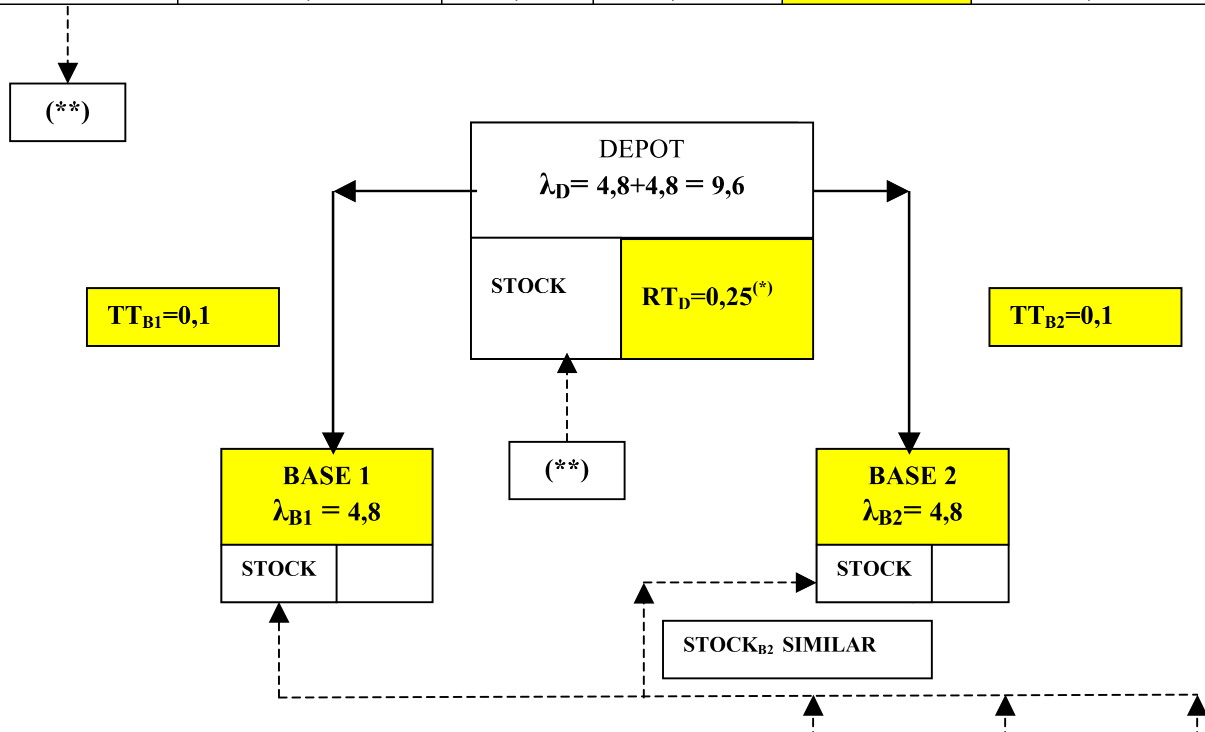
# VARI-METRIC

| $STOCK_D$ | PIPELINE= $\lambda_D$ x $RT_D$ | $BO_D$ | $EBO_D$ | VAR | $EBO_D/\lambda_D$ (time) |
|---|---|---|---|---|---|
| 0 | 2,4 | 2,4 | 2,400 | 2,400 | 0,250 |
| 1 | 2,4 | 1,4 | 1,491 | 2,046 | 0,155 |
| 2 | 2,4 | 0,4 | 0,799 | 1,341 | 0,083 |
| 3 | 2,4 | 0,0 | 0,369 | 0,675 | 0,038 |
| 4 | 2,4 | 0,0 | 0,148 | 0,272 | 0,015 |
| 5 | 2,4 | 0,0 | 0,052 | 0.091 | 0,005 |




