Inter American University of Puerto Rico
Bayamón Campus
School of Engineering
Department of Electrical Engineering

ELEN 3301 – Electric Circuits I

Quiz 2 Key

Wednesday, November 10

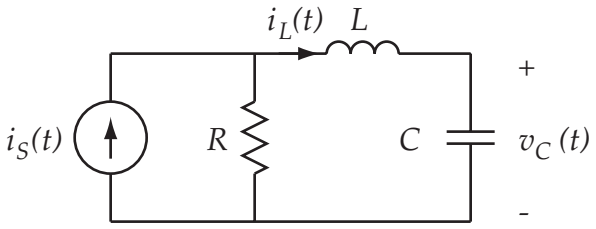
Name: _____

Enter all your work and your answers directly in the spaces provided on the printed pages. Full credit will only be given to answers that you neatly transfer to the spaces on the printed pages. Answers must be derived or explained, not just simply written down. Use the spaces to write the answers where provided. The quiz is closed book, and calculators or notes are **not** allowed.

Problem	Points	Grade
1	25	
2	10	
3	10	
4	55	
Total	100	

Problem 1: *(25 points)*

(A) Find the differential equation that relates $v_C(t)$ and $i_S(t)$ in the circuit shown below.



$$\frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = \frac{R}{LC}i_S$$

(B) Assuming $R < 2\sqrt{L/C}$, what is the exponential decay constant α of any response? Give your answer in terms of the component values of the circuit R, L and C.

$$\alpha = \frac{R}{2L}$$

(C) Assuming $R < 2\sqrt{L/C}$, what is the angular frequency ω_d of any response? Give your answer in terms of the component values of the circuit R, L and C.

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

(D) Assuming $i_S(t)$ is a step function of the form $I_S u(t)$, what is the value v_{final} of $v_C(t)$ as $t \to \infty$?

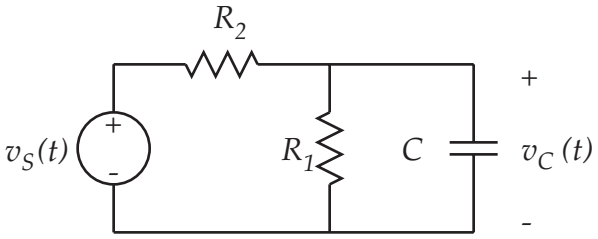
$$v_{\text{final}} = I_S R$$

(E) *(10 bonus points)* Find the differential equation that relates $i_L(t)$ and $i_S(t)$.

$$\frac{d^2i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC}i_L = \frac{R}{L}\frac{di_S}{dt}$$

Problem 2: *(10 points)*

(A) What is the exponential decay time constant τ in the circuit shown below? You don't need to find the differential equation.



$$\tau = \frac{R_1 R_2}{R_1 + R_2} C$$

(B) Assuming $v_S(t)$ is a step function of the form $V_S u(t)$, what is the final value v_{final} of $v_C(t)$ as $t \to \infty$?

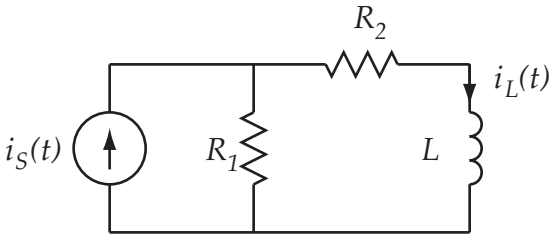
$$v_{\text{final}} = \frac{R_1}{R_1 + R_2} V_S$$

(C) *(5 bonus points)* Assuming $v_C(0) = 0$, find an algebraic expression for $v_C(t)$. You may use τ and v_{final} in your answer.

$$v_C(t) = v_{\text{final}}(1 - e^{-t/\tau})$$

Problem 3: *(10 points)*

(A) What is the exponential decay time constant τ in the circuit shown below? You don't need to find the differential equation.



$$\tau = \frac{L}{R_1 + R_2}$$

(B) Assuming $i_S(t)$ is a step function of the form $I_S u(t)$, what is the final value i_{final} of $i_L(t)$ as $t \to \infty$?

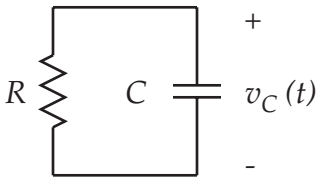
$$i_{\text{final}} = \frac{R_1}{R_1 + R_2} I_S$$

(C) *(5 bonus points)* Assuming $i_L(0) = 0$, find an algebraic expression for $i_L(t)$. You may use τ and i_{final} in your answer.

$$i_L(t) = i_{\text{final}}(1 - e^{-t/\tau})$$

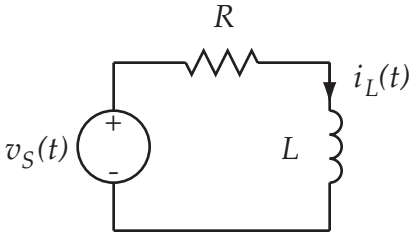
Problem 4: *(55 points)* For each of the circuits below, choose the best graph that represents the answer from the graphs in the last page. You may tear the page with the graphs off. You don't need to hand it in. The arrow on each graph represents the initial slope. The axes intersect at the origin. Each graph may represent a voltage or a current as needed. Write the letter of each graph in the space provided. You may write the answer directly if you're confident it is correct. All variables V_S, I_S, i_0 and v_0 represent real, positive numbers.

