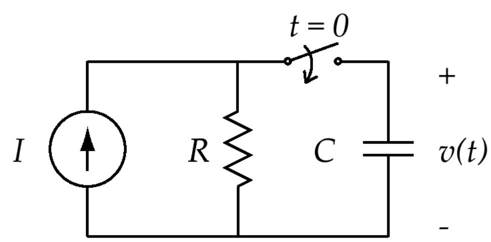
Inter American University of Puerto Rico
Bayamón Campus
School of Engineering
Department of Electrical Engineering

ELEN 3301 – Electric Circuits I

Problem Set 5 Solutions

Due Wednesday, October 6

Problem 1: The capacitor C has an initial voltage of v_0 at time $t = 0$. At this instant, the switch closes. Find an algebraic expression and sketch $v(t)$ for $t > 0$.



KCL at the topmost node after the switch has closed yields

$$I = \frac{v}{R} + C\frac{dv}{dt}$$

The differential equation in standard form is

$$\frac{dv}{dt} + \frac{1}{RC}v = \frac{I}{C}$$

The homogeneous solution is the solution to the equation

$$\frac{dv}{dt} + \frac{1}{RC}v = 0$$

which takes the form $v_H(t) = Ae^{st}$. Its derivative is $\frac{dv_H}{dt} = Ase^{st}$. Substitution into the differential equation yields

$$Ase^{st} + \frac{1}{RC}Ae^{st} = 0$$

The function e^{st} is never zero and can be eliminated. $A = 0$ solves the equation, but is of no interest since this is the *trivial* solution. Thus, assume $A \neq 0$ and cancel it. The resulting algebraic equation is

$$s + \frac{1}{RC} = 0 \Rightarrow s = -\frac{1}{RC}$$

and the homogeneous solution is

$$v_H(t) = Ae^{-\frac{t}{RC}} = Ae^{-t/\tau} \quad \tau = RC$$

The particular solution takes the form of the input function. In this case, it is a constant, and the particular solution is assumed to be a constant so that $v_P(t) = K$ and $\frac{dv_P}{dt} = 0$. Substitution into the differential equation yields

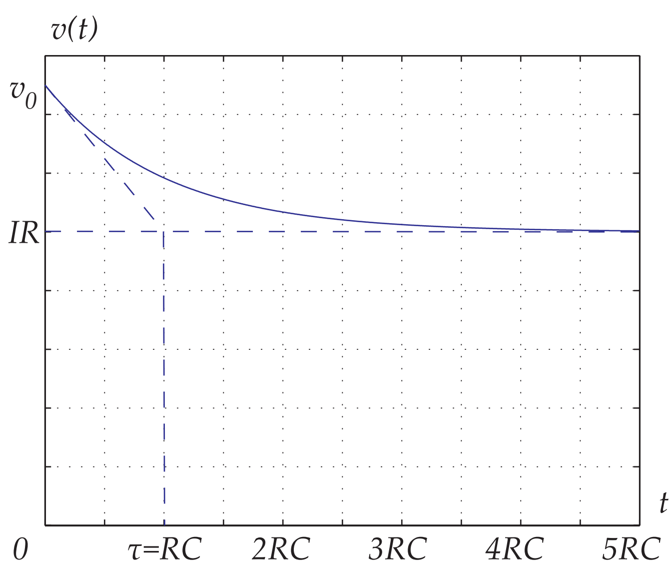
$$0 + \frac{1}{RC}K = \frac{I}{C} \Rightarrow K = IR$$

The general solution is the sum of the particular and homogeneous solutions:

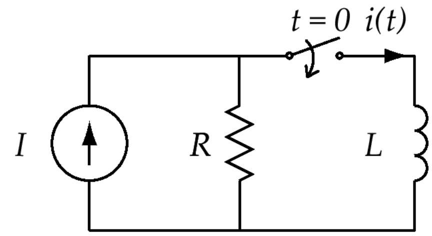
$$v(t) = v_P(t) + v_H(t) = IR + Ae^{-t/\tau} \quad \tau = RC$$

The initial condition implies $v(0) = IR + A = v_0 \Rightarrow A = v_0 - IR$. Back-substitution yields the solution to the problem:

$$v(t) = IR + (v_0 - IR)e^{-t/\tau} = v_0 e^{-t/\tau} + IR(1 - e^{-t/\tau}) \quad \tau = RC$$



Problem 2: The inductor L has an initial current of i_0 at time $t = 0$. At this instant, the switch closes. Find an algebraic expression and sketch $i(t)$ for $t > 0$.



Again, KCL at the topmost node after the switch has closed yields

$$I = \frac{v_L}{R} + i$$

where we note that $v_L = v_R$. Further $v_L = L\frac{di}{dt}$, so that

$$I = \frac{L\frac{di}{dt}}{R} + i$$

The differential equation in standard form is

$$\frac{di}{dt} + \frac{R}{L}i = \frac{RI}{L}$$

The homogeneous solution is the solution to the equation

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

which takes the form $i_H(t) = Ae^{st}$. Its derivative is $\frac{di_H}{dt} = Ase^{st}$. Substitution into the differential equation yields

$$Ase^{st} + \frac{R}{L}Ae^{st} = 0$$

The function e^{st} is never zero and can be eliminated. $A = 0$ solves the equation, but is of no interest since this is the *trivial* solution. Thus, assume $A \neq 0$ and cancel it. The resulting algebraic equation is

$$s + \frac{R}{L} = 0 \Rightarrow s = -\frac{R}{L}$$

and the homogeneous solution is

$$i_H(t) = Ae^{-\frac{Rt}{L}} = Ae^{-t/\tau} \quad \tau = \frac{L}{R}$$

The particular solution takes the form of the input function. In this case, it is a constant, and the particular solution is assumed to be a constant so that $i_P(t) = K$ and $\frac{di_P}{dt} = 0$. Substitution into the differential equation yields

