to allow for different variations in curvature. Again, for association in the V - T plane the value of the new association factor α_t would be 1.0.

The “theoretical” angle of the vector normal to the plastic potential g (Eqn 5.25) is the same as α (given by Eqn 5.24) with f replaced by g , i.e. with partial directional derivatives, $\partial g / \partial V$ and $\partial g / \partial (T/2R)$ the same as Eqns 5.22 and 5.23 respectively (with f replaced by g , V_0 replaced by V_0' , β_1 and β_2 replaced by β_3 and β_4 respectively, and the addition of $\alpha_t = 1.0$) as shown below:

$$\partial_w^P = \lambda \frac{\partial g}{\partial V} = \lambda \left(-2 \left[\frac{(\beta_3 + \beta_4)(\beta_3 + \beta_4)}{\beta_3 \beta_3 \beta_4 \beta_4} \right]^2 \left(\frac{V}{V_0'} \right)^{2\beta_3} \left(1 - \frac{V}{V_0'} \right)^{2\beta_4} \left(\frac{\beta_3}{V} - \frac{\beta_4}{V_0'} \left(1 - \frac{V}{V_0'} \right)^{-1} \right) \right) \quad (5.26a)$$

$$\partial_{\theta_t}^P = \lambda \frac{\partial g}{\partial (T/2R)} = \lambda \left(2 \left(\frac{1}{\alpha_t t_0 V_0'} \right)^2 \left(\frac{T}{2R} \right) \right) \quad (5.26b)$$

$$\alpha = \tan^{-1} \left(\frac{\frac{\partial g}{\partial (T/2R)}}{\frac{\partial g}{\partial V}} \right) \quad (5.26c)$$

β_3 and β_4 (of Eqn 5.25) are adjusted empirically and found to equal 0.45 and 0.26 respectively so that the “theoretical” angle (Eqn 5.26c) corresponds to the “experimental” incremental plastic displacement vector with an angle of $\alpha = \eta = 12.2^\circ$.

Note that no change is needed for β_3 , i.e. $\beta_3 = \beta_l = 0.45$ since these factors control the curvature on the left of $(V/V_0)_{\text{peak}}$ whereas β_4 is reduced to 0.26 (from $\beta_2 = 0.55$) since this factor corresponds to the curvature on the right of $(V/V_0)_{\text{peak}}$ resulting in a steeper slope of the tangent line to g in order to obtain a “theoretical” normal vector at an angle of approximately 12.2° . This is shown in Fig. 5.35. With β_3 and β_4 empirically chosen, the resulting plastic potential given by Eqn 5.25 (now “skewed to the right”) coincides with the yield surface of the “experimental” results for the radial displacement test at those four points (as shown in Fig. 5.35). Obviously more radial displacement tests are needed to confirm the tentative curvature factors (β_3 and β_4) proposed (as this proposal is made based solely on one radial displacement test performed in this research). Further, the $d^2R\theta_t^p/dw^p$ ratio is very small and experiments on other ratios should be conducted.

5.3.2 Why Only One Radial Displacement Test?

At this point, one may ask why only one radial displacement test performed? The reason is because after a re-calibration of the loading leg (to check the influence of axial load on the torsion strain gauge, detailed in section 4.2.4.2), a substantial bending of the loading leg was seen, most likely due to too much loads applied. This bending occurred near the torsion strain gauge hence any attempt to bend the loading leg back to the original straight position would snap the strain gauge. Thus, although more radial displacement tests were planned, this could not be carried out since erroneous results would be obtained. Fortunately, at least one radial displacement test was conducted along with all the pure vertical loading and swipe tests performed prior to the re-calibration procedure.

5.3.3 Adjusting Plastic Potential by Association Factor (α_t)

Alternatively, rather than “skewing” the plastic potential (by curvature factors β_3 and β_4), the plastic potential can be adjusted by an association factor (α_t), analogous to the horizontal and moment association factors (α_h and α_m respectively) in the three degrees of freedom plastic potential given by Eqn 2.13. The torsion association factor, α_t , can therefore be incorporated into the plastic potential function as follows:

$$g(V, T/2R) = \left(\frac{T/2R}{\alpha_t t_0 V_0'} \right)^2 - \left[\frac{(\beta_1 + \beta_2)^{(\beta_1 + \beta_2)}}{\beta_1 \beta_1 \beta_2 \beta_2} \right]^2 \left(\frac{V}{V_0'} \right)^{2\beta_1} \left(1 - \frac{V}{V_0'} \right)^{2\beta_2} = 0 \quad (5.27)$$

with the same curvature factors β_1 and β_2 obtained previously by experimental best-fit for the yield surface of Eqn 2.19 (since this alternative method assumes no curvature factor adjustment).

