A STUDY ON THE BID-ASK SPREAD

FOR PRICING STRATEGY OF B2B EXCHANGES

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I. Different Market Making Scenarios and EnronOnline

If it were not for the market maker, buyers and sellers would have to wait for each other, and negotiate prices every time they make a transaction. A market maker eliminates such burdens and facilitates trading, by being ready to buy and sell and by posting an appropriate price. There are many ways of market making.

In the NYSE, there is a market maker called the specialist, who makes a market for an individual stock. The specialist manages inventory of a stock on his own account in order to buy and sell immediately. He also announces the price at which he is willing to buy (i.e. bid price) and sell (i.e. ask price) based on information on the market, his inventory status and accumulated limit orders. The specialist in the NYSE receives and matches limit orders, and announces a quote (i.e. bid-ask price) exclusively.

In the NASDAQ, there are many market makers called dealers, for a stock. Dealers are similar to the specialist in the NYSE, in the sense that they manage inventory on their own account to provide liquidity to the market. They also receive limit orders and announce a bid and ask price. However, they are different from the specialist, in the sense that they are not exclusive. There are many competing dealers for a stock who make the market collectively. In NASDAQ, a central market system, called SOES, collects all bid-ask prices from dealers and posts the best bid and the best ask as a market bid-ask.

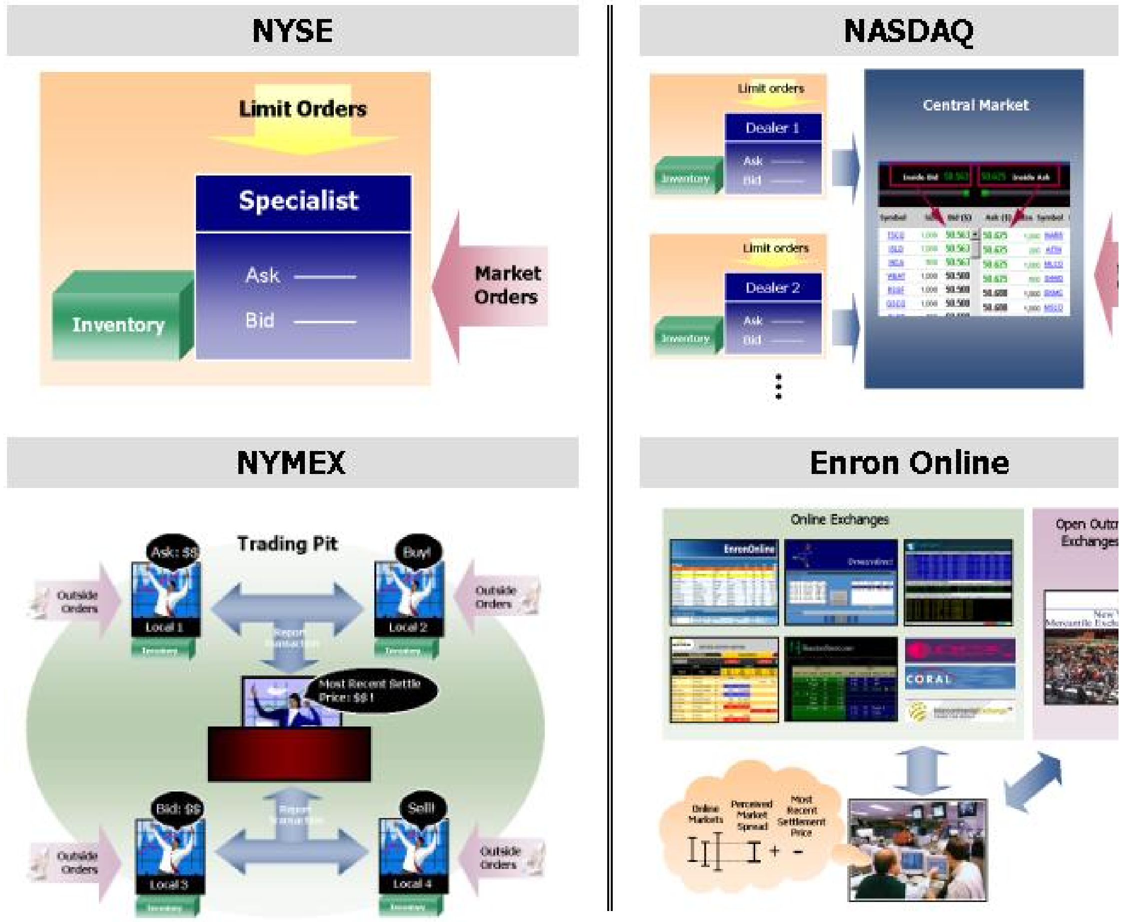
In the NYMEX, there is a trading pit where traders called locals trade futures contracts. Locals make transactions on their own account. By doing that, they collectively make the market. Locals are competing for buying and selling. If a local wants to sell a contract, he yells the ask price. If there is any local who wants to buy at the ask price, he yells to buy. Then a transaction is made between those two locals and the settlement price is reported to the center and announced to every local.

To a rational commodity trader who compares price posted on EnronOnline and prices posted on competing market sites such as DynegyDirect and Altra Energy, EnronOnline is like a dealer in the NASDAQ in the sense that its posted price is one of many prices that comprise the market price. The rational trader will compare prices and construct the market bid-ask by herself. However there are two important differences between the NASDAQ dealer and EnronOnline. One is that there is no central system that compares prices and posts the best one. Another difference is that there are strong competing pit-based markets such as NYMEX and CME, that announces only the most recent settlement price and do not post current bid-ask price.

Exhibit 1

Market Making Scenarios

In the NYSE, the specialist exclusively makes a market by trading on his account or matching limit orders against market orders. In the NASDAQ, there are many competing dealers who post bid-ask and the central market posts the best bid and the best ask. In the NYMEX, there are locals who buy and sell futures contracts by open out-cry auction. EnronOnline is similar to a NASDAQ dealer to a rational trader who compares prices.



II. Review of Study in Academia

Why does one get different prices for buying and selling when he trades at the exchange? One simple and correct answer is "because running an exchange, in other words market making, is costly." Then, what are the cost of market making components and how much is each one? Theoretical study on the bid-ask spread tries to answer such questions. Briefly, there are two big avenues in theoretical study. One avenue is the inventory-based approach and the other avenue is the information-based approach.

The inventory based approach notes that inventory is necessary for market making but incurs some cost to the market maker. The market maker needs to compensate such cost by buying at low price and selling at high price, and this makes the bid-ask spread.

The information based approach notes that traders are not the same. Some traders accept the price and trade with the market maker because they have better information on the true value. The market maker tries to compensate for the loss by charging different prices for buying and selling, and this makes the bid-ask spread. There are several branches of information-based study. One of them is the option-based approach. The quote provided by the market maker is a kind of free option to informed traders. That is, to an informed trader, the ask price may be regarded as an exercise price of a call option and the bid price may be regarded as an exercise price of a put option. To compensate the cost of option, the market maker gets profit from the margin, and this makes bid-ask spread. Another branch of information-based study is the learning approach. A learning market maker who has inferior information will regret that he did not set the price higher after he sold an asset and will regret that he did not set the price lower after he bought an asset. To minimize such regret, the learning market maker sets the selling price higher and the buying price lower than the value he believes to be true. This "regret free" strategy of the market maker makes the bid price and ask price different.

III. Recommendations: Framework

The problem we consider is the price setting strategy of EnronOnline. It turns out that it is a very complex problem and we found many different approaches we can use. The approach we took is first, to divide the problem into three parts: **1) *setting the size of the bid-ask spread,* 2) *setting and updating the price* and 3) *setting the quantity (i.e. setting the depth)***. We found there are many factors affecting decisions in each part. We divided such factors into two classes: ***average factors*** and ***intra-day factors***. Examples of average factors are average trading volume, average volatility of prices etc. Examples of intra-day factors are daily volatility, daily trading volume etc. We propose the basic decisions to be based on average factors and those decisions to be adjusted based on intra-day factors. We also propose to divide time into appropriate intervals and make decisions based on time intervals. For example, a day can be divided into the start, the middle, the end, and overnight and a year can be divided into seasons. Decisions on the size of the bid-ask spread, price updating, and quantity need to be different for different time intervals.

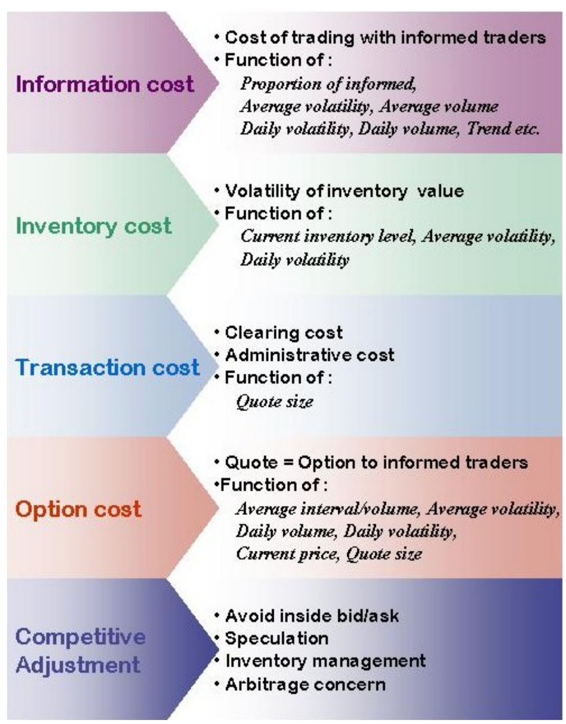
Setting the size of the bid-ask spread

Considering all reasonable arguments from academic research, we propose that the difference between ask price and bid price must be enough to cover the following costs: information cost, inventory cost, transaction cost and option cost. As explained earlier, information cost is the cost due to trading with informed traders. Inventory cost is the cost due to holding inventory such as risk of value change and uncertainty of demand and supply. Transaction cost includes the cost of clearing and order processing etc. Option cost is the cost of providing a free option to informed traders. In addition to that, we try to incorporate the wisdom we got from empirical studies and our understanding of EnronOnline's position in the market. As mentioned earlier, EnronOnline is like a dealer under a competitive market making scenario like a NASDAQ dealer. Study on NASDAQ dealers shows that dealers adjust their bid-ask spread flexibly to get in or to avoid being in the inside bid-ask (i.e. market bid-ask) depending on its situation. Similarly EnronOnline will

Exhibit 2

Components of Bid-Ask Spread

The bid-ask spread is composed of four basic components. In addition to them, in a competitive market making, dealer's bid-ask spread may need to be adjusted depending on the market bid-ask spread.



need to adjust its bid-ask depending on information, inventory position, etc. after looking at competing market places' prices.

Setting and updating price

Exhibit 3

Learning Speed and Price Discovery

In case the price of underlying asset changes, fast learning reduces the loss necessary for the price discovery.

