Math 53 Lecture: Related Rates

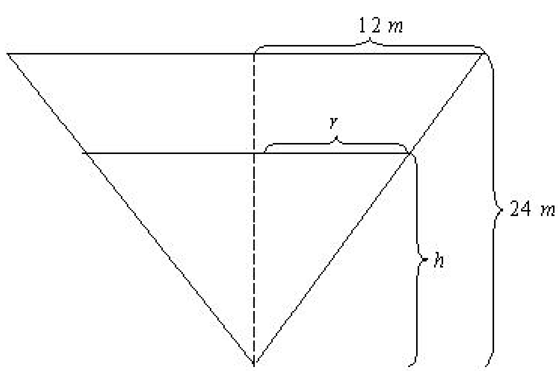
Lecturer: Jose Maria L. Esaner IV, Ph.D.
Lecture 4

Related rates usually involve rates of changes with respect to time. We assume here that each x, y, or other variable-represented quantity are all functions of time. Chain rule is usually employed as a technique in this case.

Examples:

1. If $x/y = 10$,and $D_t x = -5$, find $D_t y$.
   It is important to understand here that both x and y are functions of t and that from the given, x and y are interdependent with each other. Now we can rewrite $x/y = 10$ as $x = 10y$ provided $y \neq 0$. Differentiating this equation with respect to t, we have, by chain rule, $\dfrac{dx}{dt} = 10\dfrac{dy}{dt}$. Substituting the value $D_t x = -5$ we see that $D_t y = -1/2$.

2. A water tank in the form of an inverted cone is being emptied at the rate of 6 m^3/in. The altitude of the cone is 24 m and the base radius is 12 m. Find how fast the water level is lowering when the water is 10 m deep.



The volume of an inverted cone is $V = \dfrac{1}{3}\pi r^2 h$. Note that the volume formula is dependent on two variables namely, the radius r and the height h. What should be done for now is to reduce this equation so that the volume is only dependent on one of the variables. Looking at the figure, we see that ratio and proportion of similar triangles may be applied:

$$\frac{h}{24} = \frac{r}{12} \text{ or } r = \frac{h}{2}.$$

Substituting this equation in our volume formula we get $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{1}{12}\pi h^3$. Differentiating both sides with respect to t we get

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}.$$

Given $h = 10$ and $\frac{dV}{dt} = -6m^3/in$ (the negative sign indicates that the volume is decreasing), then

$$\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt} = \frac{4}{100\pi}(-6) = -\frac{6\pi}{25} \text{ m/s}.$$

Again the negative sign indicates that the height is decreasing.

Exercises:

1. Sand is being emptied from a hopper at a rate of 10 ft^2. The sand forms a conical pile whose height is always twice its radius. At what rate is the radius of the pile increasing when its height 5 $ft.$?

2. The width of a rectangle is half its length. At what rate is its area increasing of its width is 10 cm and is increasing at 0.5 cm/s?

3. A circular oil slick of uniform thickness is caused by a spill of 1 m^3 of oil. The thickness of the oil slick is decreasing at a rate of 0.1 cm/hr. At what rate is the radius of the slick increasing when the radius is 8 m.