Again, the “theoretical” angle of the vector normal to the plastic potential g (Eqn 5.27) is the same as α (given by Eqn 5.24) with f replaced by g , i.e. with partial directional derivatives, $\partial g / \partial V$ and $\partial g / \partial (T/2R)$ the same as Eqns 5.22 and 5.23 respectively (with f replaced by g , V_0 replaced by V_0' , and the addition of α_t) as shown below:

$$\partial_w^P = \lambda \frac{\partial g}{\partial V} = \lambda \left(-2 \left[\frac{(\beta_1 + \beta_2)^{(\beta_1 + \beta_2)}}{\beta_1 \beta_1 \beta_2 \beta_2} \right]^2 \left(\frac{V}{V_0'} \right)^{2\beta_1} \left(1 - \frac{V}{V_0'} \right)^{2\beta_2} \left(\frac{\beta_1}{V} - \frac{\beta_2}{V_0'} \left(1 - \frac{V}{V_0'} \right)^{-1} \right) \right) \quad (5.28a)$$

$$\partial_{\theta_t}^P = \lambda \frac{\partial g}{\partial (T/2R)} = \lambda \left(2 \left(\frac{1}{\alpha_t t_0 V_0'} \right)^2 \left(\frac{T}{2R} \right) \right) \quad (5.28b)$$

$$\alpha = \tan^{-1} \left(\frac{\frac{\partial g}{\partial (T/2R)}}{\frac{\partial g}{\partial V}} \right) \quad (5.28c)$$

Just as with the previous method, the torsion association factor (α_t) is also adjusted empirically so that the “theoretical” angle (Eqn 5.28c) corresponds to the “experimental” incremental plastic displacement vector with an angle of $\alpha = \eta = 12.2^\circ$. The value of α_t obtained is 5.6 (i.e. > 1) with the resulting plastic potentials shown in Fig. 5.36. This is analogous to α_h , $\alpha_m > 1$, where an ‘elongated plastic potential’ is seen with $V_0' < V_0$ for the three degrees of freedom model. Similarly, this observation can also be seen for $\alpha_t > 1$, when comparing V_0 obtained from the yield surface of Eqn 2.19 with V_0' obtained from the plastic potential of Eqn 5.27 as shown in Fig. 5.37 (for instance, the 950 N surface selected shows that the plastic V_0' is less than the yield V_0). Therefore, with α_t empirically chosen, the resulting plastic potential given by Eqn 5.27 coincides with the yield surface of the “experimental” results for the radial displacement

test at the four points (as shown in Fig. 5.36). Again, further tests are needed to confirm the tentative torsion association factor (α_t) proposed. With more radial displacement tests performed, a third method incorporating both torsion association factor (α_t) and curvature factors (β_3 and β_4) in the plastic potential equation (for ‘simultaneous’ empirical adjustments) is recommended.

6 NUMERICAL MODELLING

Numerical simulations were performed for a number of representative experiments in the V - T plane to investigate the capabilities of the newly defined Model C for six degrees of freedom. This exercise also demonstrated that Model C could be implemented numerically and used to simulate footing behaviour. The simulations were conducted for these experimental tests using a 50 mm radius footing on dense super fine silica sand. The retrospective simulations were carried out on Dr Mark Cassidy's existing FORTRAN program named ISIS, which was based on the theory of a previous FORTRAN program named OXC. Reference to OXC can be found in Cassidy (1999). In these simulations, the measured values of quantities such as vertical and torsional (rotational) displacements were taken as input, with the corresponding loads calculated.