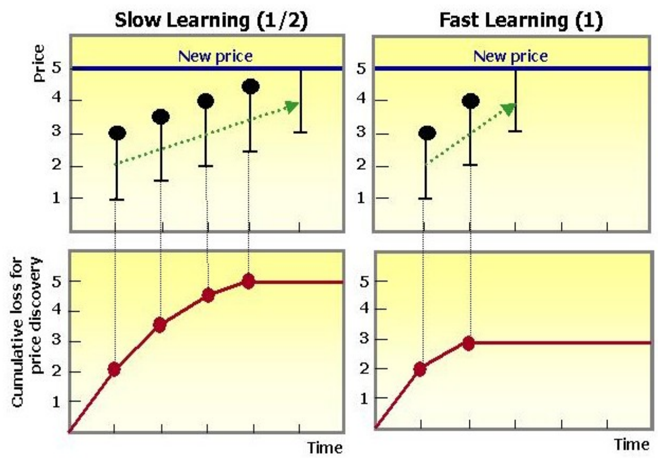
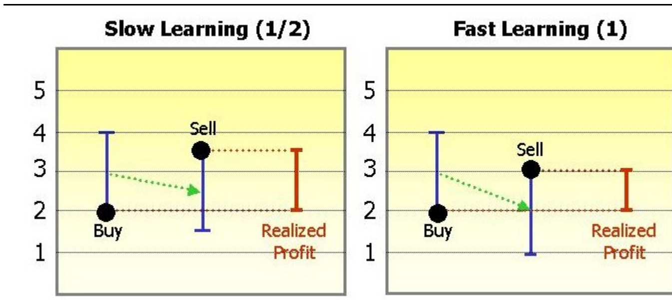


Exhibit 4

Learning and Profit

In case the price of underlying asset does not change, fast learning reduces profit from a cycle transaction (i.e. buy and sell the same quantity)



Once the size of bid-ask spread is determined, we need to consider how to position the level, that is, 'pricing'. However, a commodity market is almost like a perfectly competitive market in economist's terms. In such market, price is taken from the market rather than set by a firm. However, when the price changes, not every market place reflects the change immediately. Thus, the question for EnronOnline becomes how we can discover the new price with minimum cost in case the price changes. A reasonable way of discovering the new price is learning after each transaction. That is, lower the price after a trader sold an asset to the market and raise the price after a trader bought an asset. Such a learning strategy is suggested in information-based studies and empirically validated. The speed of learning can be defined as the amount of change after a transaction. That is, fast learning means the market maker changes price by a large amount and a slow learning means he changes by a small amount. As can be seen in Exhibit 3, to minimize the loss in the price discovery process, it is better to learn fast than to learn slowly. However, as can be seen in Exhibit 4, in case the price does not change, fast learning will reduce profit. Thus, the critical factors for deciding the speed of learning are the frequency and degree of price changes. There is a great measure for them: the price volatility. In the volatile market, the price usually changes frequently and we will need to discover the changed price frequently. Thus, in the volatile market fast learning is recommended. Similarly, slow learning is recommended for stable markets.

Setting the quantity

Exhibit 5

The Depth and the Learning Speed

For the most volatile market, fast learning with shallow quote is the most appropriate and for the most stable market, slow learning with deep quote is the most appropriate.

Usually, the bid-ask price is posted with quantity and the posted price does not change until orders reach the posted quantity. The quantity posted along with the bid-ask price is called the depth. A shallow quote is a quote with small quantity and a deep quote is a quote with large quantity. Obviously, the depth of quote affects the frequency of price changes. The price will change more frequently with a shallow quote than with a deep quote. Thus, the shallow quote will minimize the loss in the price discovery process and will be appropriate for volatile markets. Similarly, the deep quote will be appropriate for stable markets. Such behavior can be seen in NASDAQ dealer's behavior. NASDAQ dealers quote a small quantity at the market opening, to discover the right price with minimal cost.

IV. Recommendations: A Simple Model

Setting the Size of Bid-Ask Spread

Exhibit 6

A Simple Model for the Bid-Ask Spread

Component	Formula	Notation
Information	$\beta_{Inf} \frac{\sigma}{v}$	σ : Average Volatility v : Average Volume
Inventory	$\beta_{Inv} I_t \sigma$	I_t : Current inventory (unit)
Transaction	cq	q : Quote size
Option	$q[C(a)+P(b)]$ $C(a)=S_0 N(d_1)-aN(d_2)$ $P(b)=-S_0 N(-d_3)+K_B N(-d_4)$ $\theta^2=\sigma^2\tau$ $d_1=\frac{\ln(S_0/a)+\theta^2/2}{\theta}$ $d_2=d_1-\theta$ $d_3=\frac{\ln(S_0/b)+\theta^2/2}{\theta}$ $d_4=d_3-\theta$	S_0 : Current price(median of bid-ask) $a=S_0+\frac{1}{2}[\beta_{Inf}\frac{\sigma}{v}+\beta_{Inv} I_t \sigma+cq]$ $b=S_0-\frac{1}{2}[\beta_{Inf}\frac{\sigma}{v}+\beta_{Inv} I_t \sigma+cq]$

The size of bid-ask spread can be determined by the sum of information cost, inventory cost, transaction cost, and option cost. In the simple model, we used volatility and volume as decision factors because their effect on bid-ask spread is validated by empirical studies. The information cost term incorporates the intuition that information cost increases with the volatility and decreases with the volume. The inventory cost term incorporates the intuition that inventory cost is dependent on current inventory level and volatility of the asset. Transaction cost per one unit is assumed to be known and its term is calculated by multiplying quoted quantity. Option cost is calculated based on current price and the bid-ask spread is determined by information cost, inventory cost and transaction cost. This option cost is added to the bid-ask spread calculated by summing information cost, inventory cost and transaction cost.

Setting and Updating the Price

The most common and well-known learning method is Bayesian learning. Briefly, it is updating a prior belief based on observations. We propose learning approaches that are simpler than Bayesian learning but still reflect the main idea of learning. The simplest one is linear learning. That is to raise the price by Δ if the market maker sold an asset, and to lower the price by Δ if the market maker bought an asset. Proportional learning is similar to the linear learning, but increase or decrease by $\Delta * P$ where P is the current price. There could be many nonlinear learning models as there are many forms of nonlinear functions. One of them, we can easily think of is to raise by Δ if it was sale after buy, to raise by 2Δ if it was sale after sale, to raise by 3Δ if it was sale after two consecutive sales etc.

Setting the quantity

As mentioned earlier, shallow quotes let prices change frequently and minimize loss in the price discovery process. However, if the market maker changes the depth frequently, traders may be confused and file claims afterwards. This will lead to dissatisfaction of traders and rising transaction cost. Thus, we propose to use a depth consistently but use different depth for different commodity markets depending on its average volatility. For example, we can think of using shallow quotes if the average volatility for the commodity is higher than k, and using deep quotes if the average volatility for the commodity is smaller than k, where k is chosen appropriately.

V. General Recommendations

While doing research on the bid-ask spread, we came up with many ideas that could be valuable to EnronOnline in general. In this part, we will mention some of them.

One general and urgent recommendation is to monitor prices of competing market prices. The price posted on EnronOnline may be sometimes too favorable or sometimes too unfavorable to traders who compare prices. If the price is too unfavorable there will be no order flow to EnronOnline until the situation changes to make the unfavorable price a favorable price. On the other hand, too favorable price means EnronOnline is giving up realizable profit to traders. Once EnronOnline posts prices based on comparison of the prices posted on competing online markets, it will be able to make more profit by avoiding too favorable prices to traders and by attracting orders going to competing market places.

Another recommendation is 24 hour operation using different market making methods. Online markets such as Altra Energy and HoustonStreet.com operate 24 hours for every commodity they deal with, while EnronOnline and DynegyDirect do not operate 24 hours. Running a market overnight is somewhat risky if the market maker makes transactions on his own account to provide liquidity. However, it has no risk if the market maker stays outside the transactions and just matches limit orders coming from outside. Because Altra Energy and HoustonStreet.com make market by matching limit orders rather than taking inventory, they can run 24 hours without risk. EnronOnline may also run 24 hours by changing into a limit order matching system during the night. Or, EnronOnline may accumulate limit orders during the night and clear them at the opening with favorable conditions, as specialists in NYSE do.

An interesting test project we propose is competitive pricing. Currently, a single trader is setting the bid-ask price for each commodity in EnronOnline. Multiple traders may be assigned to each commodity. They will manage inventory on their account and will be obliged to announce bid and ask price. The best bid price and the best ask price are posted on EnronOnline. As can be seen, it is like the NASDAQ market making system where dealers are internal Enron traders. The expected benefit of using such a mechanism is active information gathering and distribution of risk. Each price setter will work hard to gather information under the pressure of competition, and the collected information can also be shared. One price setter's bad decision on price may be compensated by other price setters' wise decisions. However, we like to point out that price setters can make myopic decisions that reduce overall profit of organization, sometimes.

We found that in EnronOnline, information on accumulated limit orders is available to traders. As can be seen in the behavior of NYSE specialist, the market maker may make the information on limit orders only available to himself and take advantage of it to make a profit. Such privatization of information may also be a possibility for EnronOnline.

Appendix A) Review of Study in Academia

1. The Inventory-based approach

The inventory-based approach tries to explain the bid-ask spread by the risk of holding inventory. According to the inventory-based approach, the bid-ask spread exists because the risk averse market maker tries to compensate for the risk. [Demsetz] [O'Hara and Oldfield]
Amihud and Mendelson suggested that the market maker has a preferred inventory position and prices are related to the current inventory level and the preferred inventory position. According to their model, when the current inventory level is higher than the preferred position, the market maker lowers the price. When the current inventory level is lower, the market maker raises the price (i.e. the current inventory level and the price has a negative correlation).
Ho and Stoll suggested a model for competitive market making (i.e. market making with multiple market makers) based on the inventory based pricing model. According to their model, the market price is determined by the inventory dispersion among market makers. If the inventory dispersion among market makers increases, the market bid-ask spread decreases due to the increased competition.
Empirical studies found little evidence on the relationship between the price and the inventory level. The positive correlation between the current inventory level and the price was found from futures contracts data that contrasts with the prediction [Manaster and Mann], and the mean-reverting price process that has been predicted by the inventory based pricing model is not consistently observed in market data [Huang and Stoll].
Also, empirical studies found that the prediction on the price and the inventory dispersion is generally not correct [Manaster and Mann] [Chan et al.]. However, near the close, it may correctly describe the reality [Chan et al.]
Empirical studies did not find counter evidence on the relationship between the size of the bid-ask spread and the risk of inventory. Futures contracts and stocks with large average volatility showed large average bid-ask spreads.

