Math 53 Lecture: Volumes of Solids Using Cylindrical Shell Method

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Lecture 20

Consider a plane region R formed by the curve $y = f(x)$ above the x-axis, the $x - axis$, and the lines $x = a$ and $x = b$, $a < b$. If the rectangular elements of the partition in a region are perpendicular to the axis of revolution, then we use either the disk method (or the washer method). If, however, we partition the region with rectangular elements parallel to the axis of revolution, then we us the cylindrical method.

If a rectangular element is parallel to the axis of revolution, we arrive at right cylindrical shell with height $f(\xi_i)$, mean radius of ξ_i and thickness $\Delta_i x$. If this is so, then the volume is given by

$$V = \lim_{||\Delta|| \to 0} \sum_{i=1}^{n} 2\pi \ \xi_i \ f(\xi_i)\Delta_i x = 2\pi \int_a^b x f(x) \ dx.$$

In general we have

$$V = 2\pi \int_a^b x[f(x) - g(x)] \ dx.$$

Exercises: Set-up and evaluate the integral that gives the volume of the solid of revolution that is generated by rotating the plane region R about the indicated axis or line.

1. $R : y = x^2, y = 0, x = 1;$ the x-axis
2. $R : y = x^2, y = 4, x = 0$ (first quadrant only); the x-axis
3. $R : y = x^2, x = y^2;$ the x-axis
4. $R : y = x^2, y = 4x;$ the line $x = 5$
5. $R : y = 6 - x^2, y = 2;$ the x-axis
6. $R : y = x^2, x = y^2;$ the line $y = -2$
7. $R : y = x^2, y = 8 - x^2;$ the line $x = 4$