Inter American University of Puerto Rico
Bayamón Campus
School of Engineering
Department of Electrical Engineering

ELEN 3311 – Electronics I

Midterm Exam Solutions

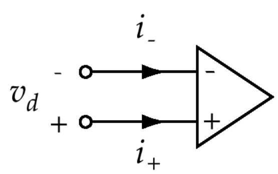
Problem 1: The operational amplifier in the circuit shown below is ideal, and characterized by the equations:

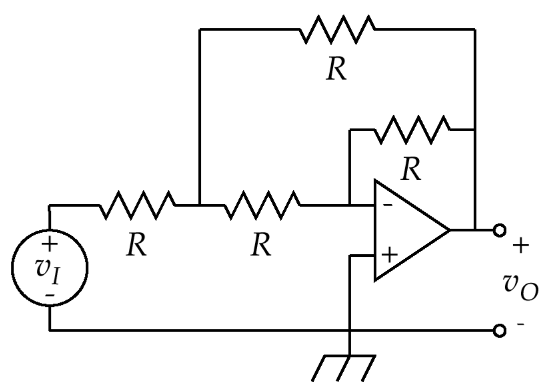
$i_+ = 0$

$i_- = 0$

$v_d = 0$

Find v_O in terms of v_I in the circuit shown below:





Let v_- and v_+ be the voltages at the + and − terminal of the operational amplifier, respectively. Then, $v_+ = 0$, since it is grounded. Since $v_d = 0$, $v_- = v_+ = 0$. Let e represent the voltage at the node to the left where three resistors unite. KCL at the v_- terminal yields

$$\frac{e-0}{R} = \frac{0-v_O}{R} \Rightarrow e = -v_O$$

KCL at node e yields

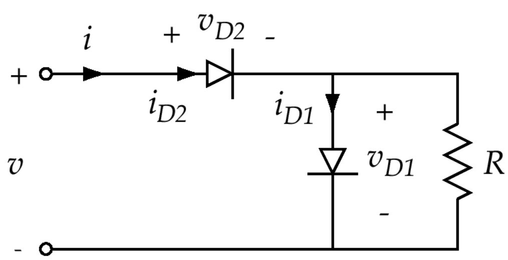
$$\frac{v_I-e}{R} + \frac{v_O-e}{R} = \frac{e-0}{R} \Rightarrow v_I + v_O - 3e = 0$$

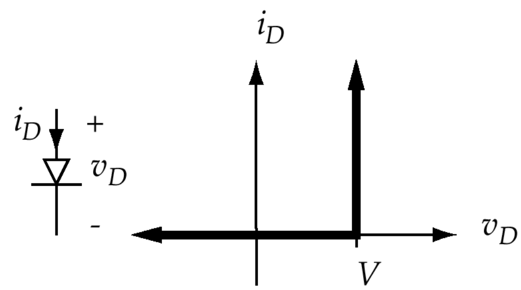
Substitution of the first equation into this equation yields

$$v_I + 4v_O = 0 \Rightarrow v_O = -\frac{1}{4}v_I$$

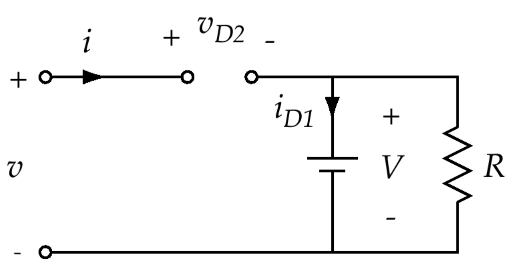
Problem 2: The diodes in the circuit shown below have a piece-wise linear voltage-current characteristic shown below, where:

$$v_D = V, \text{ for } i_D \geq 0$$
$$i_D = 0, \text{ for } v_D \leq V$$





(A) Show that the state where $v_{D1} = V$ and $i_{D2} = 0$ is impossible.



Substitution of the conditions for this state yields the circuit shown above. For this state to hold,

$$i_{D1} \geq 0$$

but

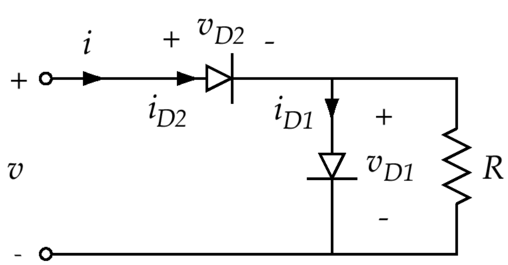
$$i_{D1} = -\frac{V}{R} < 0$$

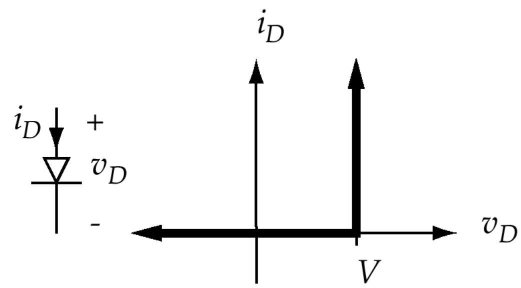
Thus, the state is impossible.

