Math 53 Lecture: Exercises on Areas of a Plane Region

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Lecture 18

In our previous discussions, we make the partition along the x-axis so that our rectangular elements are parallel to the y-axis. This gives a basic formula for the area $A = \int_a^b f(x)\,dx$. However there may be cases that a plane region may be bounded two curves such that the height component may differ from one subregion to another subregion of the whole region. If this is the case, we have to consider the subregions separately and the area of the whole region is the sum of these subregions.

In some cases, also, it might be easier to consider rectangular elements that are parallel to the x-axis. That is, we make the partition along the y-axis. If this is so then the area is set-up with respect to y.

In working with areas of a plane region, keep in mind that there are three main stages: (1) sketching of the plane region; (2) setting up of the integral; and (3) evaluating the integral.

Exercises:

1. Find the area of the plane region using rectangular elements parallel to the y-axis.
   (a) $y = 12 - 3x^2$ and $y = 4 - x^2$
   (b) $y = 4 - x^2$ and $y = 3x^2 - 12$
   (c) $y = 6$ and $y = x^2 - 3x$
   (d) $y = x$ and $y = x^2 - 3x$
   (e) $y = \sin x$ from $x = 0$ to $x = 2\pi$
   (f) $x = y^2$ and $x = 2$
   (g) $y = x + 1$, $y = 5 - x$ and the x-axis
2. Find the area of the plane region using rectangular elements parallel to the x-axis.
   (a) $y = 4 - x^2$ in the first quadrant
   (b) $y = x^2$, $y = 4x$
   (c) $y = x^2$, $x = y^2$
   (d) $x = y^2$ and $x = 2$
   (e) $y = x + 1$, $y = 5 - x$ and the x-axis