Inter American University of Puerto Rico
Bayamón Campus
School of Engineering
Department of Electrical Engineering

ELEN 3302 – Electric Circuits II

Revised Class Content

1. Transfer function $H(s)$

Impedance of elements:

$$\begin{aligned} R &\to R \\ L &\to Ls \\ C &\to \frac{1}{Cs} \\ v(t) &\to V(s) \\ i(t) &\to I(s) \end{aligned}$$

Treat all impedances as resistors.

Special transfer functions:

(A) Impedance: $\dfrac{V_{in}(s)}{I_{in}(s)}$ (at same terminal)

(B) Admittance: $\dfrac{I_{in}(s)}{V_{in}(s)}$ (at same terminal)

(C) Transmission: $\dfrac{V_{out}(s)}{V_{in}(s)}$ (at different terminals)

Example 1.1:

Find transmission transfer function $\dfrac{V_{out}(s)}{V_{in}(s)}$ for circuit (a):

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{RCs + 1}$$

Example 1.2:

Find transmission transfer function $\dfrac{V_{out}(s)}{V_{in}(s)}$ for circuit (b):

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{Cs}} = \frac{RCs}{RCs + 1}$$

Example 1.3:
Find transmission transfer function $\dfrac{V_{out}(s)}{V_{in}(s)}$ for circuit (f):

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{Cs} + Ls} = \frac{RCs}{LCs^2 + RCs + 1}$$

2. Sinusoidal steady state

$$\text{Find } H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$
$$\text{Let } s = j\omega$$
$$\text{Then, if } v_{in}(t) = V\sin(\omega_1 t),$$
$$v_{out}(t) = V \cdot |H(j\omega_1)| \sin\left[\omega_1 t + \angle H(j\omega_1)\right]$$

where V, sin and ω_1 are given by the input.

Example 2.1:
Find $v_{out}(t)$ in circuit (a) if $v_{in}(t) = V\cos(\omega_1 t)$:

Find transfer function $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$ (from Example 1.1):

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{RCs + 1}$$

Let $s = j\omega_1$,

$$H(j\omega_1) = \frac{1}{1 + jRC\omega_1}$$

Find $|H(j\omega_1)|$:

$$|H(j\omega_1)| = \frac{|1|}{|1 + jRC\omega_1|} = \frac{1}{\sqrt{1 + (RC\omega_1)^2}}$$

Find $\angle H(j\omega_1)$:

$$\angle H(j\omega_1) = \angle 1 - \angle(1 + jRC\omega_1) = 0 - \tan^{-1}\left(\frac{RC\omega_1}{1}\right) = -\tan^{-1}(RC\omega_1)$$

Then,

$$v_{out}(t) = V \cdot \frac{1}{\sqrt{1 + (RC\omega_1)^2}} \cos\left[\omega_1 t - \tan^{-1}(RC\omega_1)\right]$$

Example 2.2:
Find $v_{out}(t)$ in circuit (b) if $v_{in}(t) = V\sin(\omega_2 t)$:

Find transfer function $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$ (from Example 1.2):

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{RCs}{RCs+1}$$

Let $s = j\omega_2$,

$$H(j\omega_2) = \frac{jRC\omega_2}{1+jRC\omega_2}$$

Find $|H(j\omega_2)|$:

$$|H(j\omega_2)| = \frac{|jRC\omega_2|}{|1+jRC\omega_2|} = \frac{RC\omega_2}{\sqrt{1+(RC\omega_2)^2}}$$

Find $\angle H(j\omega_2)$:

$$\angle H(j\omega_2) = \angle jRC\omega_2 - \angle(1+jRC\omega_2) = \frac{\pi}{2} - \tan^{-1}(RC\omega_2)$$

Then,

$$v_{out}(t) = V \cdot \frac{RC\omega_2}{\sqrt{1+(RC\omega_2)^2}} \sin\left[\omega_2 t + \frac{\pi}{2} - \tan^{-1}(RC\omega_2)\right]$$

Example 2.3:
Find $v_{out}(t)$ in circuit (f) if $v_{in}(t) = V\cos(\omega_3 t + \phi)$:

Find transfer function $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$ (from Example 1.3):