(B) Plot the voltage-current characteristic at the terminals shown in the circuit below, where the diode equations are:

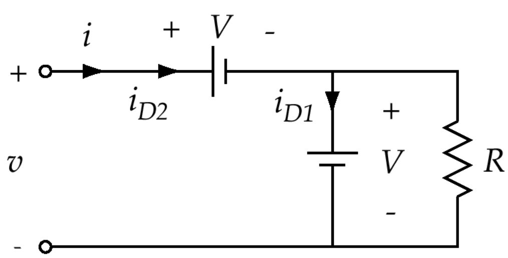
$$v_D = V, \text{ for } i_D \geq 0$$
$$i_D = 0, \text{ for } v_D \leq V$$

Use the axes provided and show all relevant voltages and currents.





State 1



Substitution of the conditions for the state where $v_{D1} = V$ *and* $v_{D2} = V$ *yields the circuit shown above. The vi characteristic for this circuit is*

$$v = 2V$$

For this state to hold,

$$i_{D1} \geq 0 \text{ and } i_{D2} \geq 0$$

For diode D2,

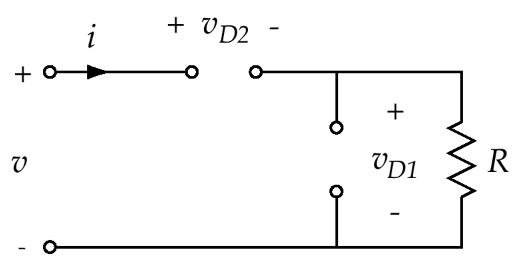
$$i_{D2} = i$$
$$\Rightarrow i \geq 0$$

which is one condition. For diode D1,

$$
\begin{aligned}
i_{D1} &= i - \frac{V}{R} \\
\Rightarrow i - \frac{V}{R} &\geq 0 \\
i &\geq \frac{V}{R}
\end{aligned}
$$

which is another, more restrictive condition. Thus, this is the condition for this state.

State 2



Substitution of the conditions for the state where $i_{D1} = 0$ *and* $i_{D2} = 0$ *yields the circuit shown above. The vi characteristic for this circuit is*

$$i = 0$$

For this state to hold,

$$v_{D1} \leq V \text{ and } v_{D2} \leq V$$

Since the current through the resistor R is zero,

$$v_{D1} = 0$$

and

$$0 \leq V$$

so that this condition is always fulfilled. Since $v_{D1} = 0$*,*

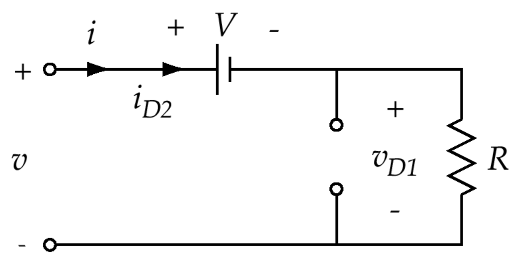
$$v = v_{D2}$$

Therefore, for $v_{D2} \leq V$ *to hold,*

$$v \leq V$$

which is the condition for this state.

State 3



Substitution of the conditions for the state where $i_{D1} = 0$ *and* $v_{D2} = V$ *yields the circuit shown above. The vi characteristic for this circuit is*