2. The Information-based approach

The information-based approach tries to explain the bid-ask spread by the cost of trading with informed traders. According to this approach, the bid-ask spread exists because the market maker tries to compensate for the loss from informed traders [Copeland and Galai] [Glosten and Milgrom]. Naturally, the bid-ask spread increases as the proportion of informed traders increases and their quality of information gets better.
Copeland and Galai suggested that option pricing based model for the bid-ask spread, based on the insights that quotes are like free options to informed traders.
Glosten and Milgrom suggested a learning based model. A learning market maker will regret he did not set the price higher after he sold an asset and will regret he did not set the price lower after he bought an asset. To minimize such regret, the learning market maker sets the selling price higher and the buying price lower than the value he believes to be true. Such a "regret free" strategy causes the bid-ask spread by making the bid price and ask price different.
Empirical studies found that market makers actually learn from each trade. Prices changed after buy and sell transactions in the predicted direction [Huang and Stoll] [Manaster and Mann].

The fact that the size of the bid-ask spread is large with stocks or futures contracts with large volatility can be interpreted as a supporting evidence of both option based models and learning based models.

3. Empirical Studies

Beside model-based studies, we can find much wisdom from empirical studies. In this section, we will present a summary of empirical studies on the NYSE specialist, NASDAQ dealers, and CME locals.

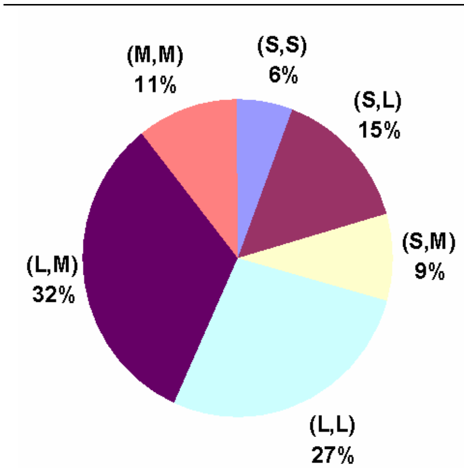
NYSE Specialist [Chung et al.]

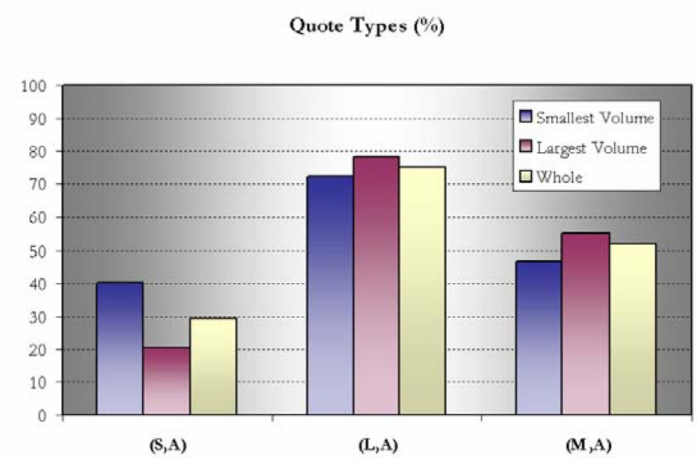
Chung, Van Ness and Van Ness studied the bid-ask spread of NYSE specialists, and found that a large portion of the posted bid-ask originates from the limit-order book without direct participation by specialists. They found that specialists participate in transactions most actively at the open with a wide spread. Consequently, the bid-ask spreads are widest at the open, narrow until late morning and then level off.

Proportion of quote types from the specialist

Most of quotes come from limit-order book. For stocks with small average volume of trading, specialists involve in transactions more frequently.

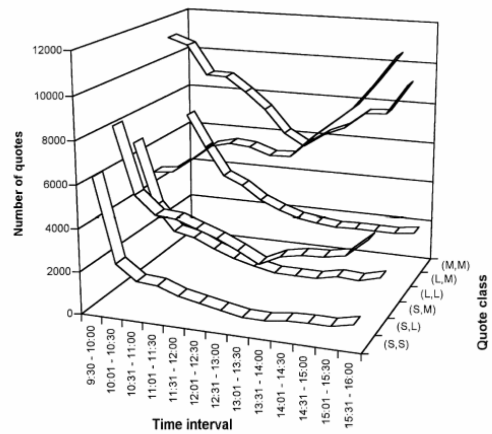
S: All quantity from the specialist
L: All quantity is from limit orders
M: Part of quantity from limit orders and part of quantity from the specialist
A: S or L or M
(S,S) means both the bid and ask price came from the specialist, (S,L) means either the bid or the ask came from limit order while the other part of the quote came from the specialist, etc. (Data: NYSE SuperDOT System, Nov. 1990 ~ Jan. 1991, 144 stocks)

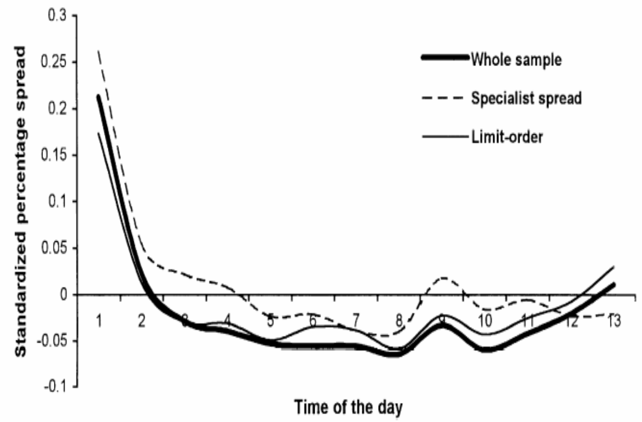




Intraday variation of quote types and size of the bid-ask spread

At the opening, the specialist is most actively involved in the transactions with wide bid-ask spread. During the day most quotes come from limit order book and the size of the bid-ask spread is relatively small. (Data: NYSE SuperDOT System: Nov. 1990 ~ Jan. 1991, 144 stocks)



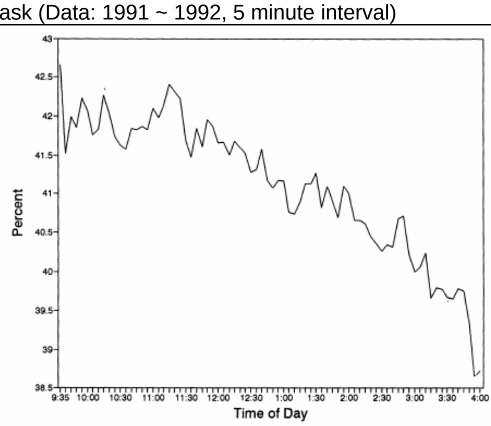


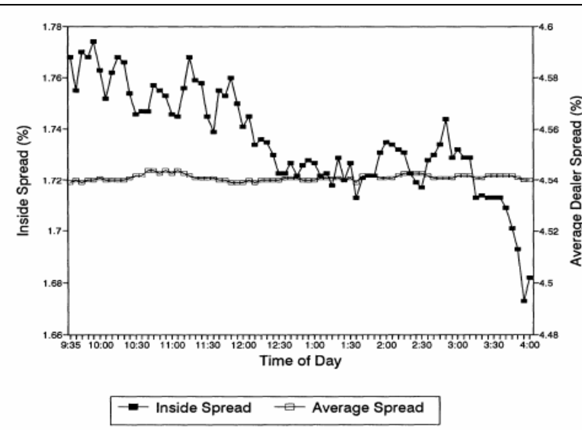
NASDAQ Dealers

Chan, Christie, and Schultz studied the bid-ask spread of NASDAQ dealers. They found that spreads are relatively stable through out the day but narrow significantly near the close. This contrasts with the U-shaped pattern for NYSE stocks, and they argued that the structural difference of those two markets (NYSE specialists are required to maintain both bid and ask prices) may explain it. Dealers try to end the market with zero inventory position, and they argue that the decline in spreads near the close for NASDAQ stocks is consistent with inventory control by individual dealers. They found that dealers rarely stay both the inside ask and the inside bid. In case, dealers stay in both, their bid-ask spread is larger than the average bid-ask spread. At the open, dealers trade frequently with small quantity to discover the price with minimum loss.

The intraday variation in the percentage of dealers that post quotes at either the inside bid or the inside ask (right)
The intraday variation of percentage inside spreads and percentage average dealer spreads (left)

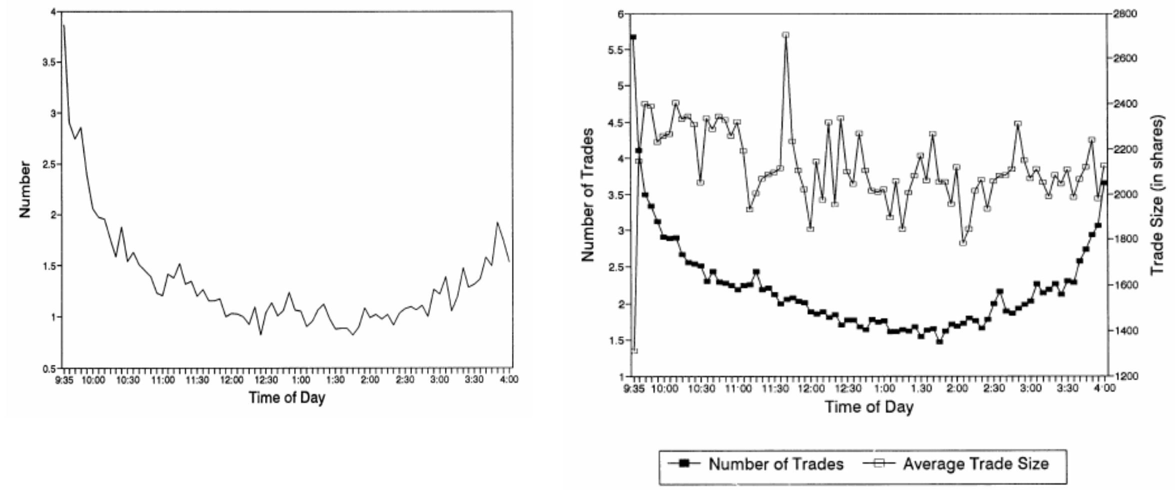
The size of market bid-ask spread becomes narrow near the end, and less proportion of dealers remain in the inside bid or ask (Data: 1991 ~ 1992, 5 minute interval)





The intraday variation in the number of quote revisions (right)
The intraday variation in the total number of trades and the average trade size (left)

Dealers revise quotes frequently at the open. The trade size is small while the number of trades is large at the open (Data: 1991 ~ 1992, 5 minute interval)

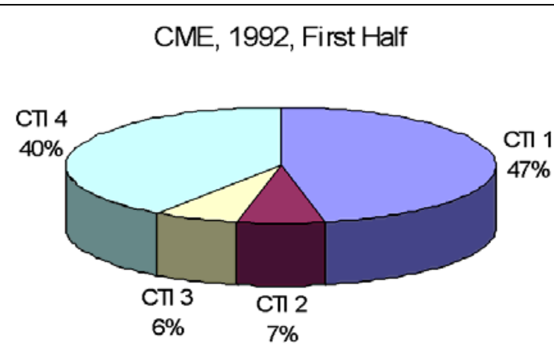


CME Locals [Manaster and Mann]

Manaster and Mann studied futures transaction data in the Chicago Mercantile Exchange (CME). They found that traders control inventory throughout the trading day. However, they found that correlations between inventory and reservation prices are positive, contradicting the prediction of typical inventory control models. They interpreted the evidence as consistent with active position taking by futures market floor traders. That is, locals change inventory position based on information or speculation. They found that locals try to end the market with zero inventory position.

