Inter American University of Puerto Rico
Bayamón Campus
School of Engineering
Department of Electrical Engineering

ELEN 3301 – Electric Circuits I

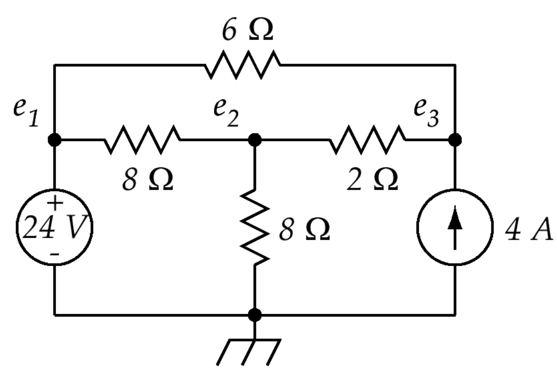
Problem Set 3 Solutions

Due Wednesday, September 15

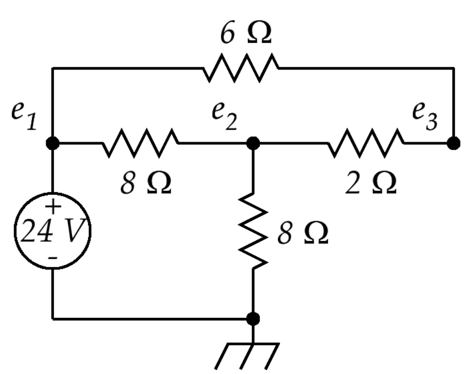
Reminder: Quiz 1 will be offered Wednesday, September 22 and will cover material given in class until September 8.

Problem 1: Find node voltages e_1, e_2 and e_3 for the networks shown below using superposition (Note that the ground node has been chosen):

(A)

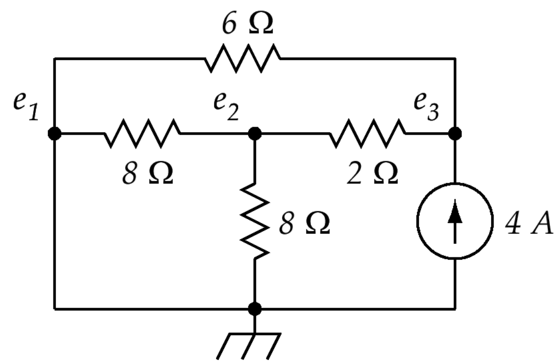


Let $I = 0$. The resulting network is shown below:



The voltage $e_{1,V} = 24V$ since it is connected to the grounded voltage source. The equivalent resistance seen by the voltage source is $(6\Omega + 2\Omega)||8\Omega + 8\Omega = 12\Omega$. Thus, the current leaving the voltage source is $24V/12\Omega = 2A$. The voltage $e_{2,V}$ can be easily calculated noting that the $2A$ must pass through the 8Ω resistor below it, creating a voltage drop of $2A \cdot 8\Omega = 16V$ with respect to ground. Furthermore, the $2A$ leaving the source must branch equally among the 8Ω upper branches. Since $1A$ is flowing through each branch, the voltage drop from $e_{3,V}$ to $e_{2,V}$ is equal to $1A \cdot 2\Omega = 2V$. Therefore the node voltage $e_{3,V} = e_{2,V} + 2V = 18V$.

Let $V = 0$. The resulting network is shown below:

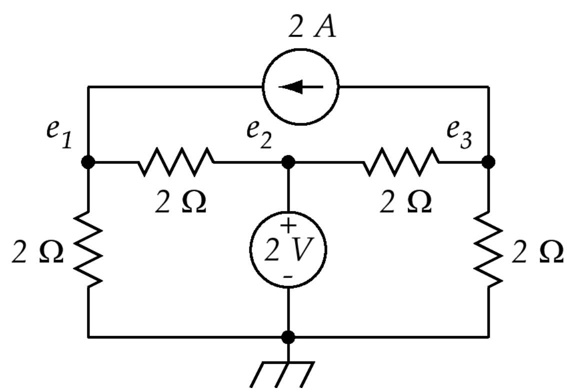


The voltage $e_{1,I} = 0$ since it is connected to ground. The equivalent resistance seen by the current source is $6\Omega||(2\Omega + 8\Omega||8\Omega) = 3\Omega$. Thus, the voltage drop across the current source is $4A \cdot 3\Omega = 12V$, which is equal to $e_{3,I}$. The voltage at $e_{2,I}$ can be calculated by noting the $4A$ must divide evenly between the two 6Ω branches. Thus, the voltage drop from $e_{3,I}$ to $e_{2,I}$ is equal to $2A \cdot 2\Omega = 4V$. Therefore the node voltage $e_{3,I} = e_{2,I} - 4V = 8V$.