6.1 Implication for Numerical Modelling

This section details the summary of the soil and other parameters for Model C used as input parameters for numerical modelling in section 6.2:

$$\gamma = 17.5 \text{ kN/m}^3 \text{ (average for dense sand, from Table 3.2(a) given earlier)}$$

$$p_a = 101.3 \text{ kPa (atmospheric pressure)}$$

$$k_v = 2.65 \text{ (from section 2.2.1.2)}$$

$$k_t = 2/3 \text{ (from section 2.2.2.2)}$$

$$G = 7.66 \text{ N/mm}^2 \text{ (for dense sand, from section 5.1.1.2)}$$

$$\Rightarrow g = 67.4 \text{ (evaluated from Eqn 2.8 with } R = 50 \text{ mm and } V_0 = 1000 \text{ N)}$$

$$\Rightarrow g = 83.7 \text{ (evaluated from Eqn 2.8 with } R = 50 \text{ mm and } V_0 = 650 \text{ N)}$$

$$\Rightarrow g = 116.9 \text{ (evaluated from Eqn 2.8 with } R = 50 \text{ mm and } V_0 = 333 \text{ N)}$$

6.2 Results of Numerical Modelling

6.2.1 Pure Vertical Loading Tests

The purpose of numerical simulations of pure vertical loading tests is to validate the implementation of the strain-hardening law described in section 2.2.1.3. The best-fit

parameters for the strain-hardening law expression given by Eqn 2.10 when conducting pure vertical loading tests on 50 mm radius circular footings are:

$$V_{peak} = 1010 \text{ N}; \quad w_{pm} = 6.9 \text{ mm}; \quad k = 40 \text{ N/mm}; \quad f_p = 0.6.$$

Rather than inputting V_{peak} , ISIS requires the back calculation of an N_γ value of Eqn 2.4 (147.0 in this case).

Based on these input parameters, the load penetration curve simulated is nearly the same as that defined by Eqn 2.10, as shown in Fig. 6.1. As expected, the program simulates the strain hardening law (Eqn 2.10).

6.2.2 Vertical-Torsion Sideswipe Tests

Vertical-torsion sideswipe tests are numerically simulated to check on the yield surface at a given penetration ($T/2R$: V space with parameters β_1 , β_2 , and t_0) and also to check on the plastic potential ($T/2R$: $2R\theta_t$ space with parameters β_3 , β_4 , and α_t), which were described in chapter 2 and parameters evaluated from experimental evidence in chapter 5. A summary of the best-fit parameters for vertical-torsion sideswipe tests (whose yield surface is given by Eqn 2.19) on both dense and loose silica sands are listed in section 5.2.3.

Retrospective simulations of vertical-torsion sideswipe tests are essentially a check of

- (1) the yield surface shape
- (2) the accuracy of the chosen plastic potential.

Two tests were chosen for numerical simulation rc0034 and rc0022, both for 50 mm radius footings and V_0 of 650 N and 333 N respectively. According to Eqn 5.5, t_0 values of 0.03 and 0.04 respectively, would be appropriate.

Two of the following methods will be employed in an attempt to fit the experiments as close as possible in both the $T/2R$: V space as well as the $T/2R$: $2R\theta_t$ space by either

1. Adjusting the plastic potential curvature parameters β_3 and β_4 defined in Eqn 5.25 (and assume $\alpha_t = 1$) or by
2. Varying the association factor α_t defined in Eqn 5.27 (while keeping $\beta_3 = \beta_1$ and $\beta_4 = \beta_2$) where α_t is formulated as a hyperbolic function in terms of plastic

displacement histories and incorporated into the FORTRAN program ISIS (in analogy to the formulation of α_m in Eqn 2.16) as

$$\alpha_t = \frac{k_t' + \alpha_{t\infty} (2R\theta_t^P / w^P)}{k_t' + (2R\theta_t^P / w^P)} \quad (6.1)$$

where α_t relates the shape of the plastic potential to the size of the yield surface and k_t' determines the rate of change of the association factor (i.e. various values of α_t , $\alpha_{t\infty}$ and k_t' are inputted into the numerical program).

One set of values for each method will be tried on both tests at different V_0 until an optimum solution is obtained. The successful numerical tests can be found below.

