Time dependent failure rate

A) Dynamic poisson process

$$p(j,t) = \frac{[\Lambda(t)]^j}{j!} e^{-\Lambda(t)}$$

$$\Lambda(t) = \int_0^t \lambda(\tau)[1 - G(t-\tau)]d\tau$$

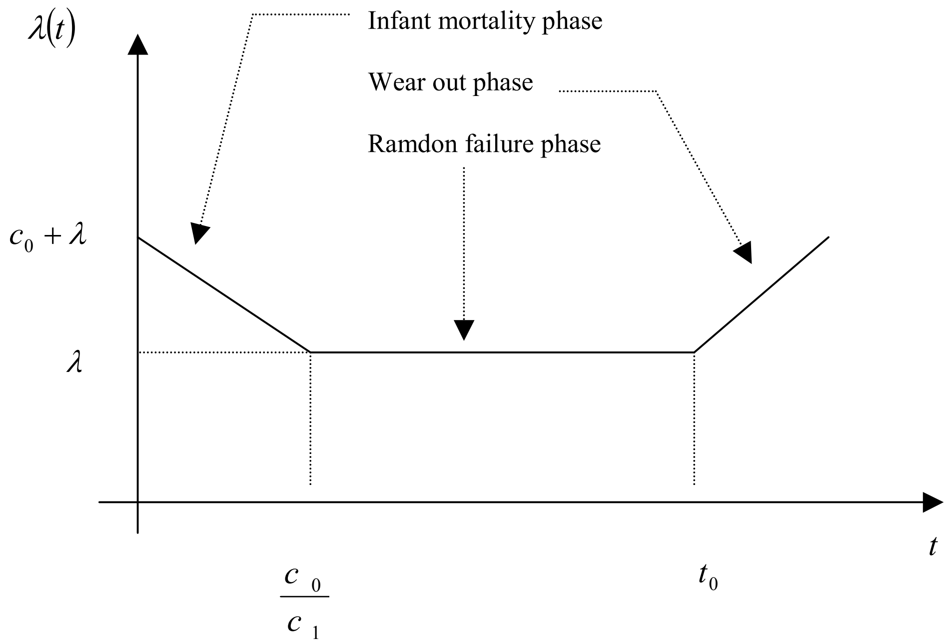
B) Dynamic failure rate

The failure rate is described by a straight lines bathtub model

$$\lambda(t) = c_0 - c_1 t + \lambda \qquad 0 \le t \le \frac{c_0}{c_1}$$

$$\lambda(t) = \lambda \qquad \frac{c_0}{c_1} < t \le t_0$$

$$\lambda(t) = c_2(t - t_0) + \lambda \qquad t > t_0$$



C) Repair time distribution (ν = mean repair time)

$$G(t) = 1 - e^{-\frac{t}{\nu}} \Rightarrow 1 - G(t-\tau) = e^{-\frac{(t-\tau)}{\nu}}$$

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D) <u>Numerical example using MS Excel</u>

D.1) Table of values

Constant	Value	Dimension	MS Excel table reference
λ	9.6	Unit / time	Cell A1
Co	7.0	Unit / time	Cell B1
C_1	1.0	Unit / square time	Cell B2
C_2	0.9	Unit / square time	Cell C1
t_o	20.0	Time	Cell C2
ν	0.25	Time	Column D
α		Unit / square time	
β		Unit / time	

D.2) Expression for pipeline Λ(t)

If $\lambda(t) = \alpha t + \beta \Rightarrow \dfrac{d[\lambda(t)]}{dt} = \alpha$ for dimensions of α and β see table above

$$\Lambda(t) = \int_0^t (\alpha\tau + \beta)\exp\left(-\frac{t-\tau}{\nu}\right)d\tau = \exp\left(-\frac{t}{\nu}\right)\int_0^t (\alpha\tau + \beta)\exp\left(\frac{\tau}{\nu}\right)d\tau = \exp\left(-\frac{t}{\nu}\right)\left[\alpha\int_0^t \tau\exp\left(\frac{\tau}{\nu}\right)d\tau + \beta\int_0^t \exp\left(\frac{\tau}{\nu}\right)\right]d\tau$$