Proportion of trade types

Locals in CME trade mostly for market making and for outside customers (Data: CME, 1992, first half)
CTI 1: Market-making trades for personal account
CTI 2: Trades executed for the account of the trader's clearing house
CTI 3: Trades executed for the account of any other exchange member
CTI 4: Trades of outside customers



Appendix B) Summary of Papers

Demsetz (1968)

Noted bid-ask is caused by providing immediacy
Economic Intuition behind this:
Supply unit price goes up if goods are provided immediately.
Demand unit price goes down if goods are purchased immediately
Intersection with immediate curve and ordinary curve: Bid-Ask spread

Conjecture: bid-ask spread for high trade volume security is smaller than low trade volume security Because value of providing immediacy is smaller. He showed this by analyzing NYSE data

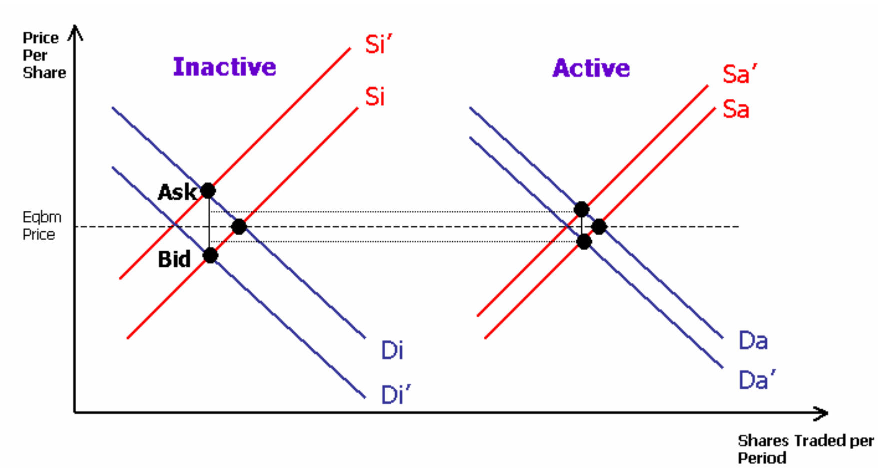


Figure 1. Bid-Ask spread model (Demsetz)

Copeland and Galai (1983)

Assumptions

- Single dealer, single security
- Two trader types: informed trader and liquidity trader
- Dealer makes commitment to buy security at b (i.e. bid price) and sell security at a (i.e. ask price) a fixed quantity to the first arrived trader
- Dealer changes prices a and b after a transaction with informed trader
- Dealer is risk neutral (expected value maximizer)
- S_0 : true value of security perceived by the dealer and liquidity traders
- S : true value, stochastic process following ~ $f(S)$
- Probability of trader type
    - P_I : probability that the arrived trader is an informed trader
    - P_L : probability that the arrived trader is a liquidity trader ($P_I + P_L = 1$)
    - P_{LT} : probability that the liquidity trader will make transaction

- o P_{LN} : probability that the liquidity trader will make transaction ($P_{LT} + P_{LN} = 1$)
- o P_{TS} : probability that the liquidity transaction is "sell"
- o P_{TB}: probability that the liquidity transaction is "buy" ($P_{LT} = P_{TS} + P_{TB}$)

- Two quotation scenarios: Instantaneous quotes and open quote interval

1) Instantaneous Quote Scenario

E [Revenue] $=(1-p_I)\{p_{BL}(a-S_0)+p_{SL}(S_0-b)+p_{NL}0\}$

E [Loss] = $p_I\left\{\int_a^\infty (S-a)f(S)dS+\int_0^b (K_B-S)f(S)dS\right\}$

Assumption (1)

$$\left.\frac{\partial p_{BL}}{\partial a}\right|_{S_0}<0 \text{ and } \left.\frac{\partial p_{SL}}{\partial b}\right|_{S_0}>0$$

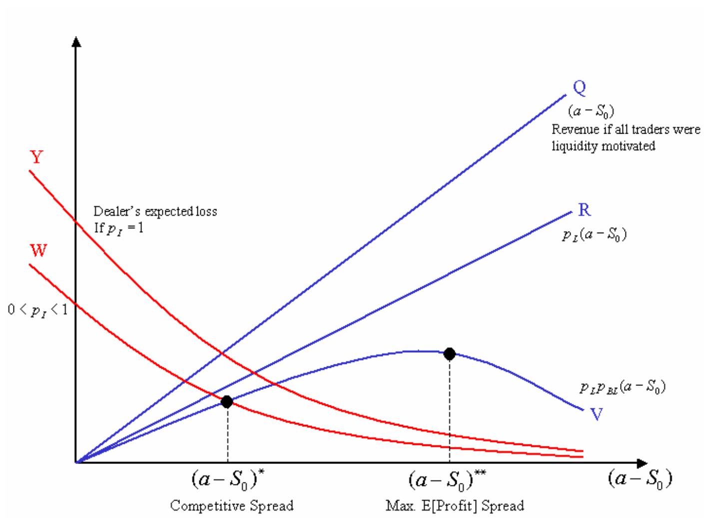


Figure 2. Ask spread determined by revenue from liquidity trader (Q,R,V) and loss from informed trader (Y,W) (Instantaneous Quote Model)

2) Open Quote Interval Scenario

E [Revenue] = $(1-p_I)\{p_{BL}(a-S_0)+p_{SL}(S_0-b)\}$

E [Loss] = $p_I[C(a)+P(b)]$

Risk neutral valuation of options when risk-neutral interest rate $r = 0$

$$C(a)=S_0N(d_1)-aN(d_2)$$

$$P(b)=-S_0N(-d_3)+K_BN(-d_4)$$

where

$$\theta^2=\sigma^2\tau,\ d_1=\frac{\ln(S_0/a)+\theta^2/2}{\theta},\ d_2=d_1-\theta,\ d_3=\frac{\ln(S_0/b)+\theta^2/2}{\theta},\ d_4=d_3-\theta$$

Analysis

1. Bid-Ask spread increases as volatility increases.
   Because as volatility increases, cost of free option becomes high and to make up that increased cost the dealer must increase the bid-ask spread
2. Bid-Ask spread increases as price per share increases.
   To see this, let's multiply λ to both S_0 and a and consider ask spread first. Expected loss becomes $\lambda\ p_I[C(a)]$ and expected revenue becomes $\lambda\ (1-p_I)\{p_{BL}(a-S_0)\}$. p_{BL} decreases with large λ (e.g. $\lambda > 1$), because the difference between new price per share $S'_0 = \lambda S_0$ and $a' = \lambda a$ increases to $\lambda(a-S_0)$ and p_{BL} decreases as the difference between price/share and ask price increases by assumption (1). Thus as price per share increases to $a' = \lambda a$, the dealer must set new ask price higher than λa to make up the loss from the informed trader because revenue with λa becomes is $\lambda\ (1-p_I)\{p_{BL}(a-S_0)\}$ and now p_{BL} is smaller. If $\lambda < 1$, similarly, due to increased p_{BL} the dealer may set ask price lower than $a' = \lambda a$ to make up the loss.

Summary

Objective

- In monopoly case, maximize
  E [Revenue from liquidity trader – Loss from informed trader]
- In competitive case, set price to recover the loss from informed trader

Modeling and problem solving technique:

- Expected value
- Option pricing model

Analysis of solution

- Instantaneous quotation scenario
  - Comparative statics
  - Competitive and monopoly case
  - Change of solution depending on the probability of informed traders(PI) and transaction(PLT and PTB)
- Open quote interval scenario
  - Option pricing model
  - Change of solution depending on price volatility and price/share in competitive case

O'Hara and Oldfield (1986)

Assumptions

- Single dealer, single security
- Each day has n trading periods
- Dealer sets ask price (a_t) and bid price (b_t) at the beginning of the trading period
- Each trading period is a call market where market orders arrive depending on the price
- At the beginning of the trading period, dealer knows submitted limit orders and has expectation of market orders
- There exists an overnight market for security
- Inventory can be negative: can be borrowed from the overnight market with paying interest r

- Positive inventory yields return through overnight market
- Dealer is risk averse expected utility maximizer

Model

Limit buy function: $\int_{a_t}^{\bar{a}} q_a(a_t)da_t = \alpha^L - \gamma^L a_t$

Limit sell function: $\int_{\underline{b}}^{b_t} q_b(b_t)db_t = \beta^L - \phi^L b_t$

Market buy function: $\tilde{A}_t^m = \alpha^m - a_t\gamma^m + \tilde{\omega}_t$

Market sell function: $\tilde{B}_t^m = \beta^m - b_t\phi^m + \tilde{\varepsilon}_t$

Total buy function:
$$\tilde{A}_t^m = \begin{cases} \alpha - a_t\gamma + \tilde{\omega}_t & if\ \alpha^L - \gamma^L a_t \ge 0 \\ \alpha^m - a_t\gamma^m + \tilde{\omega}_t & \end{cases}$$

$$\tilde{B}_t^m = \begin{cases} \beta - b_t\phi + \tilde{\varepsilon}_t & if\ \beta^L - \phi^L b_t \ge 0 \\ \beta^m - b_t\phi^m + \tilde{\varepsilon}_t & \end{cases}$$

Objective:

$$\max_{\substack{(a_1,\cdots a_n)\\(b_1,\cdots,b_n)}} \quad E[U(\sum_{t=1}^{n}\tilde{\pi}_t) + V(\tilde{I}_n)]$$
$$s.t. \quad \alpha^L - \gamma^L a_t \ge 0 \qquad \forall t = 1,\cdots,n$$
$$\beta^L - \phi^L b_t \ge 0$$

Last Period

$$\max_{(a_n,b_n)} \quad E[U(\sum_{t=1}^{n-1}\tilde{\pi}_t + a_n(\alpha - a_n\gamma + \tilde{\omega}_n) - b_n(\beta - b_n\phi + \tilde{\varepsilon}) + r\tilde{p}(I_{n-1} + \beta + b_n\phi + \tilde{\varepsilon} - \alpha + a_n\gamma - \tilde{\omega}_n))$$
$$+V(I_{n-1} + \beta + b_n\phi + \tilde{\varepsilon} - \alpha + a_n\gamma - \tilde{\omega}_n)]$$
$$s.t. \quad \alpha^L - \gamma^L a_n \ge 0$$
$$\beta^L - \phi^L b_n \ge 0$$

From Kuhn-Tucker Condition

$$a_n = \alpha/2\gamma + E[U'\tilde{\omega}_n]/E[U']2\gamma + rE[U'\tilde{p}]/2E[U'] + E[V']/2E[U']$$
$$b_n = -\beta/2\phi - E[U'\tilde{\varepsilon}_n]/E[U']2\phi + rE[U'\tilde{p}]/2E[U'] + E[V']/2E[U']$$

where

$U' \equiv \partial U/\partial\pi_n$

$V' \equiv \partial V/\partial\pi_n$

U' : function of I_{n-1}, a_n, b_n

a_n, b_n : function of I_{n-1}

Bid-Ask Spread

$$a_n - b_n = (\alpha\phi + \beta\gamma)/2\phi\gamma + (\phi E[\omega_n] + \gamma E[\varepsilon_n])/2\gamma\phi + (\phi Cov(U', \omega_n) + \gamma Cov(U', \varepsilon_n))/2\phi\gamma E[U']$$

For risk averse dealer,

$Cov(U', \omega_n) < 0$

$Cov(U', \varepsilon_n) > 0$

Analysis

The bid-ask spread may increase or decrease depending on many parameters because $\phi Cov(U', \omega_n) + \gamma Cov(U', \varepsilon_n)$ can either increase or decrease for risk averse dealer.