6.2.2.1 Retrospective Simulation of Test rc0034

The numerical simulation of the vertical-torsion swipe at 650 N is carried out in two stages, first stage being load controlled to $V_0 = 650$ N and second stage being displacement controlled for the swipe. The second stage takes the measured torsional (rotational) displacement as input and the loads of T and V calculated as output whereas the first stage takes the vertical load as input from which the vertical displacement is calculated as output. By applying the “first method” described previously, i.e. by varying the plastic potential curvature factors β_3 and β_4 , an optimum best-fit solution of $\beta_3 = 0.16$ and $\beta_4 = 0.99$ is obtained to fit the experiment quite well, whose results are shown in two plots, Fig. 6.2(a) being a plot in the $T/2R: V$ space and Fig. 6.2(b) being a plot in the $T/2R: 2R\theta_t$ space. Based on the optimum best-fit curvature factors obtained, the plastic potential given by Eqn 5.25 is plotted as shown in Fig. 6.3. The plastic potential defined by these curvature factors obtained from the numerical swipe test differs considerably from those obtained earlier from the experimental radial displacement test in section 5.3.1 (with $\beta_3 = 0.45$ and $\beta_4 = 0.26$). This difference is demonstrated graphically with the plastic potential skewing to the left for the numerical swipe test in Fig. 6.3 compared with the plastic potential skewing to the right for the experimental radial displacement test previously shown in Fig. 5.35. The incremental plastic displacement vector with an angle of 87.8° is theoretically evaluated for the numerical swipe test, also shown in Fig. 6.3. Again, the difference in the behaviours

observed is exhibited by the difference in angles obtained, with an angle of $\eta = 12.2^\circ$ for the experimental radial displacement test. However, with only one radial displacement test conducted, no conclusion can be drawn as to the accuracy of the results obtained. Nevertheless, the numerical program tracks the yield surface of the experiments very well as seen from Fig. 6.2(a) and additionally, Fig. 6.2(b) shows Model C predicting a very similar displacement path to the experiments, verifying the flow rule for this case. However, note that the “swipe” of Model C seems to stop at a point slightly beyond that shown by the experiment whereby a critical state line is reached. This point is known as the “parallel point”, corresponding to a transition between settlement and heave of the footing and where sliding failure occurs (Houlsby and Cassidy, 2001).

Similar contradicting results are obtained by applying the “second method” described previously, i.e. by varying the torsion association factor α_t . At least two sets of optimum best-fit solutions were found to fit the experimental swipe quite well with the first being $\alpha_t = 0.2$, $\alpha_{t\infty} = 2$, and $k'_t = 2.7$, the second being $\alpha_t = 0.7$, $\alpha_{t\infty} = 1.6$, and $k'_t = 4.9$ whose results are shown in Figs. 6.4 and 6.5 respectively, each in three plots with:

- (a) being a plot in the $T/2R: V$ space
- (b) being a plot in the $T/2R: 2R\theta_t$ space and
- (c) being a “close-up” scaled version of (b).

A number of observations can be noted from these tests:

- (1) There can be more than one set of best-fit solutions that fits the experimental swipe test quite well, showing that swipe tests are not ideal for obtaining plastic potential parameters to be used in a structural analysis package, whereby unique parameters are required.
- (2) The best-fit parameters obtained, namely $\alpha_t = 0.2$ or 0.7 is very much different from the experimental radial displacement test result with $\alpha_t = 5.6$. This shows a totally different behaviour in that $\alpha_t > 1$ shows an ‘elongated plastic potential’ corresponding to the dilative behaviour of dense sand, as expected, whereas $\alpha_t < 1$ shows a ‘flattened plastic potential’, which should not be the case for tests on dense sand.

Therefore, again with only one radial displacement test conducted (and this is at a low level of T with V), more tests of this type at higher levels of T with V is recommended before any sound conclusions can be drawn with respect to the accuracy of the different parameters obtained. Nevertheless, the numerical program again tracts the yield surface of the experiments quite well as seen from Figs. 6.4(a) and 6.5(a) and additionally, Figs. 6.4(b) and 6.5(b) shows Model C predicting a very similar displacement path to the experiments, verifying the flow rule for these two cases. However, Model C locates the “parallel point” slightly lower than the experiment for the case in Fig. 6.4(a) and even lower for the case in Fig. 6.5(a).