$$\Lambda(t) = \exp\left(-\frac{t}{\nu}\right)\left[\alpha\nu^2\left[\left(\frac{\tau-\nu}{\nu}\right)\exp\left(\frac{\tau}{\nu}\right)\right]_0^t + \beta\left[\nu\exp\left(\frac{\tau}{\nu}\right)\right]_0^t\right] = \exp\left(-\frac{t}{\nu}\right)\left[\alpha\nu\left[(t-\nu)\exp\left(\frac{t}{\nu}\right)+\nu\right] + \beta\nu\left[\exp\left(\frac{t}{\nu}\right)-1\right]\right]$$

$$\Lambda(t) = \nu\exp\left(-\frac{t}{\nu}\right)\left[\alpha\left[(t-\nu)\exp\left(\frac{t}{\nu}\right)+\nu\right] + \beta\left[\exp\left(\frac{t}{\nu}\right)-1\right]\right] = \nu\exp\left(-\frac{t}{\nu}\right)\left[\alpha\left[(t-\nu)\exp\left(\frac{t}{\nu}\right)+\nu\right] + \beta\left[\exp\left(\frac{t}{\nu}\right)-1\right]\right]$$

$$\Lambda(t) = \nu\exp\left(-\frac{t}{\nu}\right)\left[\alpha(t-\nu)\exp\left(\frac{t}{\nu}\right) + \alpha\nu + \beta\exp\left(\frac{t}{\nu}\right) - \beta\right] = \nu\left((\alpha t - \alpha\nu + \beta)\exp\left(\frac{t}{\nu}\right) + \alpha\nu - \beta\right) = \nu\left[(\alpha t + \beta - \alpha\nu) + (\alpha\nu - \beta)\exp\left(-\frac{t}{\nu}\right)\right]$$

$$\Lambda(t) = \nu\left[(\alpha t + \beta - \alpha\nu) - (\beta - \alpha\nu)\exp\left(-\frac{t}{\nu}\right)\right]$$

$$\Lambda(t = 0) = 0$$

$$\lambda(t) = -c_1 t + (\lambda + c_0) \Rightarrow \frac{d\lambda}{dt} = \alpha = -c_1 = -1.0; \alpha\nu = -0.25$$

$$\beta = c_0 + \lambda = 7.0 + 9.6 = 16.6$$

$$\lambda(t) = \lambda \Rightarrow \frac{d\lambda}{dt} = \alpha = 0; \alpha\nu = 0; \beta = \lambda = 9.6$$

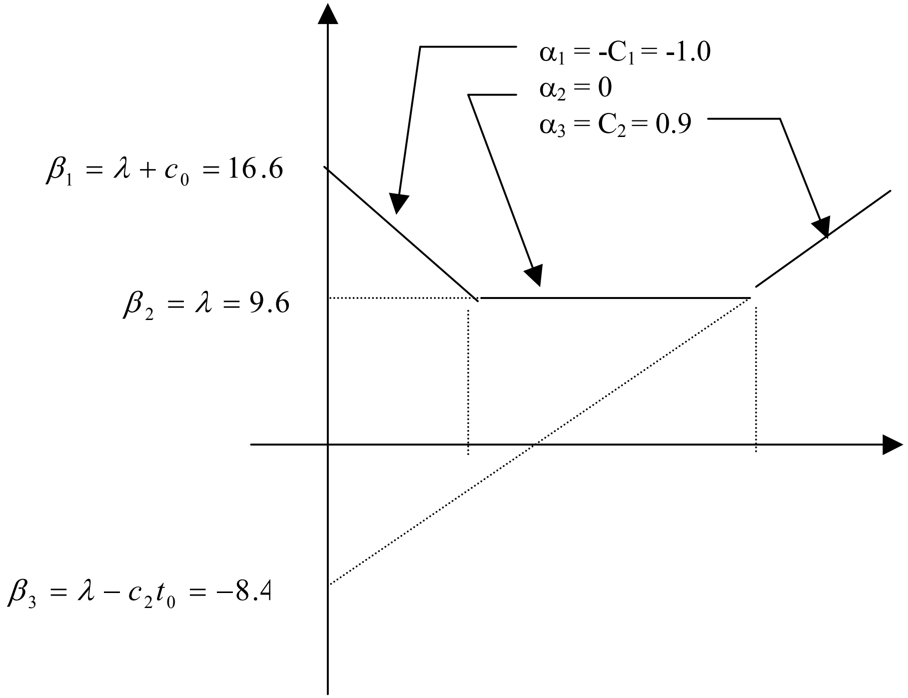
$$\lambda(t) = c_2 t + (\lambda - c_2 t_0) \Rightarrow \frac{d\lambda}{dt} = \alpha = c_2 = 0.9; \alpha\nu = 0.225$$