Garman(1976)

Assumptions

(A) Single market-maker setting an ask price a, at which he will sell one unit (i.e. fill a buy order), and a bid price b, at which he will buy one unit (i.e. fill a sell order)

(B) Arrivals of buy and sell orders: independent Poisson processes

  a. Buy process ~ $PP(D(a))$
  b. Sell process ~ $PP(S(b))$

(C) $D'(\bullet) < 0, S'(\bullet) > 0$

(D) The objective: "maximize expected average profit per unit time (profit=net cash flow)"

$\lambda = S(b)$, $\mu = D(a)$

Revenue per unit time: $R(\mu) = \mu\, a(\mu) = \mu\, D^{-1}(\mu)$

Cost per unit time: $C(\lambda) = \lambda\, a(\lambda) = \lambda\, S^{-1}(\lambda)$

Model and Solution

Market maker pursues inventory independent pricing strategies.

Market maker sets ask price *a* and bid price *b*, and as a result set λ and μ.

1) Derived ***probabilities of failure*** and the ***necessary conditions to avoid a sure failure***

  a) Objective

$$\max_{\lambda,\mu} R(\mu) - C(\lambda)$$

  b) Objective

$$\max_{\lambda,\mu} \quad R(\mu) - C(\lambda)$$

$$s.t. \quad \mu = \lambda$$

2) Examined ***the nature of failure*** and ***the duration of process*** under zero price-spread.

Amihud and Mendelson(1980)

Assumptions (Modification of Garman's Model)

(B') Inter-arrival time $\tau = \min(\tau_a, \tau_b)$ where $\tau_a \sim \exp(D(a))$ and $\tau_b \sim \exp(S(b))$ i.e.

Inter-arrival time ~ $\exp(D(a)+S(b))$

Prob(arrival is ***buy***) = $D(a)/(D(a)+S(b))$

Prob(arrival is ***sell***) = $S(b)/(D(a)+S(b))$

(E) The permissible stock inventory levels are {-K, -K+1, -K+2, … , L-2, L-1, L}, by renumbering this, {0,1, … , M-1, M} where M=L+K

Model as a Birth and Death process

λ_k = Birth rate in state k (Sell order when inventory is k-L) = $S(b_k)$

μ_k = Death rate in state k (Buy order when inventory is k-L) = $D(a_k)$

$\lambda_M = \mu_0 = 0$

(F) $R''(\mu) < 0, C''(\lambda) > 0$

(G) No transaction cost

Objective:

$$g(\underline{\lambda}, \underline{\mu}) = \sum_{k=0}^{M} \phi_k q_k$$

where

$\underline{\lambda} = (\lambda_0, ..., \lambda_{M-1})$ and $\underline{\mu} = (\mu_1, ..., \mu_M)$

$q_k = R(\mu_k) - C(\lambda_k)$

ϕ_k : the limiting probability of finding the process in state k

$\lambda_k \phi_k = \mu_{k+1}\phi_{k+1}$ k=0,…,M-1

$$\phi_k = \phi_0 \frac{\lambda_0 \cdots \lambda_{k-1}}{\mu_1 \cdots \mu_k}$$

Optimal Solution

F.O.C.

$$\lambda_k : \sum_{j=k+1}^{M} \phi_j [R(\mu_j) - C(\lambda_j)] - \lambda_k \phi_k C'(\lambda_k) = g(\underline{\lambda}, \underline{\mu}) \sum_{j=k+1}^{M} \phi_j$$

$$\mu_k : \sum_{j=k}^{M} \phi_j [R(\mu_j) - C(\lambda_j)] - \mu_k \phi_k R'(\mu_k) = g(\underline{\lambda}, \underline{\mu}) \sum_{j=k}^{M} \phi_j$$

Optimal Solution

$$C(\lambda_0) - \lambda_0 C'(\lambda_0) + g(\underline{\lambda}, \underline{\mu}) = 0 \qquad (11^0)$$

$$C(\lambda_k) - \lambda_k C'(\lambda_k) + g(\underline{\lambda}, \underline{\mu}) = R(\mu_k) - \mu_k R'(\mu_k) \qquad (11^k)$$

$$g(\underline{\lambda}, \underline{\mu}) = R(\mu_M) - \mu_M R'(\mu_M) \qquad (11^M)$$

Behavior of Optimal Solution

The optimal (λ_k, μ_k) are aligned along the curve defined by

$$[R(\mu) - \mu R'(\mu)] - [C(\lambda) - \lambda C'(\lambda)] = g \qquad (12)$$

$(\lambda_0, 0)$ and $(0, \mu_M)$ are the intersection of the curve with the axes.

Lemma 3.1 (Relationship between birth rate and death rate)

Along (12), for $\lambda, \mu > 0$, $d\mu / d\lambda < 0$

Theorem 3.2 (Price and Inventory)

Let b_k and a_k, respectively, be the optimal bid and ask prices at state *k*. Then $b_0 > b_1 > \cdots > b_{M-1}$ and $a_1 > a_2 > \cdots > a_M$. Equivalently, $\lambda_0 > \lambda_1 > \cdots > \lambda_{M-1}$ and $\mu_1 < \mu_2 < \cdots < \mu_M$.

Corollary 3.3 (Positive Bid-Ask Spread)

$a_k - b_k > 0$

Theorem 3.4 (Comparison of optimal profit with Garman's paper)

Let r^* be the (unique) solution of $R'(r^*) = C'(r^*)$. Then

$g(\underline{\lambda}, \underline{\mu}) < R(r^*) - C(r^*)$

Theorem 3.5 (Positive rate for every inventory position except extreme position)

The optimal policy $(\underline{\lambda}, \underline{\mu})$ satisfies $\lambda_k > 0$ for k=0,1,…,*M*-1, and $\mu_k > 0$ for k=1,…,M

Lemma 3.6 (Comparison of optimal rate with Garman's paper)

For all k=0,1,…,*M*-1

$\min\{\lambda_k, \mu_{k+1}\} \le r^* \le \max\{\lambda_k, \mu_{k+1}\}$,

with equality if and only if $\lambda_k = r^* = \mu_{k+1}$

Theorem 3.7 (Preferred Inventory Position)

The distribution $\{\phi_k\}_{k=1}^{M}$ is unimodal. The mode *J* is such that

$\lambda_k \ge r^*$ for $k < J$, $\lambda_k < r^*$ for $k \ge J$,

$\mu_k \le r^*$ for $k \le J$, $\mu_k > r^*$ for $k > J$

Theorem 3.8 (Comparison of ideal rate with Garman's paper)

$\lambda^I = \mu^I < r^*$

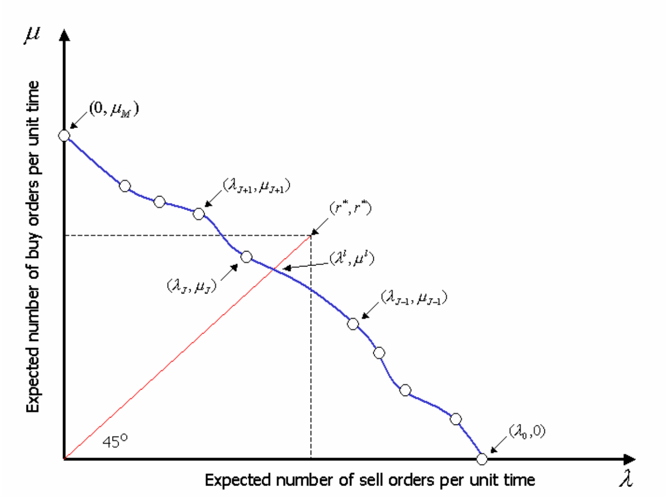


Figure 3. Optimal Solution [Amihud & Mendelson]

Optimal Solution in Linear Supply/Demand Rate Case

Linear Supply/Demand Rate

$D(a) = \gamma - \delta a$

$S(b) = \alpha + \beta b$

General Result Holds

- $b_0 > b_1 > \cdots > b_{M-1}$ and $a_1 > a_2 > \cdots > a_M$. Equivalently, $\lambda_0 > \lambda_1 > \cdots > \lambda_{M-1}$ and $\mu_1 < \mu_2 < \cdots < \mu_M$. (Theorem 3.2)
- Preferred inventory position (Theorem 3.7)
- etc

Specific result

- Bid-ask spread increases as the inventory position deviates from preferred inventory position
- Transaction-to-transaction price behavior is dependent (mean reverting)

Bid price never becomes larger than ask price (Traders cannot make profit by speculating price)

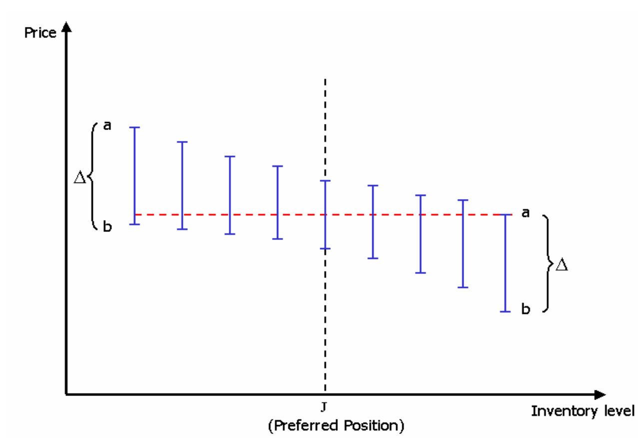


Figure 4. Linear Demand Case [Amihud & Mendelson]

Intuition

- Explanation of the fact from the empirical study: "the market-maker has preferred inventory position" (Theorem 3.7)
    - This can be explained using cash flow maximization under uncertainty
- Near preferred inventory position demand rate and supply rate becomes close
- Ask price increases and bid price decreases as inventory increases (Theorem 3.2)
- For linear demand and supply functions
    - Bid-ask spread increases as the inventory position deviates from the preferred inventory position
    - Transaction-to-transaction price behavior is inter-temporally dependent (mean reverting)
    - Bid price never becomes larger than ask price
    - Traders cannot make profit by speculating

Milgrom and Glosten (1985)

Assumptions

Sequence of Events

- The specialist sets bid and ask price
- An investor arrives and either buy or sell at the price or leave (all orders are market orders)
- Specialist is free to change the bid and ask price at any time after an arriving investor has made a decision and before the next arrival of an investor
- Only unit trades take place for each arrival
- At some time T_0 in the future, some random dollar value *V* per share will be realized.
    - The distribution of *V* is known to all participants.

