Exercises in Math 162: (taken from Mathematics of Investment and Credit, 2nd ed by Broverman)

1. Smith needs to borrow 5000 for one year. He is offered the loan at an annual rate of 5%. He is also offered a loan of 10,000 at a lower rate of interest. If he borrows the 10,000, he can invest the excess 5,000 for one year at 4%. How low must be the rate on the 10,000 loan in order for Smith to prefer it to the 5,000 loan?

2. Jones invests 100,000 in a 180-day short term guarantee investment certificate at a bank, based on simple interest at an annual rate of 7.5%. After 120 days, interest have risen to 9% and Jones would like to redeem the certificate early and reinvest in a 60-day certificate at the higher rate. In order for there to be no advantage in redeeming early and reinvesting at the higher rate, what early redemption penalty (from the accumulated value of the investment certificate to that point) should the bank charge at the time of early redemption?

3. Jones wishes to invest funds for a one-year period. Jones can invest in a one-year guaranteed investment certificate at a rate of 8%. Jones can also invest in a 6-month GIC at annual rate of 7.5%, and then reinvest the proceeds at the end of 6 months for another 6-month period. Find the minimum annual rate needed for a 6% month deposit at the end of the first 6-month period so that Jones accumulates at least the same amount with two successive 6-month deposits is she would with the one-year deposit.

4. Smith deposits 1000 in Bank A on January 1. Bank A credits interest at annual rate $i = 0.15$. If Smith closes his account, he receives a simple interest up to the time of withdrawal. Smith visits Bank B across the street and is told that he can open an account anytime that year and receive a simple interest at annual rate $i = 0.145$, paid from the date of deposit to December 31. Smith consults his math of finance text and realizes that if he chooses the right day to close his account at Bank A and immediately redeposit the proceeds in a new account in Bank B, he will maximize the return on his 1000 over that year. What is that day?

5. Suppose $i > 0$. Show that $(1+i)^n(1+i)^m > 2(1+i)^{\frac{m+n}{2}}$ if $m > n \geq 0$.

6. 2,500 is invested. Find the accumulated value of the investment 10 years after it is made for each of the following rates.
   (a) 4% annual simple interest
   (b) 4% effective annual compound interest
   (c) 6-month interest rate of 2%
   (d) 3-month interest rate of 1%

7. At time 0, a balance of amount B_0 is in an account earning an interest rate i per period. Various deposits and withdrawals are made during the period, with a transaction of amount a_k made ant time t_k for $k = 1, 2, \ldots, n$, where $0 < t_k < 1$. ($a_k > 1$ indicates a deposit to the account, while $a_k < 1$ indicates a withdrawal from the account.) Assume that $t = 1$ is the reference point for the account and interest on deposits and withdrawals to accre at the time of deposit or withdrawal.
   (a) Find an expression for B_1, the account balance at $t = 1$.
   (b) Find an expression for $\bar{B}$, the average balance in the account during the period from 0 to 1.
   (c) Show that $B_1 = B_0 + \displaystyle\sum_{k=1}^{n} a_k + \bar{B}i$.

8. Smith borrows 1200 at an annual rate 6%. He makes a payment of 500 one year later ($t = 1$). One year after that ($t = 2$), Smith borrows an additional 600, still a the 6% rate. Assuming simple interest, what amount would be required to completely repay the loan at time $t = 3$?

9. A person has debts of 200 due July 1, 1989 and 300 due July 1, 1991. A payment of 100 was made on July 1, 1986 to reduce those debts. What additional amount payable on July 1, 1990 would cancel the remaining debts, assuming effective annual interest at rate 4%?

10. A magazine offers a one-year subscription at a cost of 15 with renewal the following year at 16.50. Also offered is a two-year subscription at a cost of 28. What is the effective annual interest rate that makes the two-year subscription equivalent to two successive one-year subscriptions?