**(*) Includes transit time from base to depot.**

| $STOCK_D$ | $\lambda_{B1}$ x $TAT_{B1}$ | $\lambda_{B1}(EBO_D/\lambda_D)$ | $PIPELINE_{B1}$ | VAR | $STOCK_{B1}$ = 0 | $STOCK_{B1}$ = 1 | $STOCK_{B1}$ = 2 |
|---|---|---|---|---|---|---|---|
| 0 | 0,48 | 1,200 | 1,680 | 1,680 | $EBO_{B1}$ =1,680 | $EBO_{B1}$ =0,866 | $EBO_{B1}$ =0,366 |
| 1 | 0,48 | 0,745 | 1,225 | 1,364 | $EBO_{B1}$ =1,225 | $EBO_{B1}$ =0,538 | $EBO_{B1}$ =0,196 |
| 2 | 0,48 | 0,400 | 0,880 | 1,015 | $EBO_{B1}$ =0,880 | $EBO_{B1}$ =0,321 | $EBO_{B1}$ =0,098 |
| 3 | 0,48 | 0,184 | 0,664 | 0,741 | $EBO_{B1}$ =0,664 | $EBO_{B1}$ =0,198 | $EBO_{B1}$ =0,048 |
| 4 | 0,48 | 0,074 | 0,554 | 0,585 | $EBO_{B1}$ =0,554 | $EBO_{B1}$ =0,137 | $EBO_{B1}$ =0,027 |
| 5 | 0,48 | 0,026 | 0,506 | 0,516 | $EBO_{B1}$ =0,506 | $EBO_{B1}$ =0,112 | $EBO_{B1}$ =0,018 |

| $STOCK_D$ | $STOCK_{B1}$ | $STOCK_{B2}$ | $EBO_D$ | $EBO_{B1}$ | $EBO_{B2}$ | COST | EBO | ORDER |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 2,400 | 1,680 | 1,680 | 0p | 5,760 | |
| 0 | 0 | 1 | 2,400 | 1,680 | 0,866 | 1p | 4,946 | |
| 0 | 0 | 2 | 2,400 | 1,680 | 0,366 | 2p | 4,446 | |
| 0 | 1 | 0 | 2,400 | 0,866 | 1,680 | 1p | 4,946 | |
| 0 | 1 | 1 | 2,400 | 0,866 | 0,866 | 2p | 4,132 | |
| 0 | 1 | 2 | 2,400 | 0,866 | 0,366 | 3p | 3,632 | |
| 0 | 2 | 0 | 2,400 | 0,366 | 1,680 | 2p | 4,446 | |
| 0 | 2 | 1 | 2,400 | 0,366 | 0,866 | 3p | 3,632 | |
| 0 | 2 | 2 | 2,400 | 0,366 | 0,366 | 4p | 3,132 | |
| 1 | 0 | 0 | 1,491 | 1,225 | 1,225 | 1p | 3,941 | |
| 1 | 0 | 1 | 1,491 | 1,225 | 0,538 | 2p | 3,254 | |
| 1 | 0 | 2 | 1,491 | 1,225 | 0,196 | 3p | 2,912 | |
| 1 | 1 | 0 | 1,491 | 0,538 | 1,225 | 2p | 3,254 | |
| 1 | 1 | 1 | 1,491 | 0,538 | 0,538 | 3p | 2,567 | |
| 1 | 1 | 2 | 1,491 | 0,538 | 0,196 | 4p | 2,225 | |

