

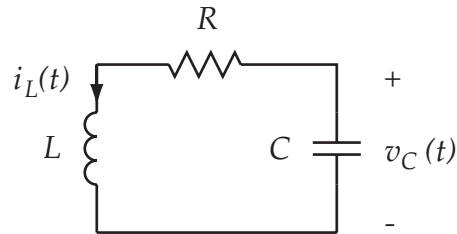
Inter American University of Puerto Rico
Bayamón Campus
School of Engineering
Department of Electrical Engineering

ELEN 3301 – Electric Circuits I

Problem Set 8 Solutions

Due Wednesday, October 27

Problem 1: For the circuit below, $v_C(0) = v_0$ and $i_L(0) = 0$. Find an algebraic expression and sketch $v_C(t)$.



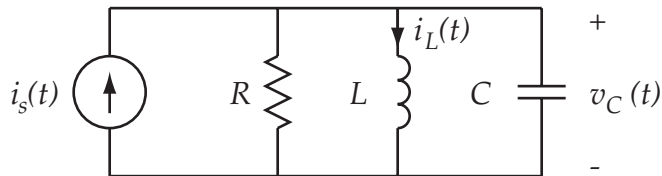
$$\frac{d^2 v_C}{dt^2} + \frac{R}{L} \frac{dv_C}{dt} + \frac{1}{LC} v_C = 0$$

$$v_C(t) = v_0 e^{-\alpha t} \left(\cos \omega_d t + \frac{\alpha}{\omega_d} \sin \omega_d t \right)$$

$$\alpha = \frac{R}{2L} \quad \omega_d = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

Problem 2:

- (A) For the circuit below, $v_C(0) = 0$ and $i_L(0) = 0$. Find an algebraic expression and sketch $i_L(t)$ if $i_S(t) = I_S u(t)$.



$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC} \frac{di_L}{dt} + \frac{1}{LC} i_L = \frac{1}{LC} i_S$$

$$i_L(t) = I_S - I_S e^{-\alpha t} \left(\cos \omega_d t + \frac{\alpha}{\omega_d} \sin \omega_d t \right)$$

$$i_L(t) = I_S - I_S \frac{\omega_0}{\omega_d} e^{-\alpha t} \cos \left[\omega_d t - \tan^{-1} \left(\frac{\alpha}{\omega_d} \right) \right]$$

$$\alpha = \frac{1}{2RC} \quad \omega_d = \sqrt{\frac{1}{LC} - \frac{1}{4R^2 C^2}}$$

- (B) Repeat part (A) if $i_S(t) = Q_S \delta(t)$.

$$Q_S \delta(t) = \frac{Q_S}{I_S} \frac{d}{dt} I_S u(t)$$

$$i_L(t) = \frac{Q_S}{I_S} \frac{d}{dt} \left[I_S - I_S e^{-\alpha t} \left(\cos \omega_d t + \frac{\alpha}{\omega_d} \sin \omega_d t \right) \right]$$

$$i_L(t) = \frac{Q_S}{LC \omega_d} e^{-\alpha t} \sin \omega_d t$$

$$\alpha = \frac{1}{2RC} \quad \omega_d = \sqrt{\frac{1}{LC} - \frac{1}{4R^2 C^2}}$$