$$v = V + Ri$$

For this state to hold,

$$v_{D1} \leq V \text{ and } i_{D2} \geq 0$$

For diode $D2$*,*

$$\begin{aligned} i_{D2} &= i \\ &\Rightarrow i \geq 0 \end{aligned}$$

which is one condition. For diode $D1$*,*

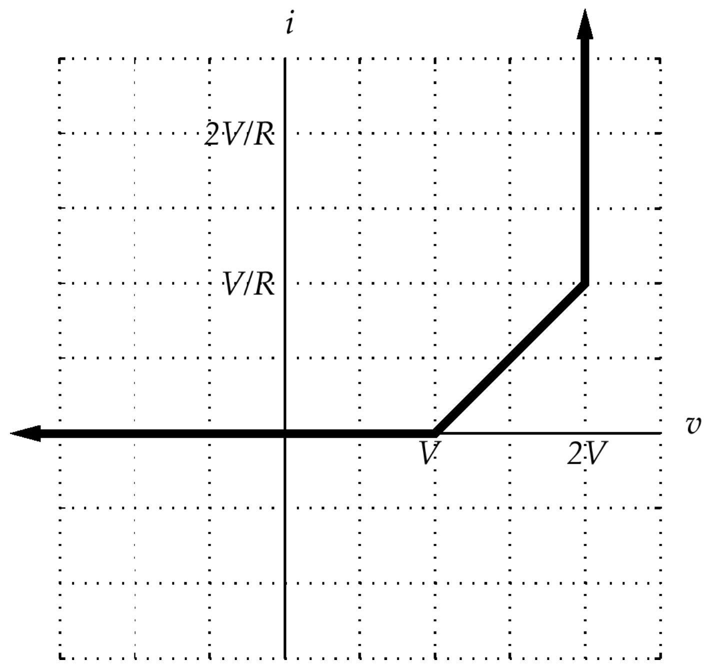
$$v_{D1} = Ri$$

Therefore, for $v_{D1} \leq V$ *to hold,*

$$\begin{aligned} Ri &\leq V \\ i &\leq \frac{V}{R} \end{aligned}$$

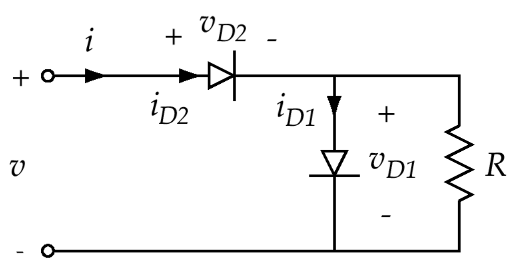
which is another condition for this state.

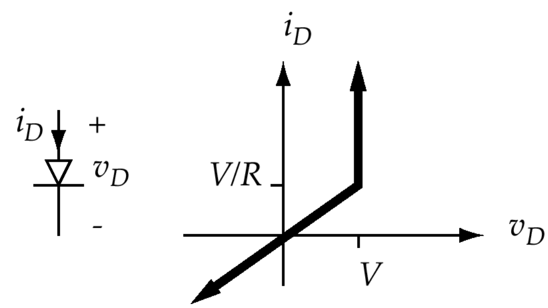
Combining all three solutions with their respective conditions yields the vi characteristic shown below.



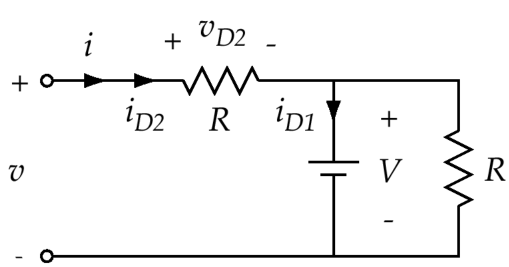
Problem 3: The new type of diodes in the circuit shown below have a piece-wise linear voltage-current characteristic shown below, where:

$$v_D = V, \text{ for } i_D \geq \frac{V}{R}$$
$$v_D = Ri_D, \text{ for } v_D \leq V \text{ or } i_D \leq \frac{V}{R}$$





(A) Show that the state where $v_{D1} = V$ and $v_{D2} = Ri_{D2}$ is impossible.



Substitution of the conditions for the state where $v_{D1} = V$ and $v_{D2} = Ri_{D2}$ yields the circuit shown above. For this state to hold,

$$i_{D1} \geq \frac{V}{R} \text{ and } i_{D2} \leq \frac{V}{R}$$

For diode D2,

$$i_{D2} = i$$
$$\Rightarrow i \leq \frac{V}{R}$$

which is one condition. For diode D1,

$$
\begin{aligned}
i_{D1} &= i - \frac{V}{R} \\
\Rightarrow i - \frac{V}{R} &\geq \frac{V}{R} \\
i &\geq \frac{2V}{R}
\end{aligned}
$$

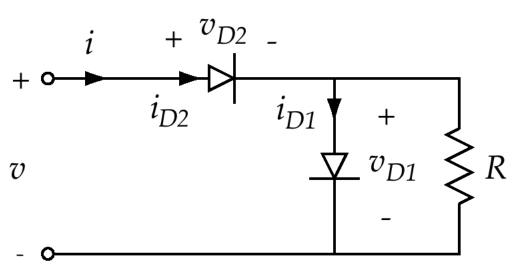
which is another condition. However, both conditions are mutually exclusive. Thus, the state is impossible.

