In Repair j = R(t)	p(j) (*)	Fill Rate R(t) < n	Available Spare s = S(t)	Waiting Spare i = W(t)	Expected Number In Repair	Expected Nr of Available Spares	P(n,s=0) PBO(n)	P[n,i=0]	Expected Number of Waited Spares E[BO(w)]	$E[BO^2(n)]$ Var[BO(n)]= $=E[BO^2(n)] - E^2[BO(n)]$
0	p(0)	p(0)	**n**	0	0*p(0)	n*p(0)		p(0)		
1	p(1)	p(0)+p(1)	**n-1**	0	0*p(0)+1*p(1)	n*p(0)+(n-1)*p(1)		p(0)+p(1)		
2	p(2)	p(0)+p(1)+ +p(2)	**n-2**	0	0*p(0)+ … + +2*p(2)	n*p(0)+(n-1)*p(1)+ +(n-2)*p(2)		p(0)+p(1)+ +p(2)		
3	p(3)	p(0)+p(1)+ +p(2)+p(3)	**n-3**	0	0*p(0)+ … + +3*p(3)	n*p(0)+(n-1)*p(1)+ +(n-2)*p(2)+(n-3)*p(3)		p(0)+p(1)+ +p(2)+p(3)		
.....	….	.....	**.....**	0		.....		.....		
n-2	p(n-2)	p(0)+p(1)+ +p(2)+p(3)+ ...+ +p(n-2)	**2**	0	0*p(0)+ … + +(n-2)*p(n-2)	n*p(0)+(n-1)*p(1)+ +(n-2)*p(2)+(n-3)*p(3)+ +....+2*p(n-2)		p(0)+p(1)+ +p(2)+p(3)+...+ +p(n-2)		
n-1	p(n-1)	$\sum_{j=0}^{n-1} p(j)$	**1**	0	0*p(0)+ … + +(n-1)*p(n-1)	n*p(0)+(n-1)*p(1)+ +(n-2)*p(2)+(n-3)*p(3)+….+ +2*p(n-2)+1*p(n-1)		p(0)+p(1)+ +p(2)+p(3)+...+ +p(n-1)		
n	p(n)		**0**	**0**	0*p(0)+ … + +(n-1)*p(n-1)+ +n*p(n)	$\sum_{j=0}^{n} (n-j)*p(j)$	p(n)	$\sum_{j=0}^{n} p(j)$		
n+1	p(n+1)		0	**1**	0*p(0)+ … + +(n+1)*p(n+1)		p(n+1)		1*p(n+1)	1^2*p(n+1)
n+2	p(n+2)		0	**2**	etc		p(n+1)+ + p(n+2)		1*p(n+1)+2*p(n+2)	1^2*p(n+1)+2^2*p(n+2)
n+3	p(n+3)		0	**3**	etc		etc		1*p(n+1)+ … +3*p(n+3)	1^2*p(n+1)+ … + +3^2*p(n+3)
.....	.....	…..	0	**.....**	……	………	….		.....	.....
∞ (t → ∞)		$\sum_{j=0}^{\infty} p(j) = 1$	0	∞	$\sum_{j=0}^{\infty} j * p(j)$	$\sum_{j=0}^{\infty} (n-j) * p(j)$	$\sum_{j=n}^{\infty} p(j)$		$\sum_{j=n+1}^{\infty} (j-n) * p(j)$ or $\sum_{i=1}^{\infty} i * p(n+i)$	$\sum_{j=n+1}^{\infty} (j-n)^2 * p(j)$ or $\sum_{i=1}^{\infty} i^2 * p(n+i)$

(*) **p(j)** is poisson or negative binomial distribution. **n** = initial spares.

$$FillRate(n) + p(n) + PBO(n) = \sum_{j=0}^{n-1} p(j) + p(n) + \sum_{j=n+1}^{\infty} p(j) = 1 \Rightarrow PBO(n) = 1 - p(n) - FillRate(n)$$

JFukuda Jan2008

EBO, expected backorder

n, initial spares quantity

p, demand probability
(poisson, negative binomial)

$$EBO(n) = \sum_{j=n+1}^{\infty} (j - n) * p(BO = j-n) = \sum_{i=1}^{\infty} i * p(n + i)$$

i, backorder quantity

p(BO=j-n)=p(j)=p(n+i)

j, demand quantity

JFukuda Dic2008

Spares Inventory Model - Simulation with MS-Excel

Failue Rate =>	**9,6**	**Time to Repair =>**	**0,25**

Initial Spare=>	**5**	**Mean # in Repair=>**	**2,400000**	**p(OH=0) =>**	**0,095869**	**E[BO] =>**	**0,051722**	**VAR[BO] =>**	**0,092932**