6.2.2.2 Retrospective Simulation of Test rc0022

Almost the same observations (as swipes at $V_0 = 650$ N) are found for swipes at $V_0 = 333$ N, with Fig. 6.6(a) highlighting the yield surface at a lower vertical load. The same best-fit parameters of $\beta_3 = 0.16$ and $\beta_4 = 0.99$ obtained (for swipes at $V_0 = 650$ N) for the “first method” are used to simulate this test (Fig. 6.6). The same two sets of best-fit parameters of $\alpha_t = 0.2$, $\alpha_{t\infty} = 2$, $k'_t = 2.7$, and $\alpha_t = 0.7$, $\alpha_{t\infty} = 1.6$, $k'_t = 4.9$ for the “second method” are also used to simulate the tests shown in Figs. 6.7 and 6.8 respectively. Overall, the numerical program tracts the yield surface of the experiments quite well, with only a slight “over-prediction” of the yield surface observed in Figs. 6.6(a), 6.7(a) and 6.8(a). Additionally, just as with the swipes at $V_0 = 650$ N, Figs. 6.6(b), 6.7(b) and 6.8(b) shows Model C predicting a very similar displacement path to the experiments, verifying the flow rule for these three cases. Again, Model C locates the “parallel point” slightly higher than the experiment for the “curvature method” for the case in Fig. 6.6(a) but locates the “parallel point” slightly lower than the experiment for the “ α_t method” for the case in Fig. 6.7(a) and even lower for the case in Fig. 6.8(a).

6.2.3 Radial Displacement Tests in V - T plane

The numerical simulation of radial displacement tests is also used to check on both the yield surface and the plastic potential just like vertical-torsion simulations, however more accurate information on the flow rule can be obtained since each radial displacement test provides information on a particular smaller range of T with V (as

opposed to one swipe test covering all values of T with V). The experiment of only one radial displacement test performed in this research, which is discussed earlier in section 5.3, is at a relatively low level of T with V . However, by performing many of these type of experiments at various levels of T with V in future research, a complete “picture” can be obtained from which one value of α_t (say) can hopefully be found to be used readily in a structural analysis package. The best-fit parameters (again for dense sand on 50 mm radius circular footings) for the “ α_t method”, whose plastic potential is given by Eqn 5.27, used in numerical modelling of radial displacement tests in the V - T plane are:

$$\alpha_t = 5.6; \quad \beta_3 = 0.45; \quad \beta_4 = 0.55; \quad t_0 = 0.025$$

(i.e. β_3 and β_4 are the same as β_1 and β_2 respectively and the t_0 value used is typical for a 50 mm radius footing with vertical loads reaching near 1000 N).

Based on these parameters, constant gradient of torsional to vertical displacement is used as input to simulate the radial displacement test in the V - T plane. The torsional to vertical displacements from the Model C simulation is plotted with the experimental torsional to vertical displacements, shown in Fig. 6.9(a). The resultant vertical and torsional loads for the numerical and experimental results are shown in Fig. 6.9(b). The rather severe fluctuation in the experimental results (shown in Fig. 6.9(a)) is due to small-recorded torsional loads, hence resulting in “noise” from the strain gauges. Nevertheless, simulations are of similar gradient in both plots (Fig. 6.9(a) and (b)), implying that the Model C flow rule is performing well. However further tests are recommended at higher levels of T with V in order to verify the tentative torsion association factor ($\alpha_t = 5.6$) proposed from this one experimental radial displacement test conducted at a relatively low level of T with V . Radial displacement tests are ideal because a unique parameter (α_t) can be obtained, which can be readily used in a structural analysis program.

7 CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions of Present Work

This thesis is primarily aimed to verify an extension of a work-hardening plasticity model from three degrees to six degrees of freedom. Particular interest is in the combined V - T load plane. The following conclusions can be drawn:

- (1) The proposed extension of the yield surface in the V - T plane has been confirmed. A refinement has been made to the existing yield surface with a piecewise function given by Eqn 5.13. This is achieved by assuming a triangular stress distribution during the pure shearing phase (i.e. $0 \leq V/V_0 \leq 0.1$) and integrating the stresses over the radius of the footing. ($0.1 \leq V/V_0 \leq 1.0$ is described by the existing yield surface). This formulation has been shown to fit the experimental results well.
- (2) Experiments showed that the expansion of the yield surface in the V - T load space (indicated by the value of t_0) is not constant but varies according to the vertical stress ratio of V_0/V_{peak} . Based on the experimental data, an empirical fit has been formulated to describe this variation as given by Eqn 5.5.
- (3) Non-association is shown in the V - T plane. Hence a different plastic potential from the yield surface has been defined by two possible expressions, either by Eqn 5.25 where the plastic potential can be skewed by curvature factors (β_3 and β_4) or by Eqn 5.27 where the plastic potential can be adjusted by an association factor (α_t).
- (4) The extended six-degree of freedom model could retrospective simulate the footing behaviour of experimental tests. However, different non-associated flow rules were required and therefore future testing in this area is required.