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{RCs}{LCs^2+RCs+1}$$

Let $s = j\omega_3$,

$$H(j\omega_3) = \frac{jRC\omega_3}{1-LC\omega_3^2+jRC\omega_3}$$

Find $|H(j\omega_3)|$:

$$|H(j\omega_3)| = \frac{RC\omega_3}{\sqrt{(1-LC\omega_3^2)^2+(RC\omega_3)^2}}$$

Find $\angle H(j\omega_3)$:

$$\angle H(j\omega_3) = \frac{\pi}{2} - \tan^{-1}\left(\frac{RC\omega_3}{1 - LC\omega_3^2}\right)$$

Then,

$$v_{out}(t) = V \cdot \frac{RC\omega_3}{\sqrt{(1 - LC\omega_3^2)^2 + (RC\omega_3)^2}} \cos\left[\omega_3 t + \phi + \frac{\pi}{2} - \tan^{-1}\left(\frac{RC\omega_3}{1 - LC\omega_3^2}\right)\right]$$

3. Frequency response

First order circuits (RC, RL):

$$\begin{aligned} \text{Find} \quad & \lim_{\omega \to 0} H(j\omega) \\ \text{and} \quad & \lim_{\omega \to \infty} H(j\omega) \end{aligned}$$

Plot magnitude vs. ω and angle vs. ω at both limits and "join".

Example 3.1: Plot the frequency response of $\dfrac{V_{out}(s)}{V_{in}(s)}$ in circuit (a):

Find transfer function $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$ (from Example 1.1):

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{RCs + 1}$$

Let $s = j\omega$,

$$H(j\omega) = \frac{1}{1 + jRC\omega}$$

Find the magnitude and angle of the transfer function when $\omega \to 0$:

$$\begin{aligned} \lim_{\omega \to 0} H(j\omega) &= \frac{1}{1} = 1 \\ |\lim_{\omega \to 0} H(j\omega)| &= |1| = 1 \\ \angle \lim_{\omega \to 0} H(j\omega) &= \angle 1 = 0 \end{aligned}$$

Find the magnitude and angle of the transfer function when $\omega \to \infty$:

$$\begin{aligned} \lim_{\omega \to \infty} H(j\omega) &= \frac{1}{jRC\omega} \\ |\lim_{\omega \to 0} H(j\omega)| &= \frac{|1|}{|jRC\omega|} = \frac{1}{RC\omega} \\ \angle \lim_{\omega \to 0} H(j\omega) &= \angle 1 - \angle jRC\omega = 0 - \frac{\pi}{2} = -\frac{\pi}{2} \end{aligned}$$

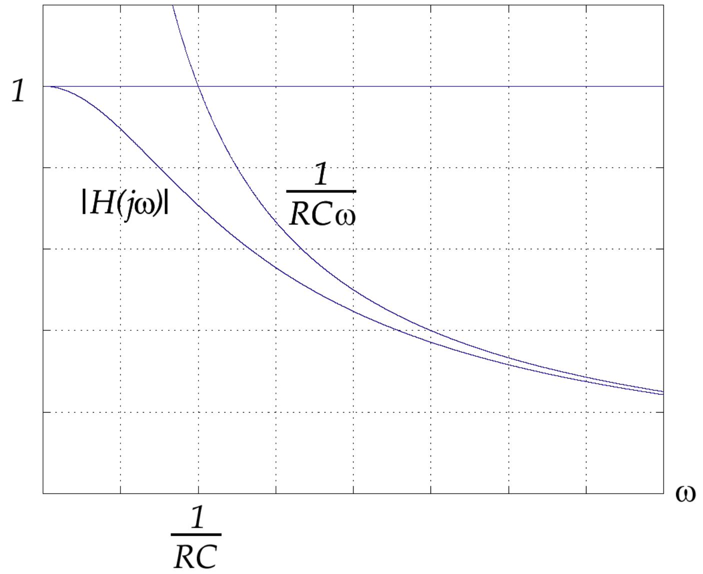
Note that the magnitude approximations will intersect at:

$$1 = \frac{1}{RC\omega} \Rightarrow \omega = \frac{1}{RC}$$

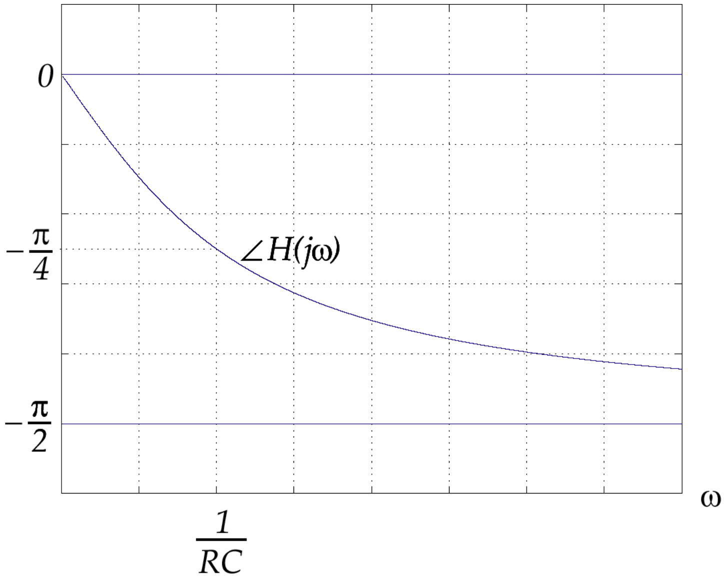
which is the same point where the angle is halfway between the two limits:

$$-\tan^{-1}(RC\omega) = -\frac{\pi}{4} \Rightarrow RC\omega = 1 \Rightarrow \omega = \frac{1}{RC}$$

Plot of $|H(j\omega)|$, $|\lim_{\omega\to 0} H(j\omega)|$, and $|\lim_{\omega\to\infty} H(j\omega)|$ versus w for circuit (a):



Plot of $\angle H(j\omega)$, $\angle \lim_{\omega\to 0} H(j\omega)$, and $\angle \lim_{\omega\to\infty} H(j\omega)$ versus w for circuit (a):



Second order circuits (LC, RLC):

$$\text{Find } \lim_{\omega\to 0} H(j\omega)$$
$$\text{and } \lim_{\omega\to\infty} H(j\omega)$$

Plot magnitude vs. ω and angle vs. ω at both limits. Find magnitude and angle at resonance $\left(\omega = \frac{1}{\sqrt{LC}}\right)$. Join limits passing through resonance values.

Example 3.2: Plot the frequency response of $\dfrac{V_{out}(s)}{V_{in}(s)}$ in circuit (f):

Find transfer function $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$ (from Example 1.1):

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{RCs}{LCs^2 + RCs + 1}$$

Let $s = j\omega$,

$$H(j\omega) = \frac{jRC\omega}{1 - LC\omega^2 + jRC\omega}$$

Find the magnitude and angle of the transfer function when $\omega \to 0$:

$$\lim_{\omega\to 0} H(j\omega) = \frac{jRC\omega}{1} = jRC\omega$$
$$|\lim_{\omega\to 0} H(j\omega)| = |jRC\omega| = RC\omega$$
$$\angle \lim_{\omega\to 0} H(j\omega) = \angle jRC\omega = \frac{\pi}{2}$$

Find the magnitude and angle of the transfer function when $\omega \to \infty$:

$$\lim_{\omega\to\infty} H(j\omega) = \frac{jRC\omega}{-LC\omega^2}$$
$$|\lim_{\omega\to 0} H(j\omega)| = \frac{|jRC\omega|}{|-LC\omega^2|} = \frac{RC\omega}{LC\omega^2} = \frac{R}{L\omega}$$
$$\angle \lim_{\omega\to 0} H(j\omega) = \angle jRC\omega - \angle(-LC\omega^2) = \frac{\pi}{2} - \pi = -\frac{\pi}{2}$$

Note that the magnitude approximations will intersect at:

$$RC\omega = \frac{R}{L\omega} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

The value of the approximations at this intersection is:

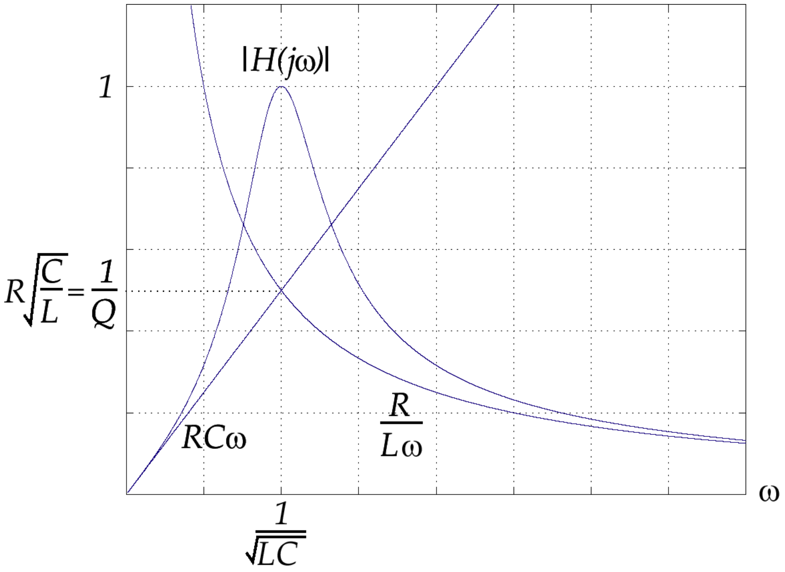
$$\left|\lim_{\omega\to 0} H\left(j\frac{1}{\sqrt{LC}}\right)\right| = RC\frac{1}{\sqrt{LC}} = R\sqrt{\frac{C}{L}}$$