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| ORD | DEPOT QTY | BASE1 QTY | BASE2 QTY | DEPOT EBO | BASE1 EBO | BASE2 EBO | COST | TOTAL EBO |
|---|---|---|---|---|---|---|---|---|
| 1 | **0** | **0** | **0** | **2,400** | **1,680** | **1,6800** | **0** | **5,760** |
| 2 | 0 | 0 | 1 | 2,400 | 1,680 | 0,8660 | 1 | 4,946 |
| 4 | 0 | 1 | 0 | 2,400 | 0,866 | 1,6800 | 1 | 4,946 |
| 10 | **1** | **0** | **0** | **1,491** | **1,225** | **1,2250** | **1** | **3,941** |
| 3 | 0 | 0 | 2 | 2,400 | 1,680 | 0,3660 | 2 | 4,446 |
| 7 | 0 | 2 | 0 | 2,400 | 0,366 | 1,6800 | 2 | 4,446 |
| 5 | 0 | 1 | 1 | 2,400 | 0,866 | 0,8660 | 2 | 4,132 |
| 11 | 1 | 0 | 1 | 1,491 | 1,225 | 0,5380 | 2 | 3,254 |
| 13 | 1 | 1 | 0 | 1,491 | 0,538 | 1,2250 | 2 | 3,254 |
| 19 | **2** | **0** | **0** | **0,799** | **0,880** | **0,8800** | **2** | **2,559** |
| 6 | 0 | 1 | 2 | 2,400 | 0,866 | 0,3660 | 3 | 3,632 |
| 8 | 0 | 2 | 1 | 2,400 | 0,366 | 0,8660 | 3 | 3,632 |
| 12 | 1 | 0 | 2 | 1,491 | 1,225 | 0,1960 | 3 | 2,912 |
| 16 | 1 | 2 | 0 | 1,491 | 0,196 | 1,2250 | 3 | 2,912 |
| 14 | 1 | 1 | 1 | 1,491 | 0,538 | 0,5380 | 3 | 2,567 |
| 20 | 2 | 0 | 1 | 0,799 | 0,880 | 0,3210 | 3 | 2,000 |
| 22 | 2 | 1 | 0 | 0,799 | 0,321 | 0,8800 | 3 | 2,000 |
| 28 | **3** | **0** | **0** | **0,369** | **0,664** | **0,6640** | **3** | **1,697** |
| 9 | 0 | 2 | 2 | 2,400 | 0,366 | 0,3660 | 4 | 3,132 |
| 15 | 1 | 1 | 2 | 1,491 | 0,538 | 0,1960 | 4 | 2,225 |
| 17 | 1 | 2 | 1 | 1,491 | 0,196 | 0,5380 | 4 | 2,225 |
| 21 | 2 | 0 | 2 | 0,799 | 0,880 | 0,0980 | 4 | 1,777 |
| 25 | 2 | 2 | 0 | 0,799 | 0,098 | 0,8800 | 4 | 1,777 |
| 23 | 2 | 1 | 1 | 0,799 | 0,321 | 0,3210 | 4 | 1,441 |
| 37 | 4 | 0 | 0 | 0,148 | 0,554 | 0,5540 | 4 | 1,256 |
| 29 | **3** | **0** | **1** | **0,369** | **0,664** | **0,1980** | **4** | **1,231** |
| 31 | **3** | **1** | **0** | **0,369** | **0,198** | **0,6640** | **4** | **1,231** |
| 18 | 1 | 2 | 2 | 1,491 | 0,196 | 0,1960 | 5 | 1,883 |
| 24 | 2 | 1 | 2 | 0,799 | 0,321 | 0,0980 | 5 | 1,218 |
| 26 | 2 | 2 | 1 | 0,799 | 0,098 | 0,3210 | 5 | 1,218 |
| 30 | 3 | 0 | 2 | 0,369 | 0,664 | 0,0480 | 5 | 1,081 |
| 34 | 3 | 2 | 0 | 0,369 | 0,048 | 0,6640 | 5 | 1,081 |
| 46 | 5 | 0 | 0 | 0,052 | 0,506 | 0,5060 | 5 | 1,064 |
| 38 | 4 | 0 | 1 | 0,148 | 0,554 | 0,1370 | 5 | 0,839 |
| 40 | 4 | 1 | 0 | 0,148 | 0,137 | 0,5540 | 5 | 0,839 |
| 32 | **3** | **1** | **1** | **0,369** | **0,198** | **0,1980** | **5** | **0,765** |
| 27 | 2 | 2 | 2 | 0,799 | 0,098 | 0,0980 | 6 | 0,995 |
| 39 | 4 | 0 | 2 | 0,148 | 0,554 | 0,0270 | 6 | 0,729 |
| 43 | 4 | 2 | 0 | 0,148 | 0,027 | 0,5540 | 6 | 0,729 |
| 47 | 5 | 0 | 1 | 0,052 | 0,506 | 0,1120 | 6 | 0,670 |
| 49 | 5 | 1 | 0 | 0,052 | 0,112 | 0,5060 | 6 | 0,670 |
| 33 | 3 | 1 | 2 | 0,369 | 0,198 | 0,0480 | 6 | 0,615 |
| 35 | 3 | 2 | 1 | 0,369 | 0,048 | 0,1980 | 6 | 0,615 |
| 41 | **4** | **1** | **1** | **0,148** | **0,137** | **0,1370** | **6** | **0,422** |
| 48 | 5 | 0 | 2 | 0,052 | 0,506 | 0,0180 | 7 | 0,576 |
| 52 | 5 | 2 | 0 | 0,052 | 0,018 | 0,5060 | 7 | 0,576 |
| 36 | 3 | 2 | 2 | 0,369 | 0,048 | 0,0480 | 7 | 0,465 |
| 42 | 4 | 1 | 2 | 0,148 | 0,137 | 0,0270 | 7 | 0,312 |
| 44 | 4 | 2 | 1 | 0,148 | 0,027 | 0,1370 | 7 | 0,312 |
| 50 | **5** | **1** | **1** | **0,052** | **0,112** | **0,1120** | **7** | **0,276** |
| 45 | 4 | 2 | 2 | 0,148 | 0,027 | 0,0270 | 8 | 0,202 |
| 51 | 5 | 1 | 2 | 0,052 | 0,112 | 0,0180 | 8 | 0,182 |
| 53 | 5 | 2 | 1 | 0,052 | 0,018 | 0,1120 | 8 | 0,182 |
| 54 | 5 | 2 | 2 | 0,052 | 0,018 | 0,0180 | 9 | 0,088 |