7.2 Recommendations for Future Work

The following recommendations for future work can be made:

- (1) There is a need for further radial displacement tests at higher levels of T with V to confirm the tentative torsion association factor (α_t) proposed. It would be hoped to achieve a consistent plastic potential.
- (2) Further validation of a constant flow rule could be advised by performing constant vertical load tests at various vertical load levels in the V - T plane (whereby once the desired vertical load level is reached, the vertical load is held constant during the course of the torsional “swipe” by further penetrating the footing vertically into the soil while “swiping”).
- (3) Further tests are needed to investigate the plastic behaviour of a circular footing under different load combinations such as T , M , H , V . It is hoped that a complete experimental verification of the full six-degree of freedom model can be achieved.
- (4) With jack-up footings over 10 m diameter being common in the offshore field, (this means 100 times larger than the 100 mm diameter footings used for testing at 1g level), centrifuge modelling at 100g is required to take into account the much larger magnitude of stress beneath the same sized footing at 100g than at 1g. Hence, 100 mm diameter footings can be used in the centrifuge at 100g as a model to the prototype (of 10 m) in the field. However, before this is attempted, the development of an improved electronic control system to control torsional (rotational) displacement is required when investigating the V - T space. This is because the present control system (with DAC-02 cards) used, is prone to rotational “drift”, hence generating too much torsion before the “swipes” even starts (as it only takes a few degrees of rotation to reach peak torsion).

Nevertheless these torsion swipes performed at 1g (as part of the experimental verification of Model C six degrees of freedom) is analogous to previous work by Gottardi *et al.* (1999) with horizontal and moment swipes performed, also at

1g (for Model C three degrees of freedom in the (V, H, M) space). Therefore, these torsion swipe tests at 1g can now serve as the base case for future centrifuge tests.

- (5) Similarly, future work in the centrifuge for pure vertical loading tests on sand (especially loose sand) is required to adequately account for footing displacements (to investigate the ‘overflow of sand’ behaviour on prototype size footings).
- (6) Model C has so far been intended to describe the drained loading of a circular foundation, however in order to accurately represent and simulate offshore conditions, undrained loading needs to be investigated. Furthermore, to further represent offshore conditions accurately, tests under cyclic loading are required since cyclic loading is known to substantially reduce the capacity of the foundation to resist the same load combinations under monotonic loading. Testing on calcareous soils, again being more representative of offshore conditions is also required.

7.3 Conclusion

The three-dimensional plain strain formulation (Model C plasticity theory for six degrees of freedom) proposed by Cassidy and Bienen (2002) has been substantially, experimentally verified (and refined) in the V - T space. This will add to the use of jack-up platforms with greater confidence in deeper and harsher waters.

REFERENCES

- Atkinson, J.H. and Bransby, P.L. (1978). "The Mechanics of Soils: An Introduction to Critical State Soil Mechanics", McGraw-Hill Book Company (UK) Limited.
- Australian Standards (1998). "AS 1289.5.5.1 – Soil Compaction and Density Tests – Determination of the Minimum and Maximum Dry Density of a Cohesionless Material – Standard Method", Standards Australia.
- Bell, R.W. (1991). "The Analysis of Offshore Foundations Subjected to Combined Loading", *M.Sc. Thesis*, University of Oxford.
- Bienen, B. (2001). "Three-Dimensional Numerical Analysis of Jack-up Structures", *Diploma Thesis*, Aachen University of Technology, Germany.
- Blinco, J. (2000). "Pipeline Modelling in Silica Sand", *Final Year Thesis*, Department of Civil and Resource Engineering, The University of Western Australia.
- Bolton, M.D. (1986). "The Strength and Dilatancy of Sands", *Géotechnique*, Vol. 36, No. 1, pp 65-78.
- Bolton, M.D. (1987). "The Strength and Dilatancy of Sands" Discussion, *Géotechnique*, Vol. 37, No. 2, pp 219-226.
- Butterfield, R., Houlsby, G.T. and Gottardi, G. (1997). "Standardized Sign Conventions and Notations for Generally Loaded Foundations", *Géotechnique*, Vol. 47, No. 5, pp 1051-1054.
- Byrne, B. and Cassidy, M.J. (2002). "Investigating the Response of Offshore Foundations in Soft Clay Soils", *Report No. C:1626*, Centre for Offshore Foundation Systems, The University of Western Australia.
- Byrne, B.W. and Houlsby, G.T. (1999). "Drained Behaviour of Suction Caisson Foundations on Very Dense Sand", *Offshore Technology Conference*, Houston, OTC 10994.
- Byrne, B.W. and Houlsby, G.T. (2001). "Observations of Footing Behaviour on Loose Carbonate Sands", *Géotechnique*, Vol. 51, No. 5, pp 463-466.
- Cassidy, M.J. (1999). "Non-Linear Analysis of Jack-Up Structures Subjected to Random Waves", *D.Phil Thesis*, University of Oxford. United Kingdom.
- Cassidy, M.J. and Bienen, B. (2002). "Three-Dimensional Numerical Analysis of Jack-up Structures on Sand", Submitted to *Proc. 12th Int. Offshore and Polar Eng. Conf.*, Kitakyushu, Japan.
- Cassidy, M.J. and Houlsby, G.T. (1999). "On the Modelling of Foundations for Jack-Up Units on Sand", *Offshore Technology Conference*, Houston, OTC 10995.