Utility of Participants

- Traders and the specialist are risk neutral utility maximizers
- Participants' utility function: $\rho xV + c$ for *x* share of stocks
- ρ : Preference on future consumption. A random variable independent of *V*.
    - Participants have different value of ρ whose value is not known to the specialist (distribution is known).
    - Informed and uninformed may have different probability distribution.
    - ρ =1 for the specialist (Normalization)

Information

H_t : Public information at time *t* (known to all participants)

J_t : Private information at time *t* (known to only informed traders)

S_t : Information available to the specialist at time *t*

F_t : Information of an arrival at time *t*; refinement of H_t including the information conveyed by bid and ask.

Trader's decision:

Buy if $Z_t > A$, sell if $Z_t < B$ (i.e. Z_t is reservation price)
Where
$Z_t = \rho_t E[V \mid H_t, A, B]$ for uninformed trader
$Z_t = \rho_t E[V \mid H_t, J_t, A, B]$ for informed trader
$Z_t = \rho_t E[V \mid F_t] = \rho_t (1-U_t) E[V \mid H_t, J_t, A, B] + \rho_t U_t E[V \mid H_t, A, B]$
U_t : 1 if the individual arriving at t is uninformed, 0 otherwise.

Objective: Specialist's expected profit
$(A - E[V \mid S_t, Z_t > A]) P\{Z_t > A \mid S_t\} - (B - E[V \mid S_t, Z_t < B]) P\{Z_t < B \mid S_t\}$

Solution: Zero expected profit equilibrium prices at time t (no regret pricing)
$A_t = E[V \mid S_t, Z_t > A_t]$
$B_t = E[V \mid S_t, Z_t < B_t]$

Revision of Expectations of V
- Upward in response to specialist sales
- Downward in response to specialist purchases

Transaction price
$p_k = E[V \mid S_k] = E[V \mid S_{T_k}^+] = E[V \mid S_{T_k}, Z_{T_k} > A_{T_k}] I(Z_{T_k} > A_{T_k}) + E[V \mid S_{T_k}, Z_{T_k} < B_{T_k}] I(Z_{T_k} < B_{T_k})$
where
$I(.)$ is indicator function
T_k: times at which trades occur
$S_{T_k}^+$ and $H_{T_k}^+$: the information available just after a trade at time T_k, including information such as whether the trade was either buy or sell and the traded price.
$S_k = S_{T_k}^+$, $H_k = H_{T_k}^+$

Proposition 1 (Existence of bid-ask spread purely caused by information asymmetry: general arrival process, zero profit equilibrium)
Suppose equilibrium bid and ask prices satisfying the zero expected profit conditions:
$A_t = E[V \mid S_t, Z_t > A_t]$
$B_t = E[V \mid S_t, Z_t < B_t]$
Then the ask price is greater and the bid price is less than the expectation of *V*. i.e.
$A_t \geq E_t[V] \geq B_t$. The inequalities are strict if adverse selection is possible; i.e. if
$P\{Z_t > E_t[V], E_t[V \mid F_t] > E_t[V]\} > 0$,
$P\{Z_t < E_t[V], E_t[V \mid F_t] < E_t[V]\} > 0$
Where
E_t: conditional expectations given the common knowledge at time *t* $E[. \mid H_t \wedge S_t]$
$A_t = E_t[A_t]$ and $B_t = E_t[B_t]$ since the bid and ask prices are common knowledge at time *t*.

Proposition 2 (No correlation between transaction prices)
The sequence of transaction prices $\{p_k\}$ forms a martingale relative to the specialist's information, $\{S_k\}$, and the public information, $\{H_k\}$.
That is $E[p_{k+1} \mid S_k] = p_k$.

Proposition 3 (Bound of average spread)
Define
$1/\gamma_k = P\{Z_k > A_k \mid S_{T_k}\}P\{Z_k < B_k \mid S_{T_k}\}$
and let γ^* be the mean value of γ_j over N+1 observations; i.e. $\sum_{k=1}^{N+1}\gamma_k /(N+1)$.
Further, define Ψ_N to be the average spread over N trades, i.e.,
$$\Psi_N = \sum_{k=1}^{N}(A_k - B_k)/N$$
If $\exists\gamma$ such that $P\{\gamma^* < \gamma\} = 1$, then
$E[(N\Psi_N^2] \le 2\,\mathrm{var}(V)\gamma$

Proposition 4 (Convergence of price to real value)
If trade is reasonably balanced, i.e., the probability of a purchase is bounded away from zero and one, then the expectations of the specialist and the traders converge as the number of trades increases, i.e., $E[V \mid S_k] - E[V \mid F_k]$ converges to zero in probability (where F_k is the information of the k th trader to arrive)

Corollary 1 (Stabilization of inventory)
The specialist's inventory of stocks tends to a driftless stochastic process; i.e.
$\lim_{k\to\infty}(P\{Z_k < B_k \mid H_k\} - P\{Z_k > A_k \mid H_k\}) = 0$

Proposition 5 (Behavior of bid and ask price)
For any given time t, the ask price A_t increases and the bid price B_t decreases when, other things being equal,

(i) the insiders' information at time *t* becomes better
(ii) the ratio of informed to uninformed arrival rates at *t* is increased
(iii) the elasticity of uninformed supply and demand at time *t* increases

Ho and Stoll (1980)

Assumptions and Notation
Bid-Ask spread is from inventory holding cost
Competing dealers know each other's inventory position
True value of security is known
One transaction occur:

- Dollar amount Q
- Buy with probability λ

- Sell with probability λ
- No transaction with probability $(1-2\lambda)$

p: Value of stock
a: Optimal reservation selling fee
b: Optimal reservation buying fee
$p_a = p + a$: Ask price
$p_b = p - b$: Bid price
$p = 1$: Normalization

Two Dealer Case

Reservation prices

- Dealer A:

$$a = R\sigma_I^2(\frac{Q}{2} - I)\,,\; b = R\sigma_I^2(\frac{Q}{2} + I)$$

- Dealer B:

$$a^0 = R^0\sigma_I^2(\frac{Q}{2} - I^0)\,,\; b^0 = R^0\sigma_I^2(\frac{Q}{2} + I^0)$$

Where
R : Coefficient of absolute risk aversion
σ_I^2 : Per period variance of the stock's return
Q : Fixed dollar transaction size
I : Dealer's inventory holding of stock

Competitive pricing in one period case

- The market bid-ask spread
  $s = \max(a, a^0) + \max(b, b^0)$
- The market reservation spread
  $s' = \min(a, a^0) + \min(b, b^0)$

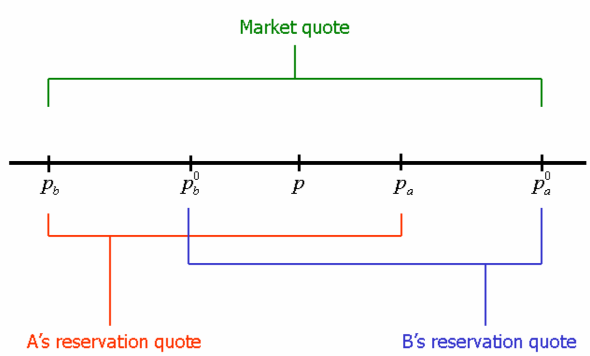


Figure 5. Quote Under Competition

Inter-dealer trading
Suppose $I - I^0 \geq 2Q$,
Seller A compares two options to decide inter-dealer trading

Option 1) To sell to a market order with probability λ to earn a fee of a^0
$EU_1 = U(W) + U'\Delta W + 1/2U''\sigma^2(W) + R(I - I^0)\lambda\sigma_I^2 U'Q$
W: the dealer's total wealth
U(*W*): the dealer's elementary utility function

Option 2) To sell to dealer B paying to B a fee of π
$EU_2 = U(\tilde{W}) + U'\Delta\tilde{W} + 1/2U''\sigma^2(\tilde{W}) + R[(I - Q) - (I^0 + Q)]\lambda\sigma_I^2 U'Q$
$\tilde{W}$: change of *W* by changing *I* to *I-Q*

- When $b = \pi$, there is no inter-dealer trading
- However, there is a negotiated fee at which A would sell B rather than take a chance on a market order. When $I > (1/2 + \lambda)$, the fee is positive.

Intuition
In competitive case, optimal strategy is dependent on inventory position and competitor's inventory position
Quotation
- Two dealer case: Quote competitor's reservation price (highest reservation asking price, lowest reservation bid price)
- More than two dealer case: Spread decreases due to competition

Inter-dealer trading in two dealer case
- No inter-dealer trading at market price (There may exist negotiated price)

In multi-period case
- Dealer must take account pricing effects on its own inventory and on competitor's inventory, because future profit is dependent on competitor's inventory position
- Bid-ask spread may increase
- No model provided

Huang and Stoll (1997)

Methodology
- Modeling + Regression
- Model based on intuition from analytical model
- Two way decomposition model
- Order processing cost + Non-order processing cost
- Three way decomposition model
- Order processing cost + Inventory cost + Information cost
- Approach 1: Time series approach
- Approach 2: Portfolio approach
- Regression of real data (with modification) to find parameter values

Notation

V_t : Unobservable fundamental value of the stock (just determined prior to the posting of the bid and ask quotes at time *t*).