**Depot Variance (Ref.: Graves, Diaz & Fu)**

$$Var\left[B_0(S_0)\right] = Var\left[B_0(S_0-1)\right] - \left\{E\left[B_0(S_0)\right] + E\left[B_0(S_0-1)\right]\right\}\left(1-\Pr\left(Q_0 \geq S_0\right)\right)$$

$$\boxed{Var\left[B_0(S_0)\right] = Var\left[B_0(S_0-1)\right] - \left\{E\left[B_0(S_0)\right] + E\left[B_0(S_0-1)\right]\right\} * POISSON\left(S_0-1, mean, TRUE\right)}$$

$$Var\left[B_0(S_0)\right] = Var\left[B_0(S_0-1)\right] - \left\{E\left[B_0(S_0)\right] + E\left[B_0(S_0-1)\right] - E\left[B_0(S_0)\right]\Pr\left(Q_0 \geq S_0\right) - E\left[B_0(S_0-1)\right]\Pr\left(Q_0 \geq S_0\right)\right\}$$

$$E\left[B_0(S_0)\right] = E\left[B_0(S_0-1)\right] - \Pr\left(Q_0 \geq S_0\right) \Rightarrow \Pr\left(Q_0 \geq S_0\right) = E\left[B_0(S_0-1)\right] - E\left[B_0(S_0)\right]$$

$$\begin{aligned} Var\left[B_0(S_0)\right] = Var\left[B_0(S_0-1)\right] - \{ & E\left[B_0(S_0)\right] + E\left[B_0(S_0-1)\right] - E\left[B_0(S_0)\right]\left(E\left[B_0(S_0-1)\right] - E\left[B_0(S_0)\right]\right) - \\ & - E\left[B_0(S_0-1)\right]\left(E\left[B_0(S_0-1)\right] - E\left[B_0(S_0)\right]\right)\} \end{aligned}$$

$$\begin{aligned} Var\left[B_0(S_0)\right] = Var\left[B_0(S_0-1)\right] & - E\left[B_0(S_0)\right] - E\left[B_0(S_0-1)\right] + E\left[B_0(S_0)\right]\cancel{E}\left[B_0(S_0-1)\right] - E\left[B_0(S_0)\right]^2 + \\ & + E\left[B_0(S_0-1)\right]^2 - E\left[B_0(S_0-1)\right]\cancel{E}\left[B_0(S_0)\right] \end{aligned}$$

$$\boxed{Var\left[B_0(S_0)\right] = Var\left[B_0(S_0-1)\right] - E\left[B_0(S_0)\right] - E\left[B_0(S_0-1)\right] - E\left[B_0(S_0)\right]^2 + E\left[B_0(S_0-1)\right]^2}$$