But the actual function evaluates to one:

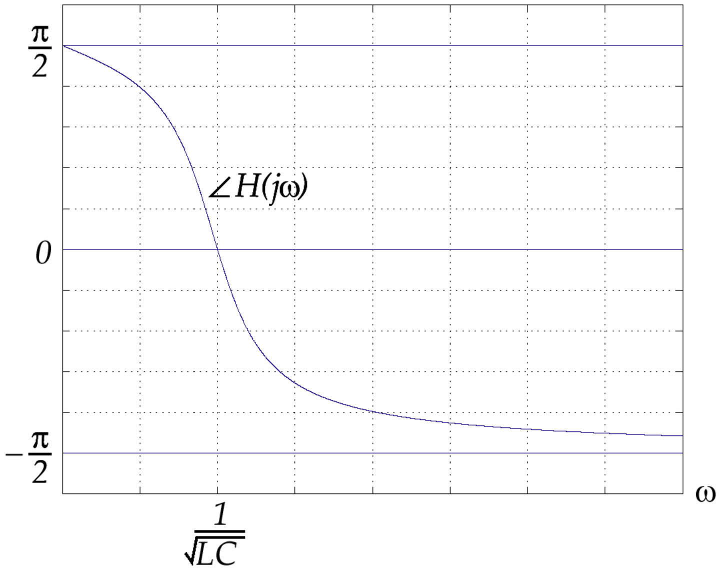
$$H\left(j\frac{1}{\sqrt{LC}}\right) = \frac{jRC\frac{1}{\sqrt{LC}}}{1 - LC\left(\frac{1}{\sqrt{LC}}\right)^2 + jRC\frac{1}{\sqrt{LC}}} = \frac{jRC\frac{1}{\sqrt{LC}}}{jRC\frac{1}{\sqrt{LC}}} = 1$$

which is the actual resonance peak. Note that if $R < \sqrt{\frac{L}{C}}$, then the peak is above the intersection of the approximations, as shown below.

Plot of $|H(j\omega)|$, $|\lim_{\omega\to 0} H(j\omega)|$, and $|\lim_{\omega\to\infty} H(j\omega)|$ versus w for circuit (f):



Plot of $\angle H(j\omega)$, $\angle \lim_{\omega\to 0} H(j\omega)$, and $\angle \lim_{\omega\to\infty} H(j\omega)$ versus w for circuit (f):



Filter types (based on frequency response):

(A) Low-pass filter: Lets low frequencies pass through, attenuates high frequencies (example 3.1).

(B) High-pass filter: Lets high frequencies pass through, attenuates low frequencies.

(C) Band-pass filter: Lets frequencies between two values (a frequency band) pass through, attenuates frequencies outside this band (example 3.2).

(D) Notch filter: Attenuates frequencies between two values (notch), lets frequencies outside the notch pass through.

4. Pole/zero plot

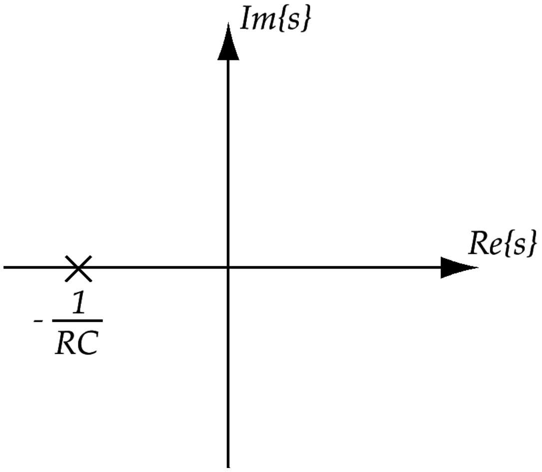
zeros: roots of the numerator of $H(s)$
poles: roots of the denominator of $H(s)$. System is stable if poles are on the left half plane (that is, if the real part is negative). Poles determine natural response (ZIR) of the system.