(A) Assuming $v_C(0) = v_0$ in the circuit shown below, which graph best represents $v_C(t)$?



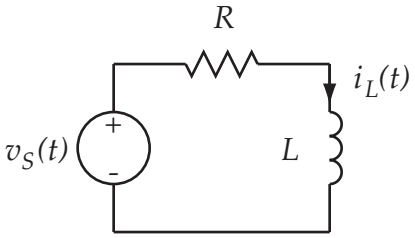
The best graph is h.

(B) Assuming $v_S(t) = V_S u(t)$ and $i_L(0) = 0$ in the circuit shown below, which graph best represents $i_L(t)$?



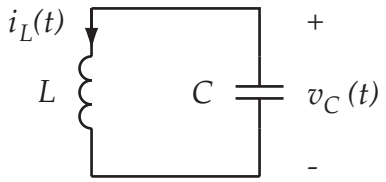
The best graph is j.

(C) Assuming $v_S(t) = V_S u(t)$, $i_L(0) = i_0$ and $i_0 > V_S/R$ in the circuit shown below, which graph best represents $i_L(t)$?



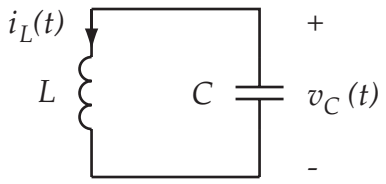
The best graph is i.

(D) Assuming $i_L(0) = i_0$ and $v_C(0) = 0$ in the circuit shown below, which graph best represents $v_C(t)$?



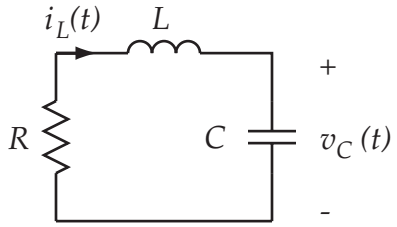
The best graph is b.

(E) Assuming $v_C(0) = v_0$ and $i_L(0) = 0$ in the circuit shown below, which graph best represents $v_C(t)$?



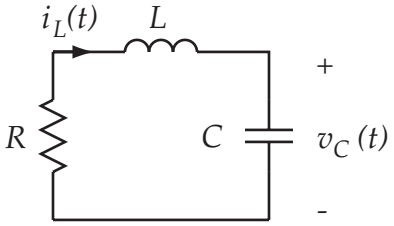
The best graph is c.

(F) Assuming $v_C(0) = v_0$, $i_L(0) = 0$ and $R < 2\sqrt{L/C}$ in the circuit shown below, which graph best represents $v_C(t)$?



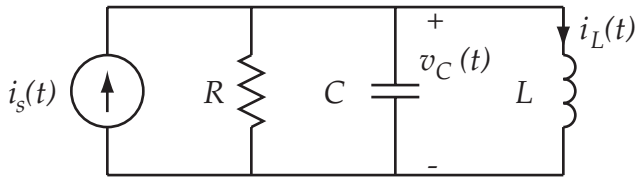
The best graph is g.

(G) Assuming $i_L(0) = i_0$, $v_C(0) = 0$ and $R < 2\sqrt{L/C}$ in the circuit shown below, which graph best represents $v_C(t)$?



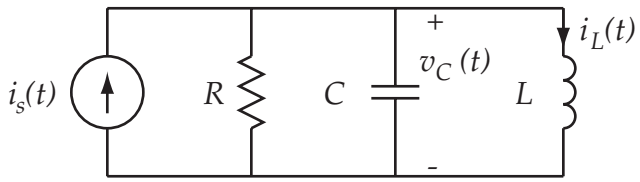
The best graph is d.

(H) Assuming $i_S(t) = I_S u(t)$, $i_L(0) = 0$, $v_C(0) = 0$ and $2R < \sqrt{L/C}$ in the circuit shown below, which graph best represents $i_L(t)$?

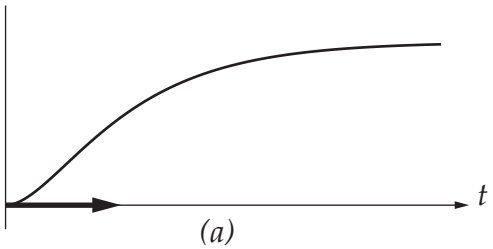


The best graph is a.