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**Depot – Variance Calculation (Ref.: Diaz & Fu)**

$$Var[B_0(S_0)] = Var[B_0(S_0-1)] - E[B_0(S_0)] - E[B_0(S_0-1)] - E[B_0(S_0)]^2 + E[B_0(S_0-1)]^2$$

$$E[B_0(S_0=0)] = Var[B_0(S_0=0)]$$

Example for $EBO_{\mathbf{D}}$ = 0.799

Var $[B_0(S_0 = 2)]$ = 2.046 – 0.799 – 1.491 – $0.799^2$ + $1.491^2$ = - 0.244 – 0.6384 + 2.223 = 1.341

**Base – Variance Calculation (Ref.: Diaz & Fu)**

$$Var\,[B_i(S_0,0)] = \left(\frac{\lambda_i}{\lambda}\right)^2 Var\,[B_0(S_0)] + \left(\frac{\lambda_i}{\lambda}\right)\left(1-\frac{\lambda_i}{\lambda}\right)E\,[B_0(S_0)] + \lambda_i L_i$$

Example for $E[B_1(S_0{=}2;\ 0)]$ = 0.799

$$Var\,[B_1(S_0 = 2;0)] = \left(\frac{4.8}{9.6}\right)^2 1.3406 + \left(\frac{4.8}{9.6}\right)\left(1-\frac{4.8}{9.6}\right)0.7999 + 4.8x0.1 = 0.33515 + 0.19975 + 0.48 = 1.015$$

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**EBO Calculation – Negative binomial distribution (Ref.: Graves)**

NB (x; r, p) = NEGBINOMDIST (x; r, p) (MS Excel notation)

x, r integer p is probability and $0 < p < 1$

$E = \frac{r(1-p)}{p}$ and $V = \frac{r(1-p)}{p^2}$ or $p = \frac{E}{V}$ $0 < p < 1$ and $r = \frac{E^2}{V - E}$ r integer

Example for $E[B_1(S_0=2; S_1=1)]$

$p = \frac{E}{V} = \frac{0.880}{1.015} = 0.8667$ and $r = \frac{E^2}{V-E} = \frac{0.880^2}{1.015 - 0.880} = 5.74$

$$NB(k,r,p) = \binom{k+r-1}{r-1} p^r (1-p)^k = \frac{(k+r-1)!}{k!(r-1)!} p^r (1-p)^k = \frac{(k+r-1)(k+r-2)!}{k(k-1)!(r-1)!} p^r (1-p)^k$$

$$NB(k-1,r,p) = \binom{k+r-2}{r-1} p^r (1-p)^{k-1} = \frac{(k+r-2)!}{(k-1)!(r-1)!} p^r (1-p)^{k-1}$$

$$NB(k,r,p) = \frac{(k+r-1)(k+r-2)!}{k(k-1)!(r-1)!} p^r (1-p)^k = \frac{(k+r-1)}{k}(1-p) * NB(k-1,r,p)$$

$$NB(k,r,p) = \frac{(k+r-1)}{k}(1-p) * NB(k-1,r,p)$$

$NB(k = 0, r, p) = p^r$ and $k = 1, 2, 3, 4, \ldots\ldots..$ } This recursion formula is valid for r not integer

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$$E\left[B_1\left(S_0 = 2, S_1 = 1\right)\right] = \sum_{k=2}^{\infty}(k - S)NB(k, r = 5.74, p = 0.8667) = 0.32084$$

