Math 53 Lecture: Exercises on Derivatives

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Lecture 0

The following is an exercises on implicit differenation. Here, our aim is to get $\dfrac{dy}{dx}$.

1. $\sin(x+y) = y^2 \cos x$
2. $x^3 + y^3 = 6xy$
3. $\sqrt{xy} = 1 + x^2 y$
4. $x \cos y + y \cos x = 1$

The next are questions on differentiability.

1. If a function is differentiable at a number x_1, does it follow that the function is continuous at x_1? Is the converse true?
2. Show that the absolute value function is not differentiable at 0. Show further that $D_x(|x|) = \dfrac{|x|}{x}$ if $x \neq 0$.

In working with one-sided derivatives, it is important to remember the definition. The usual "short-cuts" may not work. For the following problems, find the value of the derivatives of f at the given number x_1.

1. $f(x) = \begin{cases} x+2 & x \leq -4 \\ -x-6 & x > 4 \end{cases} \qquad x_1 = -4$
2. $f(x) = \begin{cases} x^2 - 4 & x < 2 \\ \sqrt{x-2} & x \geq 2 \end{cases} \qquad x_1 = 2$

This last part is devoted on solving problems involving the tangent line to the curve at a point using the concept of derivatives.

1. Find the equation of the tangent line to the curve $y = 3x^3 - 4x^2 - 5$ at the point $(0, 5)$.
2. Find the equation of the tangent line to the curve $y = 2x^2 + 3$ that is parallel to the line $8x - y + 3 = 0$.
3. Find the equation of the tangent line to the curve $y = 3x^2 - 4$ that is parallel to the line $3x + y = 4$.
4. Find the equation of the line tangent to the curve $y = 2x^2 + x - 4$ that is perpendicular to the line $x - 2y + 3 = 0$.
5. Show that there is no line passing through the point $(1, 5)$ that is tangent to the curve $y = 4x^2$.
6. Prove that there is no line through the point $(1, 2)$ that is tangent to the curve $y = 4 - x^2$.
7. Find two points on the circle $x^2 + y^2 = 1$ at which the slope of the tangent line is -2. Find their corresponding equations of the tangent line.