- Cassidy, M.J. and Houlsby, G.T. (2002). "Vertical Bearing Capacity Factors for Conical Footings on Sand", *Report No. C:1673*, Centre for Offshore Foundation Systems, The University of Western Australia.
- Cheong, J. (2002). "Physical Testing of Circular Footings on Dense Silica Sand", *Dec 2001-Feb 2002 Report*, Centre for Offshore Foundation Systems, The University of Western Australia.
- Dufty, B. (2001). "Capacity of Circular Foundations Under Combined Loading", *Final Year Thesis*, Department of Civil and Resource Engineering, The University of Western Australia.
- Gottardi, G. and Houlsby, G.T. (1995). "Model Tests of Circular Footings on Sand Subjected to Combined Loads", *Report No. 2071/95*, Oxford University Engineering Laboratory.
- Gottardi, G. and Houlsby, G.T. and Butterfield, R. (1999). "The Plastic Response of Circular Footings on Sand Under General Planar Loading", *Géotechnique*, Vol. 49, No. 4, pp 453-470.
- Hansen, J.B. (1970). "A Revised and Extended Formula for Bearing Capacity", *Bulletin of our Danish Geotechnical Society*, Vol. 28, pp 5-11.
- Houlsby, G.T. and Cassidy, M.J. (2001). "A Plasticity Model for the Behaviour of Footings on Sand Under Combined Loading", *Report No. C:1569*, Centre for Offshore Foundation Systems, The University of Western Australia.
- Martin, C.M. (1994). "Physical and Numerical Modelling of Offshore Foundations Under Combined Loads", *D.Phil Thesis*, University of Oxford, United Kingdom.
- Martin, C.M. and Houlsby, G.T. (1994). "Combined Loading Tests of Scale Model Spudcan Footings on Soft Clay: Experimental Data", *Report No. 2029/94*, Oxford University Engineering Laboratory.
- Ngo-Tran, C.L. (1996). "The Analysis of Offshore Foundations Subjected to Combined Loading", *D.Phil. Thesis*, University of Oxford.
- Platt, R. (1998). "The Behaviour of Sand When Subjected to Combined Vertical and Torsional Loading", *Final Year Report*, Oxford University.
- Poulos, H.G. and Davis, E.H. (1974). "Elastic Solutions for Soil and Rock Mechanics", *John Wiley*, New York.
- Tan, F.S.C. (1990). "Centrifuge and Numerical Modelling of Conical Footings on Sand", *PhD Thesis*, University of Cambridge.
- Vesic, A.S. (1975). "Bearing capacity of shallow foundations", *Foundation Engineering Handbook* (edited by Winterkorn, H.F. and Fang, H.Y.), Van Nostrand, New York, pp. 121-147.

- Vlahos, G., Martin C.M. and Cassidy, M.J. (2001). "Experimental Investigation of a Model Jack-Up Unit on Clay", *The International Society of Offshore and Polar Engineers*, The University of Western Australia.
- Whitlow, R. (1995). "Basic Soil Mechanics", Third Edition, Addison Wesley Longman Limited.
- Young, A.G., Remmes, B.D. and Meyer, B.J. (1982). "Foundation Performance of Offshore Jack-up Drilling Rigs", *ASCE of Geotechnical Engineering*, 110, No. 7, pp. 841-859.