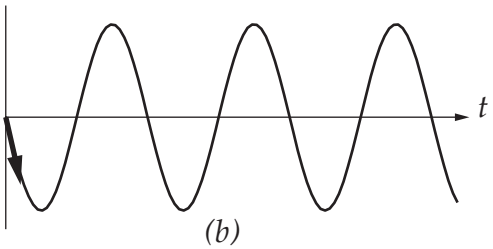
(I) Assuming $i_S(t) = I_S u(t)$, $i_L(0) = 0$, $v_C(0) = 0$ and $2R > \sqrt{L/C}$ in the circuit shown below, which graph best represents $i_L(t)$?



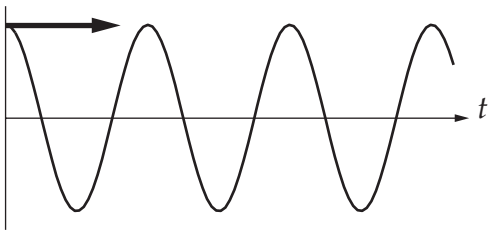
The best graph is f.



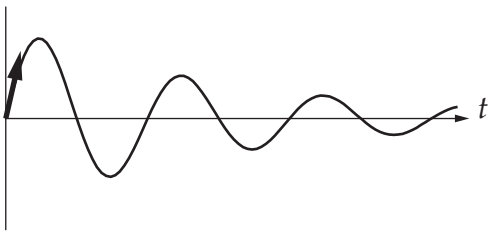
(a)



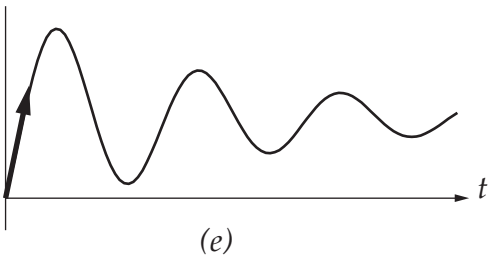
(b)



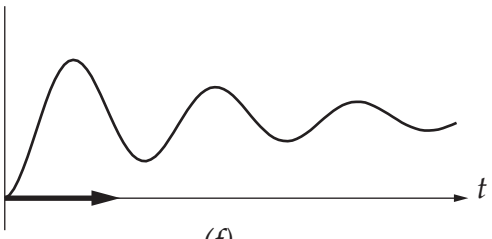
(c)



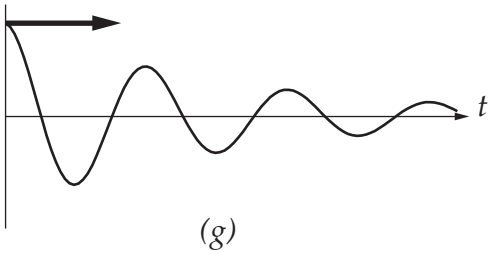
(d)



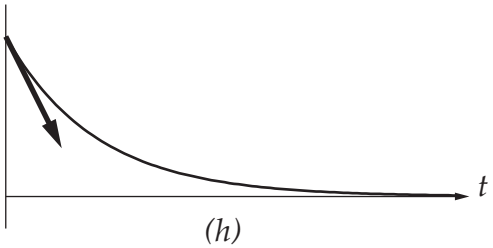
(e)



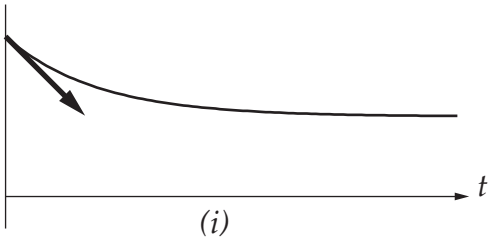
(f)



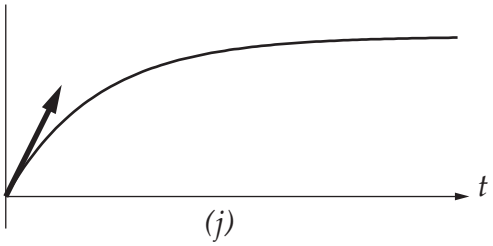
(g)



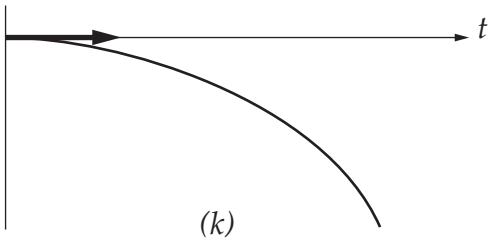
(h)



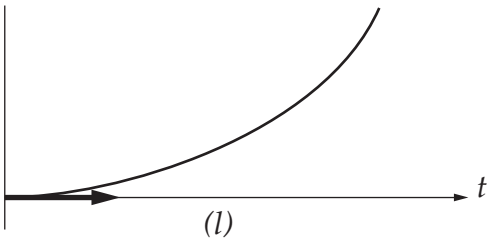
(i)



(j)



(k)



(l)