Math 100 Lecture: Additional Exercises on Optimization and Graphing Functions

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Lecture 22

This section is devoted on the working on exercises dealing with maximizing or minimizing quantities, as well as a practice in sketching graphs and making tables as guides. Using EVT, we can get the absolute extrema, while the FDT or SDT give us relative extrema. The following theorem relates relative extrema and absolute extrema.

Theorem 1 *Suppose the funciton f is continuous on the interval I containing the number c. If $f(c)$ is a relative extremum value of f on I and c is the only number in I for which f has a relative extremum, then $f(c)$ is an absolute extremium value of f on I.*

The theorem means that we can use the FDT or SDT to find relative extrema and if it the sole relative extremum then it becomes an absolute extremum.

Exercises: Find the absolute extremum value using the FDT or the SDT.

1. From a thin sheet of metal 40 cm by 40 cm, square corners are cut out so that the sides can be folded up to form a box. What are the dimensions of the box that will yield its maximum volume?

2. Two posts, one 12 ft. high and the other 28 ft. high stand 30 feet apart. They are to be stayed by wires attached to a single stake, running from ground level to the tops of the posts. Where should the stake be placed to use the least wire?

3. Suppose a company has determined that its total revenue R for a particular product is given by $R = -x^3 + 450x^2 + 52,500x$ where R is measured in dollars and x is the number of units produced. What production level will yield a maximum volume?

4. The marketing department for a business has determined that the demand for a certain product is given by the model $p = \dfrac{50}{\sqrt{x}}$. The cost of producing x items is given by $C = 0.5x + 500$. What price p will yield a maximum profit?

5. Find the area of the largest rectangle that can be inscribed in a right triangle with base 30 cm and altitude 18 cm if the base of the rectangle lies along the base of the triangle.

6. Find the point on the graph of the equation $y = x^2 + 2x + 3$ that is closest to the point $(14, 9)$.

7. A man on an island 16 km north of a straing shoreline must reach a point 20 km east of the closest point on the shore to the island. If he can row at a speed of 3 kph and walk at a speed of 5 kph, where should he land on the shore in order to reach is destination as soon as possible?

8. A bus company will transport 100 passengers or fewer on an excursion trip for Php 120 each. If there are more than 100 passengers, the company agrees to reduce the price of every ticket by 50 centavos for each passenger in excess of 100. What number of passengers will produce the greates revenue?

Exercises on Sketching Graphs: Make a table indicating all relative extrema , points of inflection and behaviours of the graph at specified intervals. Sketch the graph based on the table.

1. $f(x) = x^4 + 2x^3 - 13x^2 - 14x + 24$
2. $f(x) = 2x^4 - 15x^3 + 32x^2 - 12x - 16$
3. $f(x) = 4x^{1/3} + x^{4/3}$
4. $f(x) = \sin x + \cos x, \quad x \in [-\pi, pi]$