Example 4.1:

Find the pole/zero plot for transmission transfer function $\frac{V_{out}(s)}{V_{in}(s)}$ in circuit (a):
Recall the transfer function from Example 1.1:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{RCs + 1}$$

zeros: none
poles: $RCs + 1 = 0 \Rightarrow s = -\frac{1}{RC}$

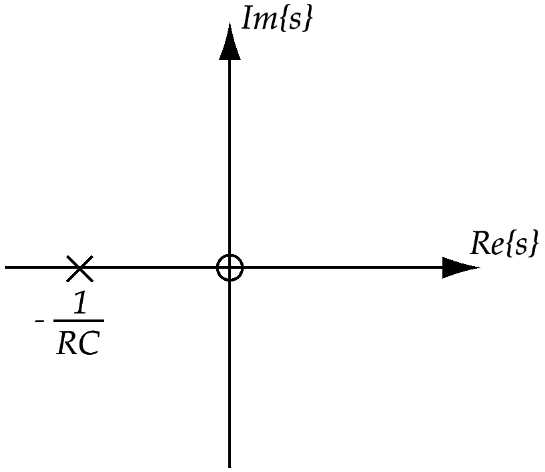


Example 4.2:

Find the pole/zero plot for transmission transfer function $\frac{V_{out}(s)}{V_{in}(s)}$ in circuit (b):
Recall the transfer function from Example 1.2:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{RCs}{RCs+1}$$

zeros: $RCs = 0 \Rightarrow s = 0$
poles: $RCs + 1 = 0 \Rightarrow s = -\frac{1}{RC}$



Example 4.3:

Find the pole/zero plot for transmission transfer function $\frac{V_{out}(s)}{V_{in}(s)}$ in circuit (f):
Recall the transfer function from Example 1.3:

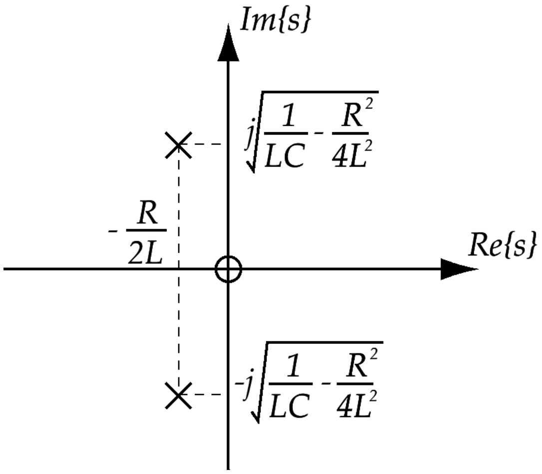
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{RCs}{LCs^2 + RCs + 1}$$

zeros: $RCs = 0 \Rightarrow s = 0$
poles: $LCs^2 + RCs + 1 = 0$
Use the fact that if $s^2 + 2\alpha s + \omega_0^2 = 0$, then $s = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$ (assuming $\omega_0 > \alpha$). In this case,

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Therefore, $\alpha = \frac{R}{2L}$ and $\omega_0^2 = \frac{1}{LC}$. Thus,

$$s = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$



5. Laplace Transform

The Laplace transform is defined as:

$$\mathcal{L}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-st}dt$$

A few Laplace transforms:

$$\begin{aligned}
\mathcal{L}\{u(t)\} &= \int_0^{\infty} e^{-st}dt = -\frac{e^{-st}}{s}\Big|_0^{\infty} = -\Big(-\frac{1}{s}\Big) = \frac{1}{s} \\
\mathcal{L}\{e^{-at}u(t)\} &= \int_0^{\infty} e^{-at}e^{-st}dt = \int_0^{\infty} e^{-(s+a)t}dt = -\frac{e^{-(s+a)t}}{s+a}\Big|_0^{\infty} = \frac{1}{s+a} \\
\mathcal{L}\{\cos(\omega t)u(t)\} &= \int_0^{\infty} \cos(\omega t)e^{-st}dt \\
&= \Big(\frac{\omega}{s^2+\omega^2}\sin(\omega t)e^{-st} - \frac{s}{s^2+\omega^2}\cos(\omega t)e^{-st}\Big)\Big|_0^{\infty} = \frac{s}{s^2+\omega^2}
\end{aligned}$$

