Math 114 Lecture: Linear Systems

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Lecture 1

Recall the following in college algebra: A linear equation in 2 variables is an equation of the form $ax + by = c$ where a, b,and c are real numbers. A solution of this equation is an ordered pair (x_0, y_0) such that $ax_0 + by_0 = c$. We now generalize these concepts.

Definition 1 *An equation of the form*

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b$$

where $a_1, a_2, \ldots, a_n, b \in \mathbb{R}$ is called a linear equation of n variables. A solution of the linear equation is an n-tuple $(s_1, s_2, \ldots, s_2)$ such that $a_1s_1 + a_2s_2 + a_3s_3 + \cdots + a_ns_n = b$. The solution set of the linear equation is the set of such n-tuples.

In general, there are infinite solutions to any given linear equation of this form. For example, $x + 2y - 3z = 6$ has a solution $x = 0, y = 0, z = -2$; another solution is $x = 0, y = 3, z = 0$. There are actually infinite solutions to this equation.

A *system of m linear equations in n unknowns* is a set of m linear equations in n variables. A form of this system is given by

$$\begin{array}{rcl} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n & = & b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n & = & b_2 \\ \vdots \quad + \quad \vdots \qquad\qquad & = & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n & = & b_m \end{array}$$

Here, the a_{ij}'s as well as the b_i's are constants. A solution of this system is an n-tuple $(s_1, s_2, \ldots, s_n)$ that satisfies all m linear equations. In our college algebra course, we have discussed cases of this type: 2 linear equations in 2 unknowns and 3 linear equations in 3 unknowns. Graphically, we have learned that 2 linear equations in 2 unknowns are represented by 2 lines which may intersect at a unique point, may coincide with each other, or may be parallel with each other. On the other hand, 3 linear equations in 3 unknowns are represented by 3 planes which may intersect at a unique point, or intersect at a unique line, or may coincide with each other, or may be parallel each other.

Algebraically, this is done by process of elimination and/or substitution. This course considers not only the case where $m = n$ but also the cases when $m < n$ and $m > n$. This lecture is devoted first on reviewing the techniques learned in solving for systems of linear equations.

Exercises: Solve for the solution set of teh following systems of linear equations.

1. $\left\{\begin{array}{rcl} x - 3y & = & -3 \\ 2x + y & = & 8 \end{array}\right.$

2. $\left\{\begin{array}{rcl} x - 3y & = & -7 \\ 2x - 6y & = & 7 \end{array}\right.$

3. $\left\{\begin{array}{rcl} x + 2y - 3z & = & 6 \\ 2x - 3y + 2z & = & 14 \\ 3x + y - z & = & -2 \end{array}\right.$