Example for $E[B_1(S_0=2; S_1=2)]$

$$E\left[B_1\left(S_0 = 2, S_1 = 2\right)\right] = \sum_{k=S_1+1}^{\infty}(k - S)NB(k; r = 5.74; p = 0.8667) = 0.09846$$

or using recursive expression:

$$E\left[B_1\left(S_0 = 2, S_1 = 2\right)\right] = E\left[B_1\left(S_0 = 2, S_1 = 1\right)\right] - \left[1 - \sum_{k=0}^{1} NB(k; r = 5.74; p = 0.8667)\right] =$$

$$= 0.32084 - 0.22238 = 0.09846$$

**<u>Base – General Variance Calculation (Ref.: Sleptchenko)</u>**

Please see next sheet

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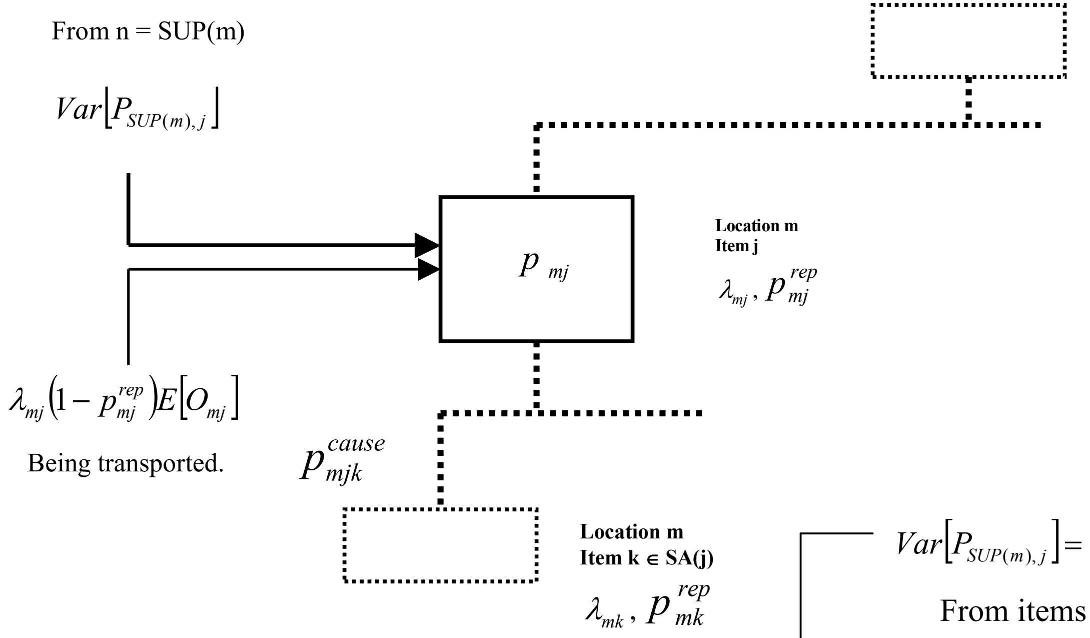


$$h_{mjk} = p_{mj}^{rep} p_{mjk}^{cause} \frac{\lambda_{mj}}{\lambda_{mk}}$$

$$f_{mj} = \left(1 - p_{mj}^{rep}\right) \frac{\lambda_{mj}}{\lambda_{SUP(m),j}}$$

$$Var\left[P_{SUP(m),j}\right] = f_{mj}\left(1 - f_{mj}\right)E\left[BO_{SUP(m),j}\right] + f_{mj}^2 Var\left[BO_{SUP(m),j}\right]$$

From items waiting at supplier n = SUP(m) for replacement.

$$Var\left[P_{mk}\right] = \sum_{k \in SA(j)} \left\{h_{mjk}\left(1 - h_{mjk}\right)E\left[BO_{mk}\right] + h_{mjk}^2 Var\left[BO_{mk}\right]\right\}$$

From items in repair at location m.

From items being transported to location m.

From all items at location m for subassembly replacement.

$$Var\left[P_{mj}\right] = \lambda_{mj} p_{mj}^{rep} E\left[STm_{mj}\right] + \lambda_{mj}\left(1 - p_{mj}^{rep}\right)E\left[O_{mj}\right] + Var\left[P_{SUP(m),j}\right] + Var\left[P_{mk}\right] =$$

$$= E\left[P_{mj}\right] + \sum_{k \in SA(j)} \left\{h_{mjk}^2\left(Var\left[BO_{mk}\right] - E\left[BO_{mk}\right]\right)\right\} + f_{mj}^2\left(Var\left[BO_{SUP(m),j}\right] - E\left[BO_{SUP(m),j}\right]\right)$$

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