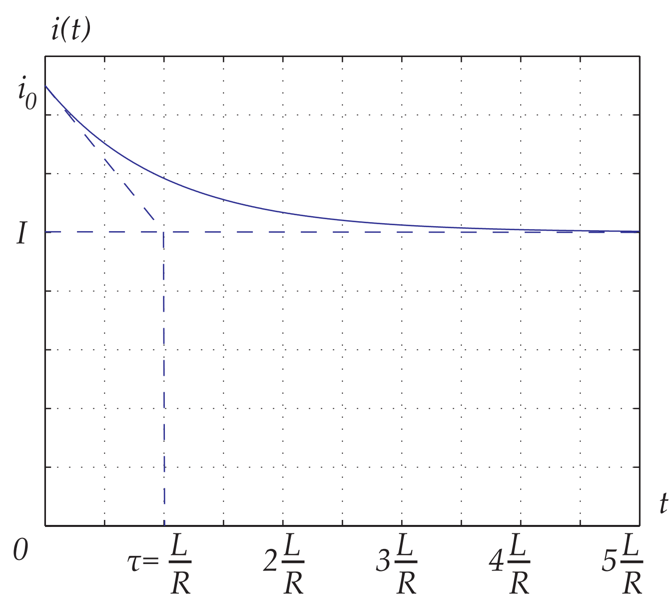
$$0 + \frac{R}{L}K = \frac{RI}{L} \Rightarrow K = I$$

The general solution is the sum of the particular and homogeneous solutions:

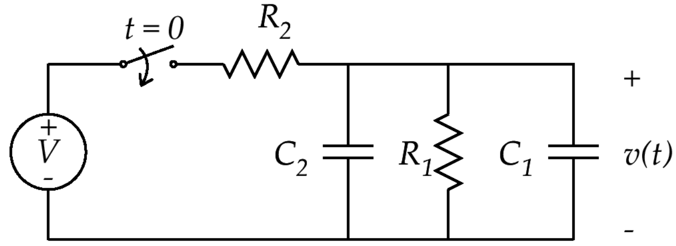
$$i(t) = i_P(t) + i_H(t) = I + Ae^{-t/\tau} \quad \tau = \frac{L}{R}$$

The initial condition implies $i(0) = I + A = i_0 \Rightarrow A = i_0 - I$. Back-substitution yields the solution to the problem:

$$i(t) = I + (i_0 - I)e^{-t/\tau} = i_0 e^{-t/\tau} + I(1 - e^{-t/\tau}) \quad \tau = \frac{L}{R}$$



Problem 3: The voltage $v(t)$ has an initial voltage of 0 at time $t = 0$. At this instant, the switch closes. Find an algebraic expression and sketch $v(t)$ for $t > 0$.



Once more, KCL at the topmost node after the switch has closed yields

$$\frac{V - v}{R_2} = C_2\frac{dv}{dt} + \frac{v}{R_1} + C_1\frac{dv}{dt}$$

The differential equation in standard form is

$$\frac{dv}{dt} + \frac{R_1 + R_2}{R_1 R_2(C_1 + C_2)}v = \frac{V}{R_2(C_1 + C_2)}$$

The homogeneous solution is the solution to the equation

$$\frac{dv}{dt} + \frac{R_1 + R_2}{R_1 R_2(C_1 + C_2)}v = 0$$

which takes the form $v_H(t) = Ae^{st}$. Its derivative is $\frac{dv_H}{dt} = Ase^{st}$. Substitution into the differential equation yields

$$Ase^{st} + \frac{R_1 + R_2}{R_1 R_2(C_1 + C_2)}Ae^{st} = 0$$

The function e^{st} is never zero and can be eliminated. $A = 0$ solves the equation, but is of no interest since this is the *trivial* solution. Thus, assume $A \neq 0$ and cancel it. The resulting algebraic equation is

$$s + \frac{R_1 + R_2}{R_1 R_2(C_1 + C_2)} = 0 \Rightarrow s = -\frac{R_1 + R_2}{R_1 R_2(C_1 + C_2)}$$

and the homogeneous solution is

$$v_H(t) = Ae^{-t/\tau} \quad \tau = \frac{R_1 R_2(C_1 + C_2)}{R_1 + R_2}$$

The particular solution takes the form of the input function. In this case, it is a constant, and the particular solution is assumed to be a constant so that $v_P(t) = K$ and $\frac{dv_P}{dt} = 0$. Substitution into the differential equation yields

$$0 + \frac{R_1 + R_2}{R_1 R_2 (C_1 + C_2)} K = \frac{V}{R_2 (C_1 + C_2)} \Rightarrow K = \frac{R_1}{R_1 + R_2} V = v_f$$

The general solution is the sum of the particular and homogeneous solutions:

$$v(t) = v_P(t) + v_H(t) = v_f + Ae^{-t/\tau} \quad \tau = \frac{R_1 R_2 (C_1 + C_2)}{R_1 + R_2}$$

The initial condition implies $i(0) = v_f + A = 0 \Rightarrow A = -v_f$. Back-substitution yields the solution to the problem:

$$v(t) = v_f - v_f e^{-t/\tau} = v_f(1 - e^{-t/\tau}) \quad \tau = \frac{R_1 R_2 (C_1 + C_2)}{R_1 + R_2}$$