The impulse function may be defined as:

$$\delta(t) = \lim_{\Delta\to 0}\delta_{\Delta}(t)$$

where $\delta_{\Delta}(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{\Delta} & 0 < t < \Delta \\ 0 & t > \Delta \end{cases}$

From this definition, two properties of the impulse function are:

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$$

Using the second property, the Laplace transform of the impulse function is

$$\mathcal{L}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t)e^{-st}dt = e^{-st}\big|_{t=0} = 1$$

The table below shows most of the Laplace transforms of interest for first and second order linear circuits:

Function name	**Function**	**Laplace transform**
impulse	$\delta(t)$	1
step	$u(t)$	$\dfrac{1}{s}$
exponential	$e^{-at}u(t)$	$\dfrac{1}{s+a}$
cosine	$\cos(\omega t)u(t)$	$\dfrac{s}{s^2+\omega^2}$
sine	$\sin(\omega t)u(t)$	$\dfrac{\omega}{s^2+\omega^2}$
decaying cosine	$e^{-at}\cos(\omega t)u(t)$	$\dfrac{s+a}{(s+a)^2+\omega^2}$
decaying sine	$e^{-at}\sin(\omega t)u(t)$	$\dfrac{\omega}{(s+a)^2+\omega^2}$

6. Unit step response

Find Laplace transform of input: $\mathcal{L}\{v_{in}(t)\} = V_{in}(s)$.
If input is $u(t)$, its transform is $1/s$. In the Laplace domain, the output is the product of the transfer function and the input:
$V_{out}(s) = H(s) \cdot V_{in}(s)$
Find inverse Laplace transform of output: $\mathcal{L}^{-1}\{V_{out}(s)\} = v_{out}(t)$.

Example 6.1:
Find $v_{out}(t)$ if $v_{in}(t) = u(t)$ in circuit (a):
Recall the transfer function from Example 1.1:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{RCs+1}$$

$$V_{in}(s) = \frac{1}{s}$$

$$V_{out}(s) = \frac{1}{s} \cdot \frac{1}{RCs+1} = \frac{1}{s(RCs+1)}$$

Use partial fraction expansion to express $V_{out}(s)$ in a form which can be easily transformed back to time:

$$\frac{1}{s(RCs+1)} = \frac{A}{s} + \frac{B}{RCs+1}$$

Multiply by $s(RCs+1)$ to eliminate all denominators:

$$1 = A(RCs+1) + Bs$$

Choose "clever" values of s to find A and B:

$$s = 0 \Rightarrow 1 = A \cdot 1 + 0 \Rightarrow A = 1$$

$$s = -\frac{1}{RC} \Rightarrow 1 = 0 + B \cdot \frac{-1}{RC} \Rightarrow B = -RC$$

Therefore,

$$V_{out}(s) = \frac{1}{s(RCs+1)} = \frac{1}{s} + \frac{-RC}{RCs+1} = \frac{1}{s} - \frac{1}{s+\frac{1}{RC}}$$

$$v_{out}(t) = u(t) - e^{-\frac{t}{RC}}u(t) = (1 - e^{-\frac{t}{RC}})u(t)$$

Example 6.2:
Find the unit step response of circuit (b):
Recall the transfer function from Example 1.2:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{RCs}{RCs+1}$$

$$V_{in}(s) = \frac{1}{s}$$

$$V_{out}(s) = \frac{1}{s} \cdot \frac{RCs}{RCs+1} = \frac{RCs}{s(RCs+1)} = \frac{RC}{RCs+1} = \frac{1}{s+\frac{1}{RC}}$$

Therefore,

$$v_{out}(t) = e^{-\frac{t}{RC}}u(t)$$

7. Unit impulse response $h(t) = \mathcal{L}^{-1}\{H(s)\}$

A very important result in linear systems is that the transfer function of a circuit is the Laplace transform of its impulse response. Thus, if the transfer function of system is known, then its impulse response is simply the inverse Laplace transform of this transfer function:

$$\begin{aligned} v_{in}(t) &= \delta(t) \\ V_{in}(s) &= \mathcal{L}\{\delta(t)\} = 1 \\ V_{out}(s) &= H(s)V_{in}(s) = H(s) \cdot 1 \\ v_{out}(t) &= \mathcal{L}^{-1}\{H(s)\} = h(t) \end{aligned}$$