$$\beta = \lambda - c_2 t_0 = 9.6 - 0.9 * 20.0 = -8.4$$

$$\Lambda(t) = \nu\left[(\lambda(t) + c_1\nu) - (\lambda + c_0 + c_1\nu)\exp\left(-\frac{t}{\nu}\right)\right]$$

$$\Lambda(t) = \lambda\nu\left[1 - \exp\left(-\frac{t}{\nu}\right)\right]$$

$$\Lambda(t) = \nu\left[(\lambda(t) - c_2\nu) - (\lambda - c_2 t_0 - c_2\nu)\exp\left(-\frac{t}{\nu}\right)\right]$$



The terms of expression obtained have dimensional homogeneity.

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$$\Lambda(t)=\nu\left[(\alpha t+\beta-\alpha\nu)-(\beta-\alpha\nu)\exp\left(-\frac{t}{\nu}\right)\right]$$

In the early failure range and the time for maximum pipeline Λ(t) is:

$$\frac{d\Lambda(t)}{dt}=\nu\left[\alpha-(\beta-\alpha\nu)\left(-\frac{1}{\nu}\right)\exp\left(-\frac{t}{\nu}\right)\right]=\alpha\nu+(\beta-\alpha\nu)\exp\left(-\frac{t}{\nu}\right)=0$$

$$(\alpha\nu-\beta)\exp\left(-\frac{t}{\nu}\right)=\alpha\nu\Rightarrow\exp\left(\frac{t}{\nu}\right)=\frac{\alpha\nu-\beta}{\alpha\nu}\Rightarrow t=\nu*LN\left(1-\frac{\beta}{\alpha\nu}\right)$$

$$t=\nu*LN\left(1-\frac{\beta}{\alpha\nu}\right)=0.25*LN\left(1-\frac{16.6}{(-0.25)}\right)=0.25*LN(67.4)\cong 0.25*4.21\cong 1.05$$

$$\Lambda(t=1.05)=0.25\left[(-1*1.05+9.6+7+1*0.25)-(9.6+7+1*0.25)\exp\left(-\frac{1.05}{0.25}\right)\right]=0.25(15.8-16.85*0.015)\cong 3.8868$$

As time t increases the exponential **exp (-t / ν)** decreases rapidly, so neglecting last portion the pipeline formula becomes

$$\Lambda(t)\cong\nu\left(\lambda(t)-\nu\frac{d(\lambda(t))}{dt}\right)$$

and

$$t\geq\nu*LN\left(1-\frac{\beta}{\alpha\nu}\right)$$

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D.3) Time dependent failure rates

$$\Lambda(t) \cong \nu\left(\lambda(t) + c_1\nu\right)$$ for infant mortality phase

$$\Lambda(t) \cong \nu\lambda$$ for random failure phase

$$\Lambda(t) \cong \nu\left(\lambda(t) - c_2\nu\right)$$ for wear out phase

For constant failure range and steady state ($t \to \infty$)

$$\Lambda(t) = \nu\beta\left(1 - \exp\left(-\frac{t}{\nu}\right)\right) \Rightarrow \Lambda(t \to \infty) = \lambda\nu$$

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