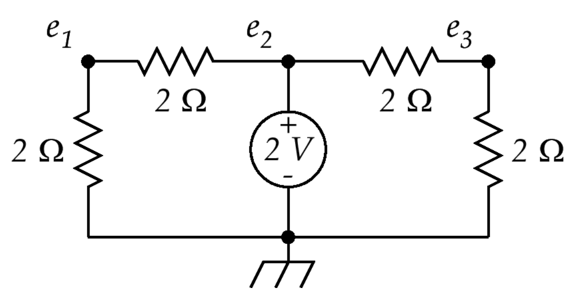
The total voltages are:

$$e_1 = e_{1,V} + e_{1,I} = 24V + 0 = 24V$$
$$e_2 = e_{2,V} + e_{2,I} = 16V + 8V = 24V$$
$$e_3 = e_{3,V} + e_{3,I} = 18V + 12V = 30V$$

(B)

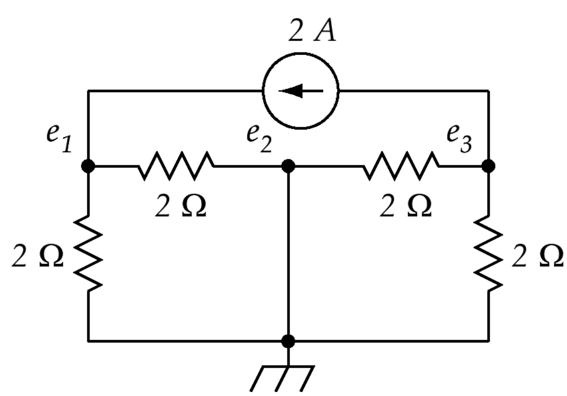


Let $I = 0$. The resulting network is shown below:



The voltage $e_{2,V} = 2V$ since it is connected to the grounded voltage source. The equivalent resistance seen by the voltage source is $(2\Omega + 2\Omega)||(2\Omega + 2\Omega) = 2\Omega$. Thus, the current leaving the voltage source is $2V/2\Omega = 1A$. The voltages $e_{1,V}$ and $e_{3,V}$ can be easily calculated noting that $1A$ must pass through each branch into the ground node, creating a voltage drop of $1A \cdot 2\Omega = 2V$ with respect to ground.

Let $V = 0$. The resulting network is shown below:

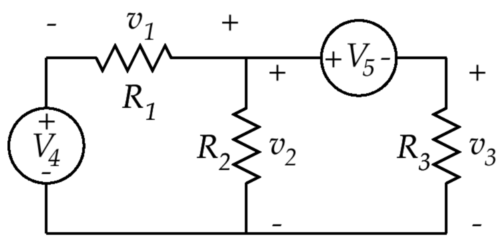


The voltage $e_{2,I} = 0V$ since it is connected to ground. The $2A$ leaving the current source divides evenly into each branch. The voltage $e_{1,I}$ is equal to the $1A$ flowing to ground through the 2Ω resistor, resulting in $2V$. Similarly, $e_{3,I}$ is equal to $-2V$, since the current is now flowing from the ground to the node.

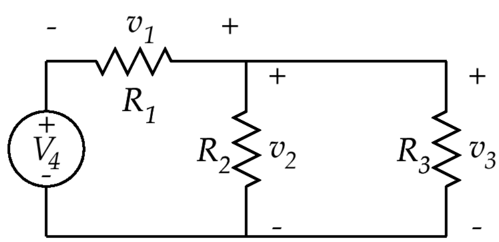
The total voltages are:

$$e_1 = e_{1,V} + e_{1,I} = 1V + 2V = 3V$$
$$e_2 = e_{2,V} + e_{2,I} = 2V + 0 = 2V$$
$$e_3 = e_{3,V} + e_{3,I} = 1V - 2V = -1V$$

Problem 2: Find voltages v_1, v_2 and v_3 in the following circuit using superposition:



Let $V_5 = 0$. The resulting network is shown below:



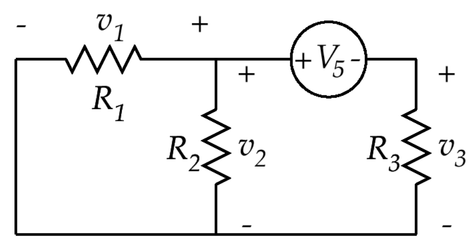
The voltages $v_{2,V_4} = v_{3,V_4}$ can be calculated with a voltage divider:

$$v_{2,V_4} = v_{3,V_4} = \frac{R_2||R_3}{R_1 + R_2||R_3} V_4 = \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} V_4$$

