

Math 114 Lecture: Matrix Representation of Linear Systems

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Lecture 4

This section is devoted in transforming linear systems into its matrix form. We will do this since this will suppose to make our solving linear systems easier, especially when we are dealing with large number of equations and/or variables. This will be our key in solving linear systems without using the elimination method and/or substitution method.

Consider the linear system of m equations in n unknowns:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\ \vdots + \vdots &= \vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

We define the following matrices:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}.$$

Then the linear system may be written in matrix form as

$$AX = B.$$

A is called the coefficient matrix. The matrix formed by adjoining B to A ,

$$[A|B] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array} \right]$$

is called the **augmented matrix** of A and B . The coefficient matrix and augmented matrices will play important roles in solving linear systems.

Exercises: Give the coefficient matrix A and the augmented matrix $[A|B]$

1. $\begin{cases} x - 3y = -3 \\ 2x + y = 8 \end{cases}$

2. $\begin{cases} x - 3y = -7 \\ 2x - 6y = 7 \end{cases}$

3. $\begin{cases} x + 2y - 3z = 6 \\ 2x - 3y + 2z = 14 \\ 3x + y - z = -2 \end{cases}$