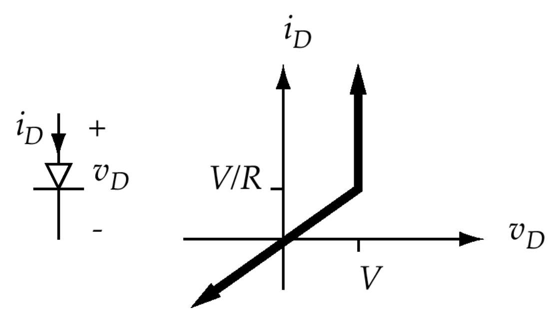
(B) Plot the voltage-current characteristic at the terminals shown in the circuit, where the diode equations are:

$$v_D = V, \text{ for } i_D \geq \frac{V}{R}$$

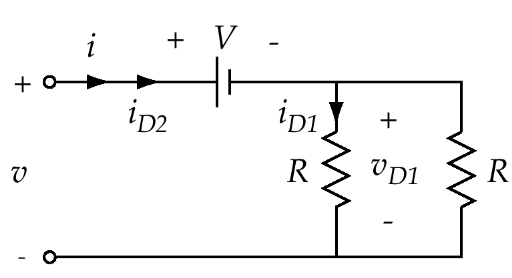
$$v_D = Ri_D, \text{ for } v_D \leq V \text{ or } i_D \leq \frac{V}{R}$$

Use the axes provided and show all relevant voltages and currents.





State 1



Substitution of the conditions for the state where $v_{D1} = Ri_{D1}$ *and* $v_{D2} = V$ *yields the circuit shown above. The vi characteristic for this circuit is*

$$v = V + \frac{R}{2}i$$

since i *splits evenly between the two resistors. For this state to hold,*

$$i_{D1} \leq \frac{V}{R} \text{ and } i_{D2} \geq \frac{V}{R}$$

For diode $D2$*,*

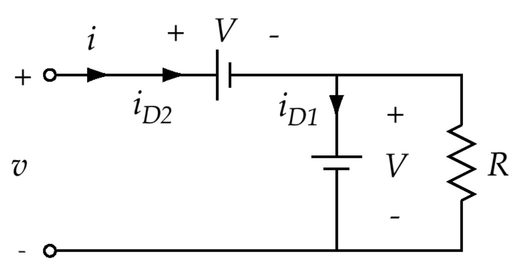
$$\begin{aligned} i_{D2} &= i \\ \Rightarrow i &\geq \frac{V}{R} \end{aligned}$$

which is one condition. For diode $D1$*,*

$$\begin{aligned} i_{D1} &= \frac{i}{2} \\ \Rightarrow \frac{i}{2} &\leq \frac{V}{R} \\ i &\leq \frac{2V}{R} \end{aligned}$$

which is another condition.

State 2



Substitution of the conditions for the state where $v_{D1} = V$ *and* $v_{D2} = V$ *yields the circuit shown above. The vi characteristic for this circuit is*

$$v = 2V$$

For diode D2,

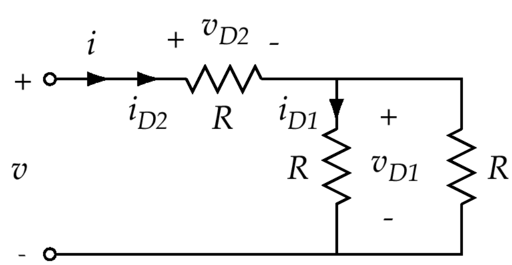
$$
\begin{aligned}
i_{D2} &= i \\
\Rightarrow i &\geq \frac{V}{R}
\end{aligned}
$$

which is one condition. For diode D1,

$$
\begin{aligned}
i_{D1} &= i - \frac{V}{R} \\
\Rightarrow i - \frac{V}{R} &\geq \frac{V}{R} \\
i &\geq \frac{2V}{R}
\end{aligned}
$$

which is another, more restrictive condition. Thus, this is the condition for this state.

State 3



Substitution of the conditions for the state where $v_{D1} = Ri_{D1}$ *and* $v_{D2} = Ri_{D2}$ *yields the circuit shown above. The vi characteristic for this circuit is*

$$v = \frac{3}{2}Ri$$

which is the combination of all three resistors. For this state to hold,

$$i_{D1} \leq \frac{V}{R} \text{ and } i_{D2} \leq \frac{V}{R}$$

For diode D2,

$$
\begin{aligned}
i_{D2} &= i \\
\Rightarrow i &\leq \frac{V}{R}
\end{aligned}
$$

which is one condition. For diode D1,

$$
\begin{aligned}
i_{D1} &= \frac{i}{2} \\
\Rightarrow \frac{i}{2} &\leq \frac{V}{R} \\
i &\leq \frac{2V}{R}
\end{aligned}
$$

However, the previous condition for diode D2 is more restrictive. Thus, the condition for diode D2, $i \leq \frac{V}{R}$, is the condition for this state.

Combining all three solutions with their respective conditions yields the vi characteristic shown below.