M_t : Quote mid-point (calculated from the bid0ask quotes that prevail just before a transaction)

P_t : Price of the transaction at time *t*

Q_t : Buy-sell indicator variable – buy/sell is from dealer's perspective

- +1: if sale transaction occurred above mid-point
- -1: if buy transaction occurred below mid-point
- 0: if transaction occurred at mid-point

S: constant "traded" spread (differ from observed posted spread, S_t)

Two way decomposition:
Order processing component + Non-order processing component

$$V_t = V_{t-1} + \alpha \frac{S}{2} Q_{t-1} + \varepsilon_t$$

α : The percentage of the half-spread attributable to adverse selection

ε_t : Serially uncorrelated public information shock

$$M_t = V_t + \beta \frac{S}{2} \sum_{i=1}^{t-1} Q_i$$

β : The proportion of the half-spread attributable to inventory holding cost

$$\Delta M_t = (\alpha + \beta) \frac{S}{2} Q_{t-1} + \varepsilon_t$$

$$P_t = M_t + \frac{S}{2} Q_t + \eta_t$$

η_t : The deviation of the observed half-spread ($P_t - M_t$) from the constant half-spread, *S*/2.

$$\Delta P_t = \frac{S}{2}(Q_t - Q_{t-1}) + \lambda \frac{S}{2} Q_{t-1} + e_t$$

$$\lambda = \alpha + \beta$$

$$e_t = \varepsilon_t + \Delta \eta_t$$

Three way decomposition:
Order processing cost $(1-\lambda)$, Inventory cost (β), Adverse selection (α)

Approach 1: Time series - serial correlation in trade flows

π : Probability of a trade flow reversal

(1- π): probability of a trade flow continuation

Serial covariance of the trade flow, { Q_t , t=1, …} depends on π .

$$E[Q_{t-1} \mid Q_{t-2}] = (1-2\pi) Q_{t-2}$$

$$\Delta V_t = \alpha \frac{S}{2} Q_{t-1} - \alpha \frac{S}{2}(1-2\pi)Q_{t-2} + \varepsilon_t$$
$$E[\Delta V_t \mid V_{t-1}, Q_{t-2}] = 0$$

$$M_t = V_t + \beta \frac{S}{2} \sum_{i=1}^{t-1} Q_i$$
$$\Delta M_t = (\alpha + \beta)\frac{S}{2} Q_{t-1} - \alpha \frac{S}{2}(1-2\pi)Q_{t-2} + \varepsilon_t$$
$$E[\Delta M_t \mid M_{t-1}, Q_{t-2}] = \beta \frac{S}{2}(1-2\pi)Q_{t-2}$$

$$\Delta P_t = \frac{S}{2} Q_t + (\alpha + \beta - 1)\frac{S}{2} Q_{t-1} - \alpha \frac{S}{2}(1-2\pi)Q_{t-2} + e_t$$

Approach 2: Portfolio - trading pressure

$$M_t^k = V_t^k + \beta^k \frac{S^k}{2} \sum_{i=1}^{t-1} Q_i^*$$

where superscript *k* designates security *k*, and Q_t^* is the aggregate buy-sell indicator variable. That is

$$Q_t^* = \begin{cases} 1 & \text{for } \sum_{k=1}^{n} Q_t^k > 0 \\ -1 & \text{for } \sum_{k=1}^{n} Q_t^k < 0 \\ 0 & \text{for } \sum_{k=1}^{n} Q_t^k = 0 \end{cases}$$

$$\Delta P_t^k = \frac{S^k}{2} \Delta Q_t^k + \alpha^k \frac{S^k}{2} Q_{t-1}^k + \beta^k \frac{S^k}{2} Q_{t-1}^* + e_t^k$$

Results

Data: 20 stocks in NYSE, in 1992

Two way decomposition

- High order processing component(98.1% to 77.7%, 88% avg.)
- Inventory+ Adverse selection(1.9% to 22.3%)

Three way decomposition

- Time series approach(serial correlation)
  - Positive correlation – Need clustering to get negative one:
  - Avg. reversal probability = 0.87
  - Order processing component (61.7% avg.)

    - Inventory component (28.7% avg.)
    - Adverse selection component (9.6% avg.)
- Cross sectional approach(portfolio)
    - Need alignment of trading time
    - Order processing component (68.9% avg.)
    - Inventory component (9.6% avg.)
    - Adverse selection component (21.5% avg.)

Bayesian Learning: Basic (Glosten and Milgrom, 1985)

$V \in \{\underline{V}, \overline{V}\}$ (true value)

$P\{V = \underline{V}\} = \delta$ (prior belief)

$E[V] = (1-\delta) \bullet \overline{V} + \delta \bullet \underline{V}$

$P\{\text{informed}\}$, $P\{\text{uninformed}\}$, $P_U(B), P_U(S)$ are known

$Q \in \{B, S\}$ (transaction)

B*: buy,* S*: sell*

$E[V \mid B] = P\{V = \overline{V} \mid B\} \bullet \overline{V} + P\{V = \underline{V} \mid B\} \bullet \underline{V}$

$E[V \mid S] = P\{V = \overline{V} \mid S\} \bullet \overline{V} + P\{V = \underline{V} \mid S\} \bullet \underline{V}$

a : ask price, b : bid price

$a = E[V \mid B] > E[V]$

$b = E[V \mid S] < E[V]$

$$P\{V = \overline{V} \mid B\} = \frac{P\{V = \overline{V}\} P\{B \mid V = \overline{V}\}}{P\{B \mid V = \overline{V}\} P\{V = \overline{V}\} + P\{B \mid V = \underline{V}\} P\{V = \underline{V}\}} = \frac{(1-\delta) P\{B \mid V = \overline{V}\}}{P\{B \mid V = \overline{V}\}(1-\delta) + P\{B \mid V = \underline{V}\}\delta}$$

$\delta(Q) = P\{V = \underline{V} \mid Q\}$ (Bayesian update of belief)

Example)

$P\{\text{informed}\} = P\{\text{uninformed}\} = 1/2$

$P_U(B) = P_U(S) = 1/2$

$\delta = 1/2$

$\underline{V} = 0, \overline{V} = 1$

$a = E[V \mid B] = P\{V = 1 \mid B\} \bullet 1 + P\{V = 0 \mid B\} \bullet 0 = P\{V = 1 \mid B\}$

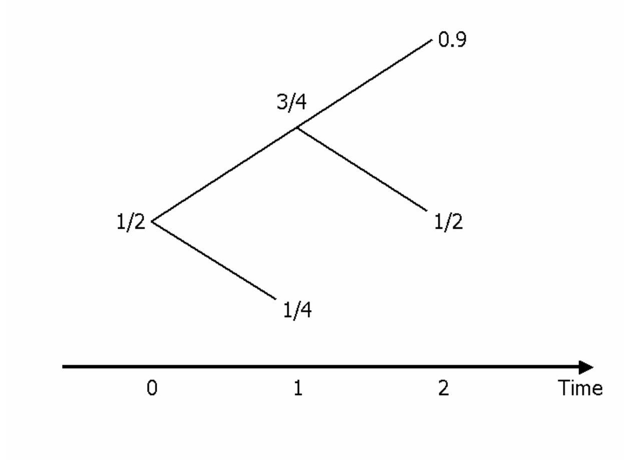
$$P\{V = 1 \mid B\} = \frac{P\{V = 1\} P\{B \mid V = 1\}}{P\{B \mid V = 1\} P\{V = 1\} + P\{B \mid V = 0\} P\{V = 0\}} = \frac{(1-\delta) P\{B \mid V = 1\}}{P\{B \mid V = 1\}(1-\delta) + P\{B \mid V = 1\}\delta}$$

$$P\{B \mid V = 1\} = P\{\text{uninformed}\} P_U\{B \mid V = 1\} + P\{\text{informed}\} P_I\{B \mid V = 1\} = \frac{1}{2} \bullet \frac{1}{2} + \frac{1}{2} \bullet 1 = \frac{3}{4}$$

$$a = P\{V = 1 | B\} = \frac{1/2 \bullet 3/4}{3/4 \bullet 1/2 + 3/4 \bullet 1/2} = \frac{3/4}{3/2} = 1/2$$

Assume the transaction is B,

Change δ to $P\{V = \underline{V} | B\} = \frac{1}{4}$



Easley and O'Hara (1987)

$Q \in \{B^1, B^2, S^1, S^2\}$

$B^1 < B^2$, $S^1 < S^2$

α : probability that new information exists

s : signal

μ : proportion of trades from informed traders

$\delta(Q) = P\{s = L | Q\} = 1 \bullet P\{s = L | Q\} + 0 \bullet P\{s = H\} + \delta \bullet P\{s = \phi | Q\}$

$$P\{s = x | Q\} = \frac{P\{s = x\}P\{Q | s = x\}}{P\{Q | s = L\}P\{s = L\} + P\{Q | s = H\}P\{s = H\} + P\{Q | s = \phi\}P\{s = \phi\}}$$

$E[V | Q]$, or { $E[V | B^1]$, $E[V | B^2]$, $E[V | S^1]$, $E[V | S^2]$ }= $\{a^1, a^2, b^1, b^2\}$

Separating equilibrium

$\delta(B^1) = \delta(S^1) = \delta$

$$\delta(S^2) = P\{V = \underline{V} | S^2\} = \delta \frac{\alpha\mu + P_U(S^2)[1 - \alpha\mu]}{\alpha\mu\delta + P_U(S^2)[1 - \alpha\mu]} \geq \delta$$

$$\delta(B^2) = P\{V = \underline{V} | B^2\} = \delta \frac{P_U(B^2)(1 - \alpha\mu)}{\alpha\mu(1 - \delta) + P_U(B^2)(1 - \alpha\mu)} \leq \delta$$

$E[V | B^1] = E[V | S^1] = E[V]$

$$E[V | S^2] = E[V] - \frac{\sigma_V^2}{[\overline{V} - \underline{V}]}\left[\frac{\alpha\mu}{P_U(S^2)(1 - \alpha\mu) + \delta\alpha\mu}\right]$$

$$E[V \mid B^2] = E[V] + \frac{\sigma_V^2}{[\overline{V} - \underline{V}]}\left[\frac{\alpha\mu}{P_U(B^2)(1-\alpha\mu) + (1-\delta)\alpha\mu}\right]$$

where σ_V^2 is the prior variance of V.