11. Calculate the following derivatives:

    (a) $\frac{d}{di}\,(1+i)^n$
    (b) $\frac{d}{di}\,v^n$
    (c) $\frac{d}{dn}\,(1+i)^n$
    (d) $\frac{d}{dn}\,v^n$

12. A loan of 1000 is made at the start of a year ($t=0$). It is repaid by 3 equal payments of 350 each, made every 4 months (i.e. at $t=\frac{1}{3}, \frac{2}{3}$ and 1).

    (a) If calculation is on the basis of simple interest with $t=1$ as the focal date, what is the annual interest rate i.
    (b) How would (a) be solved if the focal date were $t=0$? How would the rate compare to the one found in (a)?
    (c) If calculation is on the basis of compound interest, how would the rate compare with those in parts (a) and (b)?

13. If $\delta_t = (1+t)^{-k}\ln B$, what is the accumulation function?

14. A person's savings earn an effective rate of $i=0.12$ on which 45% income tax is paid. If the inflation rate is 10% per year, what is the annual after-tax real rate of return?

15. If the force of interest for the coming year will be δ and the force of inflation will be δ_r, show that the inflation-adjusted force of interest is $\delta - \delta_r$.

16. Since June 30, 1986 Smith has been making deposits of 100 each into a bank account on the last day of each month. For all of 1986 and 1987 Smith's account earned nominal interest compounded monthly at an annual rate of 9%. For the first 9 months of 1988 the account earned $i^{(12)} = 0.105$, and since then the account has been earning $i^{(12)} = 0.12$. Find the balance in the account on February 1, 1989. Find the amount of interest credited on February 28, 1989.

17. If $s_{\overline{n}|} = 70$ and $s_{\overline{2n}|} = 210$, find the values of $(1+i)^n$, i and $s_{\overline{3n}|}$.

18. If $s_{\overline{3n}|} = X$ and $s_{\overline{n}|} = Y$, express v^n in terms of X, Y and constants.

19. If $s_{\overline{n}|} = 48.99$, $s_{\overline{n-1}|} = 36.34$, and $i>0$, find i.

20. An $m+n$ year annuity of 1 per year has $i=7\%$ during the first m years and has $i=11\%$ during the remaining n years. If $s_{\overline{m}|0.07} = 34$ and $s_{\overline{n}|0.07} = 128$, what is the accumulated value of the annuity just after the last payment?

21. Given $s_{\overline{10}|0.10} = S$, find the value of $\sum_{t=1}^{10} s_{\overline{t}|0.10}$ in terms of S.

22. Find expressionsfor $\frac{d}{dn}s_{\overline{n}|}$ and $\frac{d}{dn}a_{\overline{n}|}$.

23. Smith borrows 5000 on January 1, 1990. She repays the loan with 20 annual payments starting January 1, 1991. The payments in even-numbered years are X each and the payments in odd-numbered years are Y each. IF $i=0.08$ and the total of all 20 loan payments is 10,233, find X and Y.

24. The force of interest has the form $\delta_t = \dfrac{0.10}{\sqrt{1 + 0.10t}}$. An annuity has payments of amount 1 each times 3 through 7 inclusive. Find the accumulated value of the annuity at time 10.

25. If i is the annual effective interest rate equivalent to force of interest δ, and i' is the annual effective equivalent to the force of interest $k\delta$, where $k > 0$, show that $a_{\overline{n}|i'} = \dfrac{a_{\overline{kn}|i}}{s_{\overline{k}|i}}$.

26. A perpetuity pays 1 every January 1 starting in 1994. The annual effective rate of interest will be i in odd-numbered years and j in even-numbered years. Find an expression for the present value of the perpetuity on January 1, 1993.

27. A perpetuity starting January 1, 1994 pays 1 every January 1 in even years and 2 every January 1 in odd years. Find an expression for the present value at rate i per year of a perpetuity in (a) January 1, 1993, and (b) January 1, 1994.