Spares In Repair N(t)	p(j) Poisson	Fill Rate Poisson Last value	Spares on Hand	Spare on Back-order	Expected Number of Spares in Repair	p(BO=0) Last value	p(BO) Column E	p(OH=0)	p(OH) Column D	Expected Number of Spares on Backorder	Expected Number of square of BO	Variance
0	0,090717953	0,090717953	5	0	0,000000000	0,0907179533	0,0000000000	0,0000000000	0,0907179533	0,0000000000	0,0000000000	0,0000000000
1	0,217723088	0,308441041	4	0	0,217723088	0,3084410412	0,0000000000	0,0000000000	0,2177230879	0,0000000000	0,0000000000	0,0000000000
2	0,261267705	0,569708747	3	0	0,740258499	0,5697087467	0,0000000000	0,0000000000	0,2612677055	0,0000000000	0,0000000000	0,0000000000
3	0,209014164	0,778722911	2	0	1,367300992	0,7787229110	0,0000000000	0,0000000000	0,2090141644	0,0000000000	0,0000000000	0,0000000000
4	0,125408499	0,904131410	1	0	1,868934986	0,9041314097	0,0000000000	0,0000000000	0,1254084986	0,0000000000	0,0000000000	0,0000000000
5	0,060196079		0	0	2,169915383	0,9643274890	0,0000000000	0,0601960793		0,0000000000	0,0000000000	0,0000000000
6	0,024078432		0	1	2,314385974		0,0240784317	0,0842745111		0,0240784317	0,0240784317	0,0234986609
7	0,008255462		0	2	2,372174210		0,0082554623	0,0925299734		0,0405893564	0,0571002810	0,0554527851
8	0,002476639		0	3	2,391987319		0,0024766387	0,0950066121		0,0480192724	0,0793900292	0,0770841787
9	0,000660437		0	4	2,397931252		0,0006604370	0,0956670491		0,0506610204	0,0899570210	0,0873904820
10	0,000158505		0	5	2,399516301		0,0001585049	0,0958255539		0,0514535448	0,0939196429	0,0912721756
11	0,000034583		0	6	2,399896713		0,0000345829	0,0958601368		0,0516610420	0,0951646266	0,0924957634
12	0,000006917		0	7	2,399979712		0,0000069166	0,0958670534		0,0517094581	0,0955035389	0,0928296708
13	0,000001277		0	8	2,399996311		0,0000012769	0,0958683303		0,0517196733	0,0955852609	0,0929103363
14	0,000000219		0	9	2,399999376		0,0000002189	0,0958685492		0,0517216434	0,0956029916	0,0929278632
15	0,000000035		0	10	2,399999901		0,0000000350	0,0958685842		0,0517219937	0,0956064940	0,0929313294
16	0,000000005		0	11	2,399999985		0,0000000053	0,0958685895		0,0517220514	0,0956071297	0,0929319591
17	0,000000001		0	12	2,399999998		0,0000000007	0,0958685902		0,0517220603	0,0956072365	0,0929320650
18	0,000000000		0	13	2,400000000		0,0000000001	0,0958685903		0,0517220616	0,0956072532	0,0929320816
19	0,000000000		0	14	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072557	0,0929320840
20	0,000000000		0	15	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320843
21	0,000000000		0	16	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320844
22	0,000000000		0	17	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320844
23	0,000000000		0	18	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320844
24	0,000000000		0	19	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320844
25	0,000000000		0	20	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320844
26	0,000000000		0	21	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320844
27	0,000000000		0	22	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320844
28	0,000000000		0	23	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320844
29	0,000000000		0	24	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320844
30	0,000000000		0	25	2,400000000		0,0000000000	0,0958685903		0,0517220618	0,0956072560	0,0929320844

JFukuda Feb2008

Input values (Poisson modeled)
Failure rate = 9.6
Time to repair = 0.25
Initial spares = IS = 5

Output values
Fill rate = FR = 0.9041 ≅ 90.41%
Probability of spares in repair = p(IR=j)
p(j=0) = 0.0907 p(1) = 0.2177 p(2) = 0.2613 p(3) = 0.2090 p(4) = 0.1254 p(5) = 0.0602 …..
Mean or expected number in repair = 2.40 = 9.6 x 0.25
Probability of spare on hand = p(OH=n)
p(n=0) = 0.0959 p(1) = 0.1254 p(2) = 0.2090 p(3) = 0.2613 p(4) = 0.2177 p(5) = 0.0907
Mean or expected number on hand = 0*p(n=0)+1*p(1)+2*p(2)+3*p(3)+4*p(4)+5*p(5) = 2.6517
Probability of backorder = p(BO=k)
p(k=0) = 0.9643 p(1) = 0.0241 p(2) = 0.0083 p(3) = 0.0025 p(4) = 0.0007 p(5) = 0.0002 …..
Expected backorder = EBO = 0.0517
Variance of backorder = VBO = 0.0929
VBO/EBO = 1.797 > 1.000

Values relationship

$$FR = \sum_{j=0}^{IS-1} p(IR = j) \qquad p(BO = 0) = \sum_{k=0}^{IS} p(IR = k) \qquad p(OH = 0) = \sum_{n=IS}^{\infty} p(IR = n)$$

E(BO) = E(IR) – IS + E(OH) Example: 0.0517 = 2.4 – 5 + 2.6517
FR + p(IR=IS) = p(BO=0) Example: 0.9041 + 0.0602 = 0.9643
FR = 1 – p(OH=0) Example: 0.9041 = 1 – 0.0959
p(BO=0) + p(OH=0) = 1 + p(IR=IS) Example: 0.9643 + 0.0959 = 1 + 0.0602

JFukuda July2008