Inter American University of Puerto Rico
Bayamón Campus
School of Engineering
Department of Electrical Engineering

ELEN 3302 – Electric Circuits II

Problem Set 1

Due Wednesday, August 25

Problem 1:

(A) Prove that $e^{jx} = \cos x + j \sin x$ by using the Taylor series expansion of e^x, $\sin x$ and $\cos x$.

(B) Using the relation proved in the previous part, derive the following relationships:

(i) $\cos x = \frac{1}{2}(e^{jx} + e^{-jx})$

(ii) $\sin x = \frac{j}{2}(e^{-jx} - e^{jx})$ *(Hint: Show that $1/j = -j$.)*

(iii) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

(iv) $\sin x \sin y = \frac{1}{2}\cos(x - y) - \frac{1}{2}\cos(x + y)$

(v) $\sin(x + y) = \sin x \cos y + \cos x \sin y$

(C) A linear system has the output $y(t) = e^{j3t}$ if the input is $x(t) = e^{j2t}$, and the output $y(t) = e^{-j3t}$ if the input is $x(t) = e^{-j2t}$. Determine the output of the system if:

(i) $x_1(t) = \cos 2t$

(ii) $x_2(t) = \cos[2(t - 1/2)]$

Problem 2:

(A) Consider a time-invariant system. Show that if input $x(t)$ is periodic, then output $y(t)$ must also be periodic. *Hint: Remember that periodicity is defined as $x(t-T) = x(t)$ for some period T.*

(B) Give an example of a time-invariant system and a nonperiodic input signal $x(t)$ such that the corresponding output $y(t)$ is periodic.

Problem 3:

(A) Consider the statement: The series interconnection of two linear time-invariant systems is a linear time-invariant system. True or false? Justify your answer.

(B) Consider the statement: The series interconnection of two nonlinear systems is a nonlinear system. True or false? Justify your answer.

Problem 4: An even signal is a signal which satisfies the condition $x_e(-t) = x_e(t)$. An odd signal satisfies $x_o(-t) = -x_o(t)$. Any signal $x(t)$ may be decomposed into even and odd parts using $x_e(t) = [x(t) + x(-t)]/2$ and $x_o(t) = [x(t) - x(-t)]/2$.

(A) Check that the equation for $x_e(t)$ yields an even signal, and that the equation for $x_o(t)$ yields an odd signal. Also check that $x_e(t) + x_o(t) = x(t)$.

(B) Show that

$$\int_{-\infty}^{+\infty} x_o(t)dt = 0$$

for any odd signal $x_o(t)$.

(C) Show that if $x_1(t)$ is an odd signal and $x_2(t)$ is an even signal, then $x_1(t)x_2(t)$ is an odd signal.

(D) Show that

$$\int_{-\infty}^{+\infty} x^2(t)dt = \int_{-\infty}^{+\infty} x_e^2(t)dt + \int_{-\infty}^{+\infty} x_o^2(t)dt$$

where $x_e(t)$ and $x_o(t)$ are the even and odd parts of $x(t)$.