Condition for separating equilibrium

$S^2[b^2 - \underline{V}] \geq S^1[b^1 - \underline{V}]$ and $B^2[\overline{V} - a^2] \geq B^1[\overline{V} - a^1]$

That is.

$$\frac{S^2}{S^1} \geq 1 + \frac{\alpha\mu\delta}{P_U(S^2)[1-\alpha\mu]} \text{ and } \frac{B^2}{B^1} \geq 1 + \frac{\alpha\mu(1-\delta)}{P_U(B^2)[1-\alpha\mu]}$$

Pooling equilibrium

$\hat{a}^1, \hat{a}^2, \hat{b}^1, \hat{b}^2$

ψ_{B^1} : proportion of informed traders who buy small quantity

ψ_{S^1} : proportion of informed traders who sell small quantity

Condition for pooling equilibrium

$$\frac{S^2}{S^1} < 1 + \frac{\alpha\mu\delta}{P_U(S^2)[1-\alpha\mu]} \text{ and } \frac{B^2}{B^1} < 1 + \frac{\alpha\mu(1-\delta)}{P_U(B^2)[1-\alpha\mu]}$$

$$\hat{b}^1 = \hat{\delta}(S^1)\underline{V} + (1-\hat{\delta}(S^1))\overline{V}$$

$$\hat{b}^2 = \hat{\delta}(S^2)\underline{V} + (1-\hat{\delta}(S^2))\overline{V}$$

where

$$\hat{\delta}(S^1) = P\{V = \underline{V} \mid S^1\} = \frac{\delta[\psi_{S^1}\mu\alpha + (1-\alpha\mu)P_U(S^1)]}{[\delta\alpha\mu\psi_{S^1} + (1-\alpha\mu)P_U(S^1)]}$$

$$\hat{\delta}(S^2) = P\{V = \underline{V} \mid S^2\} = \frac{\delta[(1-\psi_{S^1})\mu\alpha + (1-\alpha\mu)P_U(S^2)]}{[\delta\alpha\mu(1-\psi_{S^1}) + (1-\alpha\mu)P_U(S^2)]}$$

Similarly,

$$\hat{a}^1 = \hat{\delta}(B^1)\underline{V} + (1-\hat{\delta}(B^1))\overline{V}$$

$$\hat{a}^2 = \hat{\delta}(B^2)\underline{V} + (1-\hat{\delta}(B^2))\overline{V}$$

where

$$\hat{\delta}(B^1) = P\{V = \underline{V} \mid B^1\} = \frac{\delta[\psi_{B^1}\mu\alpha + (1-\alpha\mu)P_U(B^1)]}{[\delta\alpha\mu\psi_{B^1} + (1-\alpha\mu)P_U(B^1)]}$$

$$\hat{\delta}(B^2) = P\{V = \underline{V} \mid B^2\} = \frac{\delta[(1-\psi_{B^1})\mu\alpha + (1-\alpha\mu)P_U(B^2)]}{[\delta\alpha\mu(1-\psi_{B^1}) + (1-\alpha\mu)P_U(B^2)]}$$

Appendix C) Market Microstructure Theory (O'Hara)

Criticism: Inventory-based model

- Effect on price is temporary (ultimately reverts to "true" levels)
- Unclear preferred inventory level
  - The desired level is not clear
  - What changes the level is not clear
  - Especially when dealer speculates, it is not clear
- Weak empirical evidence
  - Existence of preferred level is verified but it departs from the level in the long run (Madhavan and Smidt[91,93], Hasbrouck and Sofianos[93])
  - Little evidence of mean reverting behavior of prices
    - Lyons[93]: found evidence in foreign exchange markets
    - Madhavan and Smidt[91,92]: little evidence in equity markets
    - Manaster and Mann[92]: little evidence in futures markets

Criticism: Information-based model

- Price adjustment path to true value
  - How long it will take is not clear
  - Different paths depending on the information the market maker sees
- Existence of new information?
- Do not allow traders to select quantity (one unit at a time)
- Cannot explain immediate price drop associated with block trades and subsequent price behavior (i.e. partial recovery)
- Virtual queue is required to explain arrival process
- Strategy of unformed and informed traders is not considered

Real Scenario: NYSE in 1992

- 400 specialist, 2089 listed firms
- Each specialist handles average 3.7 stocks
- Specialists participate in average 19.4 % of transactions (greater involvement with less frequently traded stocks)
- Limit orders (contingent on price, time etc) are kept in the specialist's book (not common knowledge)
- Trading occurs between 9:30 to 4:00 E.S.T
- Trading Mechanism: call auction + continuous auction
- Program trading: 11.5 % or all orders (SuperDOT system)
- Inter-market trading system(ITS):
- 9 US Markets: NYSE, American Stock Exchange, Boston, Midwest, Cincinnati, Pacific, Philadelphia, the Chicago Board Options Exchange, the NASD
- Block Trade
  - Transaction involving 10,000 shares or more
  - Explains 50.7 % of total volume

  Cf) 27% of block trades use "upstairs market maker" who forms a syndicate of buyers to take the other side of the trade, not specialist.

- Price continuity
- 96.4 % of all transactions occurred with a price change of 1/8 or less
- Specialist is expected to stabilize market: ex) no sale into a falling market
- Circuit breakers
- Introduced after the market crash in 1987
- DJIA: 250 points, 400 points declines: trading halt
- Rule 80A: restricts index arbitrage – execution of sell orders only on a plus tick, buy orders only on a minus tick when DJIA 50 points declines
- Trading halt in response to firm-specific events

What did we miss?

- Existence of new information
- Selection of transaction quantity: block trading or small trading
- Learning based on public information such as
  - Price process
  - Trading volume
  - Transaction time
- Order book: limit order, stop order
- Market failure issue
- Stability of market
- Relationships between markets
  - Stock market, and upstairs market, futures, derivatives market, index product
- Requirements of market
- Market efficiency: definition of efficiency, dynamic efficiency

Information-based model: Easley and O'Hara[1987]

Assumptions

- Dual uncertainty: existence of information and information content – dual learning
- Trader selects quantity: large quantity or small quantity

Two possible equilibria

- Separating: Informed traders choose large quantity, uninformed choose both large and small
- Pooling: Informed traders choose both, uninformed choose both

Spread could vary across trade sizes

- Large trade with larger spread
- Large trade at worse price

Uninformed trade can have information value

- Signal that no new information exists

Partial price recovery after block trading

- If following quantity is small, dealer updates belief on no information

Examining only the small-trade spread can be misleading

Learning

Difficulty of modeling learning process

- Learning depends on traders' risk preferences, endowments, the nature and extent of uncertainty, market structure itself etc.
- Many traders learn from sequence of prices, trading volume, trading time beside current price

Sequence of prices

- Sequence of prices can be informative beyond the information provided by the individual prices
- Model: private information and supply uncertainty
- Grundy and McNichols[1989], Brown and Jennings[1989]

Trading volume

- Empirical link is strong:
  - Volume is larger when prices move up than when they move down
  - Strong link between volume and the absolute value of price changes
- Wang[1994]: Volume is decreasing in the amount of the information asymmetry
- Blume, Easley, and O'Hara[1994]: Volume conveys information about signal quality (small volume when the quality is very high or very low)

Trading time

- Diamond and Verrecchia[1987]
  - Traders have various short sale constraints
  - Observing an absence of trade: signal of bad news
  - Fast convergence under no constraints
- Easley and O'Hara
  - Trades provide signals on the direction of any new information
  - The lack of trade provides a signal of the existence of any new information: high probability of no new information
- Contrasted response of market maker to lack of trade
  - DV: increase spread, move prices down
  - EO: decrease spread, price may go up or down depending on the cumulated volume

Order forms: limit order, stop order

Limit order

- Provide liquidity to the market
- "Free option" from trader
- Usually from uninformed traders

- Submitted for hedging or expecting profit from market maker's inventory position adjustment process
- Limit order strategy depends on available information: structure of the book, market maker's inventory position etc.

Stop order
- Typically used to sell stock when prices are falling
- Take liquidity from the market
- Usually from uninformed traders

Effect of introducing order book
- Market order flow is more likely to come from informed traders
- Large spreads with faster adjustment to "true" value (high volatility)
- Price converge faster to full-information values

Order type uncertainty is inevitable
- Bidders may use complex portfolio strategy that does not use book explicitly

Market failure issue

Adverse selection issue

Too many informed traders, too large bid-ask spread, no transaction, no quote update, no transaction, no quote update …..

Solutions
- Halt trading
- Monopoly position of specialist: Glosten[1989]
    - Sets prices that maximize profits on average not transaction-by-transaction
- Call market: Madhavan[1992]
    - No quote, price is not known when traders submit bid
    - Cost: price discovery is hard due to the lack of continuity (especially so, for infrequently traded stock)
- Larger traders are better off, small traders are worse off

Possibility of cream skimming (3rd market for small traders)
- Restrict orders to small amount
- Match the best bid or offer
- Arizona Stock Exchange, POSIT, Instinet etc.
    - NYSE estimates about 20% of total volume and 35% of small trade volume (less than 2,100 shares) was diverted to third party providers[1993]

Stability of market

Explanation of market fall in 1987: uncertainty of hedging amount (Gennotte and Leland[1990], Jacklin et al.[1992])

"Sunshine policy": Grossman[1988], Admati and Pfleiderer[1991]

- Uninformed price-contingent orders are revealed to the market prior to their execution
- Increase liquidity by inducing more risk averse liquidity providers enter
- Large traders benefit more from increased liquidity than do small traders

Relationships between markets

Possibility of arbitrage
- Bid-ask spread increases due to the risk of trading with arbitrageurs (technical uninformed traders)
- A market may have effect on the liquidity of other markets

Different types of information
- Information common to all securities, systematic information, security-specific information etc.

Discretionary liquidity traders
- Informed trading increases variance of order flow
- Discretionary liquidity traders prefer market with lowest variance
- Adding discretionary order flow decreases variance of market
- Every discretionary liquidity trader goes to the same market in equilibrium..?
- There are many factors affecting overall volatility

Upstairs market
- 27% of blocks in 1992 used upstairs market, why?
- Large size of transaction is beyond risk bearing capacity of the specialist
- The lack of anonymity in upstairs market to avoid signaling to market

Index products
- Reduces liquidity of the individual securities
- Increase sensitivity of individual securities to systematic information

Derivative market
- Increasing volatility of the stock increases option liquidity
- Informed traders may move to the derivative market

Futures market
- Information in the futures market on the aggregate value is more informative: Kumar and Seppi, 1990
- Futures markets respond more strongly to information than do the underlying stock markets

Requirements of Market: Commodity Exchange Act of 1974

Reliable price discovery
- Market clearing price can always be found

- Single setting providing market-clearing price or multiple setting provide different market prices for different quantities?

Broad-based price dissemination
- Transparency: expedite price discovery process
- Third market who "free ride" price discovery elsewhere

Effective hedging against price risks
- General role of markets: provide insurance to liquidity traders
- Greater liquidity, reduction of anonymity, prohibition of dual trading and front running

Market Efficiency

Market Efficiency
- How well and how quickly a market aggregates and impounds information into the price?
    - Strong form efficient: prices reflect all private and public information
    - Semi-strong form efficient: prices reflect all public information
    - Weak form efficient: prices reflect the information in their own past values

Dynamic efficiency
- Speed with which prices reflect full information
- There is volatility-efficiency trade off
- The goals of minimizing uninformed trading costs and increased price efficiency conflict
- However, in general, market efficiency benefits society directly by reducing the cost of capital for firms

Summary

- **Different environment and assumptions**: Information-based model with uncertainty of information existence and volume selection
- **Learning with public information beside current price**: Learning by price sequence, trading volume and transaction time
- **Market structure and price behavior**
    - Effect of order book
    - Market failure and stability
- **Relationship between markets**: Relationship between stock market and upstairs market, index product, derivative, futures market
- **Market performance**: requirements, efficiency