Another way to write the same equation:

$$\begin{aligned}
v_{out}(t) &= \mathcal{L}^{-1}\{V_{out}(s)\} \\
&= \mathcal{L}^{-1}\{H(s)\cdot V_{in}(s)\} \\
&= \mathcal{L}^{-1}\big\{H(s)\cdot \mathcal{L}\{v_{in}(t)\}\big\} \\
&= \mathcal{L}^{-1}\big\{H(s)\cdot \mathcal{L}\{\delta(t)\}\big\} && \text{since } v_{in}(t)=\delta(t) \\
&= \mathcal{L}^{-1}\{H(s)\cdot 1\} && \text{since } \mathcal{L}\{\delta(t)\}=1 \\
&= \mathcal{L}^{-1}\{H(s)\} \\
&= h(t)
\end{aligned}$$

Example 7.1:
Find the unit impulse response of circuit (b):
Recall the transfer function from Example 1.2:

$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{RCs}{RCs+1}$$

Since $h(t)=\mathcal{L}^{-1}\{H(s)\}$, express $H(s)$ in a form which can be inverse-transformed easily:

$$H(s)=\frac{RCs}{RCs+1}=1-\frac{1}{RCs+1}=1-\frac{\frac{1}{RC}}{s+\frac{1}{RC}}=1-\frac{1}{RC}\cdot\frac{1}{s+\frac{1}{RC}}$$

Therefore,

$$h(t)=\delta(t)-\frac{1}{RC}e^{-\frac{t}{RC}}u(t)$$

Example 7.1:
Find the unit impulse response of circuit (h) if $L=1\text{H}$, $C=0.2\text{F}$ and $R=2\Omega$:
First, find the transfer function:

$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{\frac{1}{Cs}}{Ls+R+\frac{1}{Cs}}=\frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

Substitute the values for R, L and C:

$$H(s)=\frac{5}{s^2+2s+5}$$

Complete the square:

$$H(s)=\frac{5}{s^2+2s+1+4}=\frac{5}{(s+1)^2+4}$$

Since the remainder in the denominator is positive, $H(s)$ must be made to match one of the sinusoidal forms of the Laplace transform:

$$H(s) = \frac{5}{(s+1)^2+4} = \frac{5}{2}\cdot\frac{2}{(s+1)^2+2^2}$$

Therefore,

$$h(t) = \frac{5}{2}e^{-t}\sin(2t)u(t)$$

Example 7.2:
Find the unit impulse response of circuit (h) if $L = 1\text{H}$, $C = 0.2\text{F}$ and $R = 6\Omega$:
First, find the transfer function:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{Cs}}{Ls + R + \frac{1}{Cs}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Substitute the values for R, L and C:

$$H(s) = \frac{5}{s^2+6s+5}$$

Complete the square:

$$H(s) = \frac{5}{s^2+6s+9-4} = \frac{5}{(s+3)^2-4}$$

Since the remainder in the denominator is negative, $H(s)$ can be separated into two real fractions:

$$H(s) = \frac{5}{(s+3)^2-2^2} = \frac{5}{[(s+3)+2][(s+3)-2]} = \frac{5}{(s+5)(s+1)} = \frac{A}{s+5} + \frac{B}{s+1}$$

Multiply by $(s+5)(s+1)$ to eliminate all denominators:

$$5 = A(s+1) + B(s+5)$$

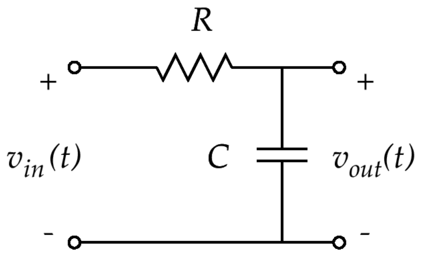
$$s = -1 \Rightarrow 5 = B\cdot 4 \Rightarrow B = \frac{5}{4}$$

$$s = -5 \Rightarrow 5 = A\cdot(-4) \Rightarrow A = -\frac{5}{4}$$

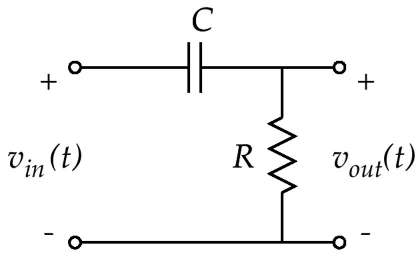
$$H(s) = \frac{-5/4}{s+5} + \frac{5/4}{s+1}$$

Therefore,

$$h(t) = \frac{5}{4}e^{-t}u(t) - \frac{5}{4}e^{-5t}u(t)$$


R
+
+
$v_{in}(t)$
C
$v_{out}(t)$
-
-

(a)


