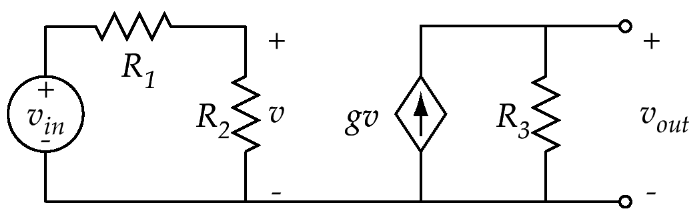
Inter American University of Puerto Rico
Bayamón Campus
School of Engineering
Department of Electrical Engineering

ELEN 3301 – Electric Circuits I

Problem Set 4 Solutions

Due Wednesday, September 29

Problem 1: Refer to the network shown below for all parts of this problem:



(A) Find v_{out}.

The voltage v can be found using a voltage divider:

$$v = \frac{R_2}{R_1 + R_2} v_{in}$$

The voltage v_{out} is simply the current gv times R_3. Substituting the expression for v,

$$v_{out} = R_3 g \frac{R_2}{R_1 + R_2} v_{in}$$

(B) Find the Thevenin equivalent circuit looking into the v_{out} terminals.

First, we find the equivalent resistance R_{eq} looking into the v_{out} terminals. To do this, we "turn off" v_{in} by replacing it with a short circuit and effectively grounding R_1. KCL at the node where R_1 and R_2 join tells us:

$$\frac{v}{R_1} + \frac{v}{R_2} = 0$$

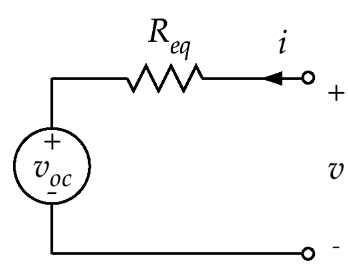
which is true only if $v = 0$. Thus, we can replace the dependent current source gv with an open circuit. Thus, the resistance seen at the v_{out} terminals is simply R_3.

$$R_{eq} = R_3$$

Then, we find the open circuit voltage v_{oc} by looking at the voltage in the v_{out} terminals due to v_{in}. However, this is exactly the same problem as solved in part (A). Thus,

$$v_{oc} = R_3 g \frac{R_2}{R_1 + R_2} v_{in}$$

Thus, the Thevenin equivalent circuit is:



where

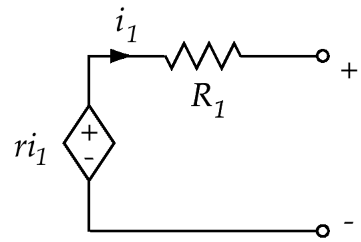
$$v_{oc} = R_3 g \frac{R_2}{R_1 + R_2} v_{in}$$

and

$$R_{eq} = R_3$$

Problem 2:

(A) Find the Thevenin equivalent circuit looking into the marked terminals of the circuit below:



To find v_{oc}, note that $i_1 = 0$. This implies that both the dependent source voltage and the voltage across R_1 are zero. Therefore, by KVL,

$$v_{oc} = ri_1 - R_1 i_1 = 0$$

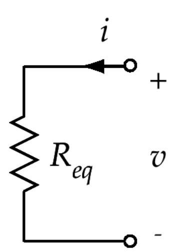
To find R_{eq}, apply a test voltage V_T with the same polarity as the marked terminals. KVL in the circuit results in:

$$V_T + R_1 i_1 = ri_1$$

Further, note that the desired test current $I_T = -i_1$. Thus,

$$V_T - R_1 I_T = -rI_T$$

and the Thevenin equivalent is

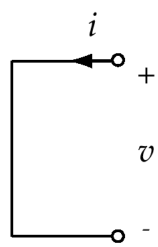


where

$$R_{eq} = \frac{V_T}{I_T} = R_1 - r$$

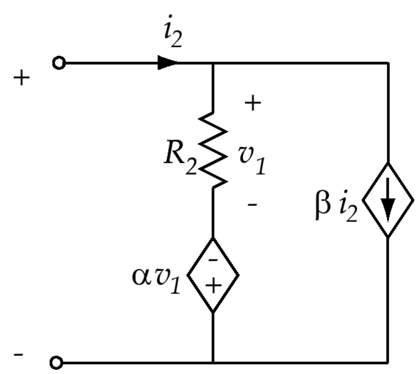
What is the Thevenin equivalent circuit if $r = R_1$? Explain.