Then,

$$v_{1,V_4} = v_{2,V_4} - V_4 = \frac{-R_1 R_2 - R_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} V_4$$

Let $V_4 = 0$. The resulting network is shown below:



The current flowing out of V_5 is

$$i = \frac{V_5}{R_1||R_2 + R_3} = \frac{R_1 + R_2}{R_1R_2 + R_1R_3 + R_2R_3}V_5$$

Then, $v_{1,V_5} = v_{2,V_5}$ can be calculated by multiplying this current by the effective resistance of $R_1||R_2$:

$$v_{1,V_5} = v_{2,V_5} = i \cdot R_1||R_2 = \frac{R_1R_2}{R_1R_2 + R_1R_3 + R_2R_3}V_5$$

Similarly, v_{3,V_5} is equal to the current flowing from the ground to V_5:

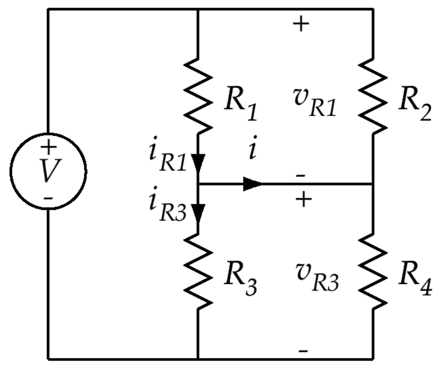
$$v_{3,V_5} = -i \cdot R_3 = \frac{-R_1R_3 - R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}V_5$$

The total voltages are:

$$v_1 = v_{1,V_4} + v_{1,V_5} = \frac{-R_1R_2 - R_1R_3}{R_1R_2 + R_1R_3 + R_2R_3}V_4 + \frac{R_1R_2}{R_1R_2 + R_1R_3 + R_2R_3}V_5$$
$$v_2 = v_{2,V_4} + v_{2,V_5} = \frac{R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}V_4 + \frac{R_1R_2}{R_1R_2 + R_1R_3 + R_2R_3}V_5$$
$$v_3 = v_{3,V_4} + v_{3,V_5} = \frac{R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}V_4 + \frac{-R_1R_3 - R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}V_5$$

Problem 3:

(A) Find the current i in the following circuit:



The voltages v_{R_1} and v_{R_3} can be found using voltage dividers:

$$v_{R_1} = \frac{R_1||R_2}{R_1||R_2 + R_3||R_4} V = \frac{R_1R_2R_3 + R_1R_2R_4}{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4} V$$

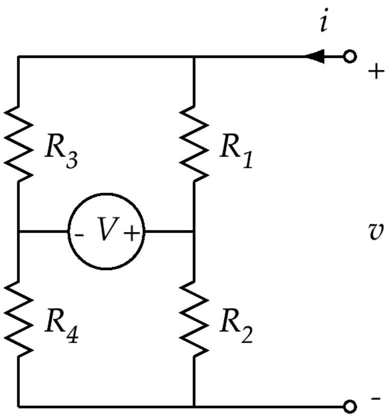
$$v_{R_3} = \frac{R_3||R_4}{R_1||R_2 + R_3||R_4} V = \frac{R_1R_3R_4 + R_2R_3R_4}{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4} V$$

Then, using KCL with i, i_{R_1} and i_{R_3}:

$$i = i_{R_1} - i_{R_3} = \frac{v_{R_1}}{R_1} - \frac{v_{R_3}}{R_3} = \frac{\frac{R_1R_2R_3+R_1R_2R_4}{R_1} - \frac{R_1R_3R_4+R_2R_3R_4}{R_3}}{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4} V$$

$$i = \frac{R_2R_3 - R_1R_4}{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4} V$$

(B) Find the Norton equivalent of the following circuit (*Hint: Part (A) of this problem is helpful*).



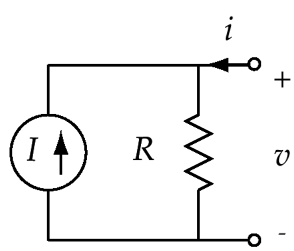
To find the equivalent resistance, let $V = 0$. Then,

$$R_{eq} = R_1||R_3 + R_2||R_4 = \frac{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4}{R_1R_2 + R_1R_4 + R_2R_3 + R_3R_4}$$

To find i_{sc}, we must short the terminals and find the current traveling from the top terminal into the bottom terminal. However, this is exactly the same problem that was solved in the previous part. Thus,

$$i_{sc} = i = \frac{R_2R_3 - R_1R_4}{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4}V$$

Finally, the Norton equivalent circuit is:



where $I = i_{sc}$ and $R = R_{eq}$.