Math 53 Lecture: Higher-Order Derivatives

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Lecture 1

Recall that for a function to be differentiable, the function has to be differentiable at every point in its domain. That is, the derivative of the function must exist at every number in the domain of the original function. If that is so, then the derivative of a function f is a function, in which case, f' (also called the *first-order derivative* or simply *first derivative*) itself may also have its derivative. This gives rise to the concept of higher-order derivatives. The following is a formal definition of the second derivative of f.

Definition 1 *Let f be a differentiable function for which its first derivative f' exists. Then the second derivative $f''(x)$ is defined as*

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

provided the limit exists.

Other notations of $f''(x)$ include $\frac{d}{dx}\left(\frac{df(x)}{dx}\right) = \frac{d^2 f(x)}{dx^2}$, $\frac{d^2 y}{dx^2}$, y'', and $D_x^2[f(x)]$. In general, we define $\frac{d^n y}{dx^n}$ as the nth derivative of the function $y = f(x)$. Other notations of the nth derivative are $\frac{d^n f(x)}{dx^n}$, $D_x^n[f(x)]$ and $y^{(n)}$ (read as "y upper n").

Exercises:

1. Find the second derivative of $f(x) = x^5 - 2x^3 + x$.
2. Find the third derivative of $y = 5\cos 2x + 6\sin 4x - x^3$.
3. Find all derivatives of $y = 3x^4 - 4x^3 - 6x^2 + 3x - 1$.
4. Find $y^{(2)}$ of $y = \frac{3x}{4x-1}$.
5. Find $\frac{d^3 y}{dt^3}$ of $y = 3\tan 6t + 8\cos^2 2t - \sin 3t \csc t$.
6. Find $f'(0)$ and $f''(0)$ of $f(x) = \begin{cases} \frac{x^2}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$.