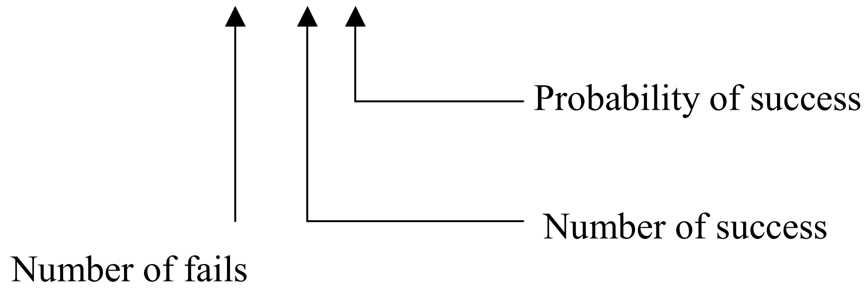
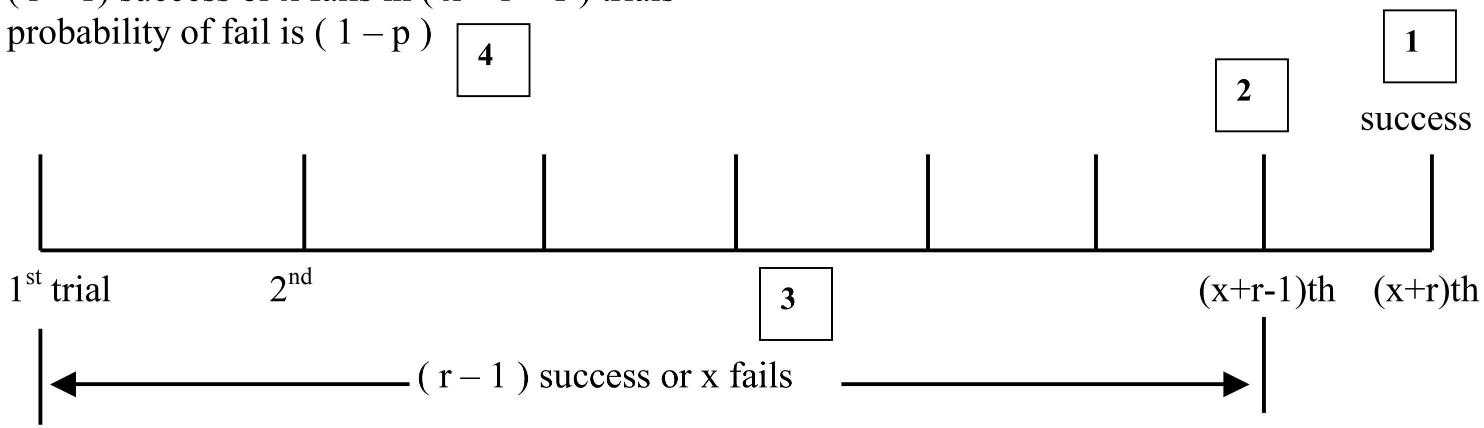
Negative Binomial Distribution

Definition: Y = **NegBinDist** (x; r, p) x, r integers and r > 0, 0 < p < 1



Concept: Negative Binomial Distribution “**NegBinDist**” gives the probability “Y” to get “x” numbers of fails before to reach “r” numbers of success with constant probability of success “p”.

Variables: number total of trials (x + r)
(x + r) th trial is a success
(r – 1) success or x fails in (x + r – 1) trials
probability of fail is (1 – p)



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NegBinDistr (x; r, p) = $\overset{\boxed{1}}{p}\left[\binom{x+\overset{\boxed{2}}{r}-1}{\underset{\boxed{3}}{r-1}}p^{\overset{\boxed{3}}{(r-1)}}(1-\underset{\boxed{4}}{p})^{\overset{\boxed{2}}{(x+r-1)}-\overset{\boxed{3}}{(r-1)}}\right] \qquad = \qquad \binom{x+r-1}{r-1}p^{r}(1-p)^{x} = \binom{x+r-1}{x}p^{r}(1-p)^{x}$

0 < p < 1 **r > 0 and integer**

With mean = $r\dfrac{1-p}{p}$ variance = $r\dfrac{1-p}{p^{2}}$ $\dfrac{\text{var}\,iance}{mean}=\dfrac{1}{p}$ and $p=\dfrac{mean}{\text{var}\,iance}$ $r=\dfrac{mean^{2}}{\text{var}\,iance-mean}$

If $q=\dfrac{1}{p}=\dfrac{mean}{\text{var}\,iance}$ then

NegBinDistr (x; r, p) = $\binom{x+r-1}{r-1}\dfrac{1}{q^{r}}(1-\dfrac{1}{q})^{x}=\binom{x+r-1}{r-1}\dfrac{1}{q^{r}}\left(\dfrac{q-1}{q}\right)^{x}=\binom{x+r-1}{r-1}\dfrac{(q-1)^{x}}{q^{x+r}}=\binom{x+r-1}{x}\dfrac{(q-1)^{x}}{q^{x+r}}$

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If $y = x + r$ then **NegBinDist(y; r, p)** = $\binom{y-1}{r-1} p^r (1-p)^{y-r}$ with y = r, r+1, r+2, r+3, …….

The following values are from MATHLAB or MS-EXCEL:

Y = **NegBinDistr** (0 =< x <= 10; r=3; p=0.5) and 1 – p = 0.5

X	0	1	2	3	4	5	6	7	8	9	10
Y	0.1250	0.1875	0.1875	0.1563	0.1172	0.0820	0.0547	0.0352	0.0220	0.0134	0.0081

For **NegBinDistr**(0; 3; 0.5) = p(success, success, success) = 0.5^3 = 0.125

For **NegBinDistr**(1; 3; 0.5) = p(fail = f, success = s, success, success) + p(s, f, s, s) + p(s, s, f, s) = $3x0.5^4$ = 0.1875

For **NegBinDistr**(2; 3; 0.5) = p(f, f, s, s, s) + p(f, s, f, s, s) + p(f, s, s, f, s) + p(s, f, f, s, s) + p(s, f, s, f, s) + p(s, s, f, f, s) =
= 6 x 0.5^5 = 0.1875 …….

……. And so on.

If r > 0 and it is not integer;

NegBinDistr (x; r, p) = $\frac{\Gamma(x+r)}{x!\Gamma(r)} p^r (1-p)^x$

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