The Inverse of a Matrix

An $n \times n$ matrix A is nonsingular (or invertible) if there exists an $n \times n$ matrix B such that $AB = BA = I_n = I$. B is called the inverse of A is denoted by A^{-1}. If no such B exists, then A is singular.

Example: $\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$ and $\begin{bmatrix} \frac{3}{8} & -\frac{1}{8} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$ are inverses to each other.

Remark: The inverse of a nonsingular matrix is unique. Indeed, suppose B and C are inverses of A. Then $BA = AC = I$. However, $B = BI = B(AC) = (AB)C = IC = C$, and hence the uniqueness.

Theorem: Let A and B be nonsingular matrices of the same size, then

1. A^{-1} and B^{-1} are nonsingular.
2. $(A^{-1})^{-1} = A$.
3. AB is nonsingular and $(AB)^{-1} = B^{-1}A^{-1}$.
4. $(A^T)^{-1} = (A^{-1})^T$.

Example: Find the inverse of the following matrices, if any: $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$

Alternative Method of finding A^{-1}

Given: $n \times n$ matrix A

1. Form the $n \times 2n$ matrix $[A{:}I_n]$.
2. Transform this matrix to its reduced row echelon form.