C
+
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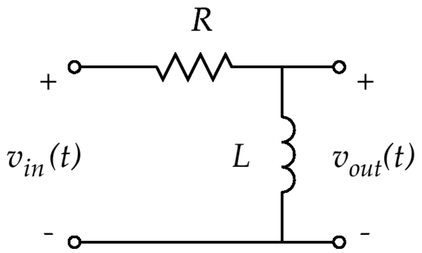
$v_{in}(t)$
R
$v_{out}(t)$
-
-

(b)


R
+
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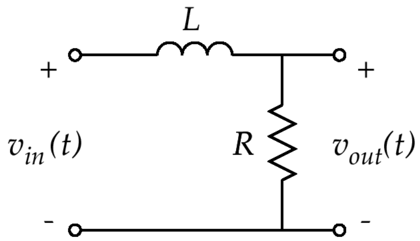
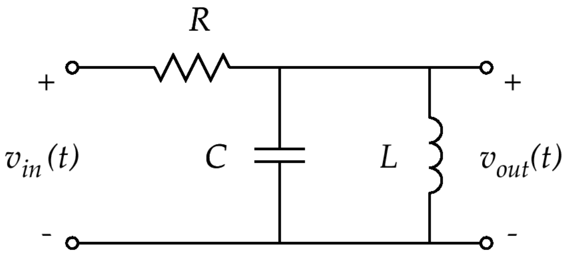
-
-

(c)


L
+
+
$v_{in}(t)$
R
$v_{out}(t)$
-
-

(d)


R
+
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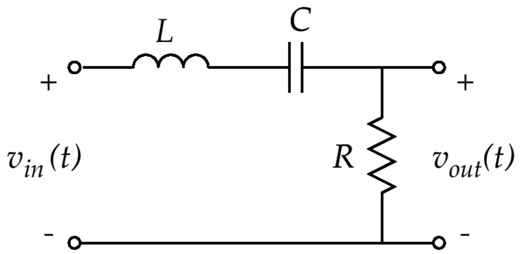
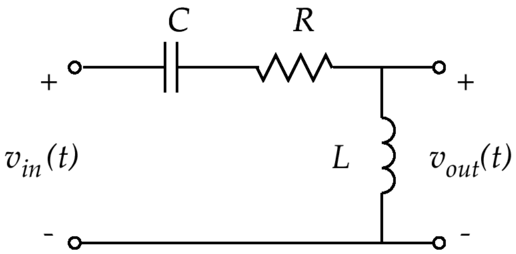
C
L
$v_{out}(t)$
-
-

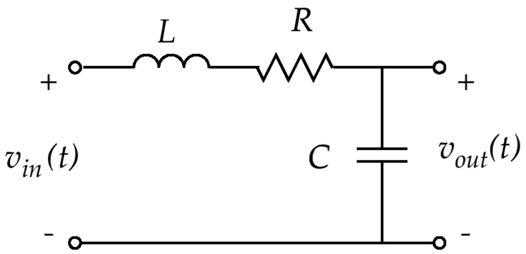
(e)


L
C
+
+
$v_{in}(t)$
R
$v_{out}(t)$
-
-

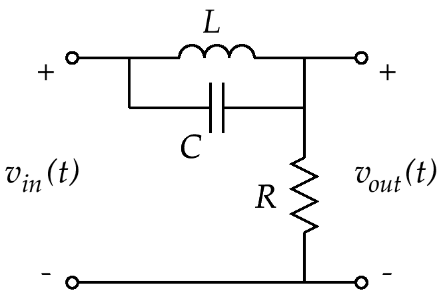
(f)


C
R
+
+
$v_{in}(t)$
L
$v_{out}(t)$
-
-

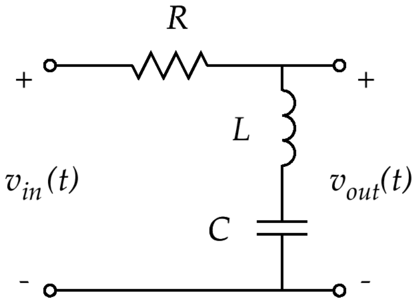
(g)


L
R
+
+
$v_{in}(t)$
C
$v_{out}(t)$
-
-

(h)


L
+
+
C
$v_{in}(t)$
R
$v_{out}(t)$
-
-

(i)


R
+
+
L
$v_{in}(t)$
C
$v_{out}(t)$
-
-

(j)