If $r = R_1$, the Thevenin equivalent is



since the dependent source will exactly cancel the voltage drop across R_1.

(B) Find the Thevenin equivalent circuit looking into the marked terminals of the circuit below:



To find v_{oc}, note that $i_2 = 0$. If $i_2 = 0$, then βi_2 is also zero. By KCL at the top node, the current entering R_2 is zero, and $v_1 = 0$. Thus, the dependent voltage source is also zero. Using KVL, $v_{oc} = v_1 - \alpha v_1 = 0$.
To find R_{eq}, apply a test voltage V_T with the same polarity as the marked terminals. KVL in the left loop yields:

$$V_T = v_1 - \alpha v_1$$

KCL at the top node yields:

$$\frac{v_1}{R_2} = i_2 - \beta i_2$$

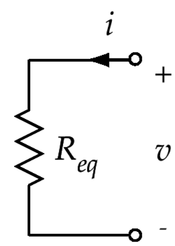
Note that i_2 is the desired test current I_T. Thus,

$$v_1 = (1 - \beta) R_2 I_T$$

and

$$V_T = (1 - \alpha)(1 - \beta) R_2 I_T$$

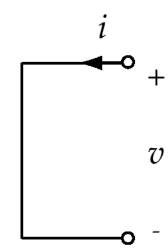
Thus, the thevenin equivalent is,



where $R_{eq} = (1-\alpha)(1-\beta)R_2$.

What is the Thevenin equivalent circuit if $\beta = 1$? Explain.

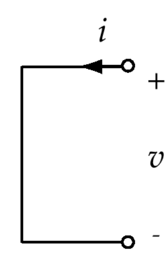
If $\beta = 1$, the Thevenin equivalent is



because all the current i_2 entering the circuit is going to be drawn by the current source. Thus, no current is going to flow down the center branch. This means $v_1 = 0$, and $\alpha v_1 = 0$. Thus, by KCL, the voltage across the terminal will be zero for all values of i_2.

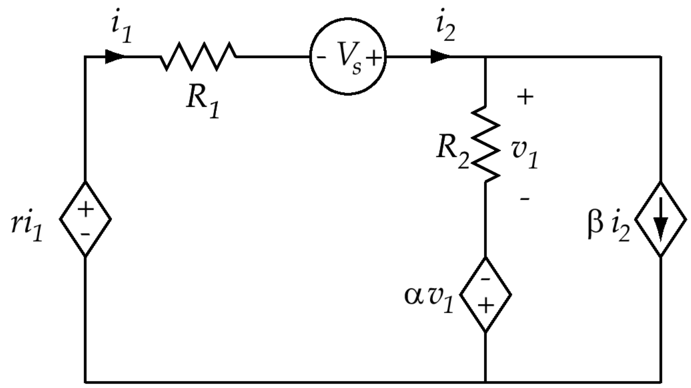
What is the Thevenin equivalent circuit if $\alpha = 1$? Explain.

Again, if $\alpha = 1$, the Thevenin equivalent is



because the dependent voltage source is going to cancel exactly the voltage drop across R_2. Thus, the voltage drop across the terminals will always be zero.

(C) The networks shown above are connected to a floating voltage source as shown below:



Find i_2.

Both networks can be replaced by their Thevenin equivalents, which consist of an equivalent resistance in each case. Thus, current i_2 is simply the source voltage V_s divided by the series combination of both equivalent resistances:

$$i_2 = \frac{V_s}{R_1 - r + (1 - \beta)(1 - \alpha)R_2}$$