A PERFORMANCE ASSESSMENT OF A BREAK-EVEN ANALYSIS IN ALGEBRA I

Joseph Ray Brillantes
Dr. Barry Mitchell
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TEACHER PACK

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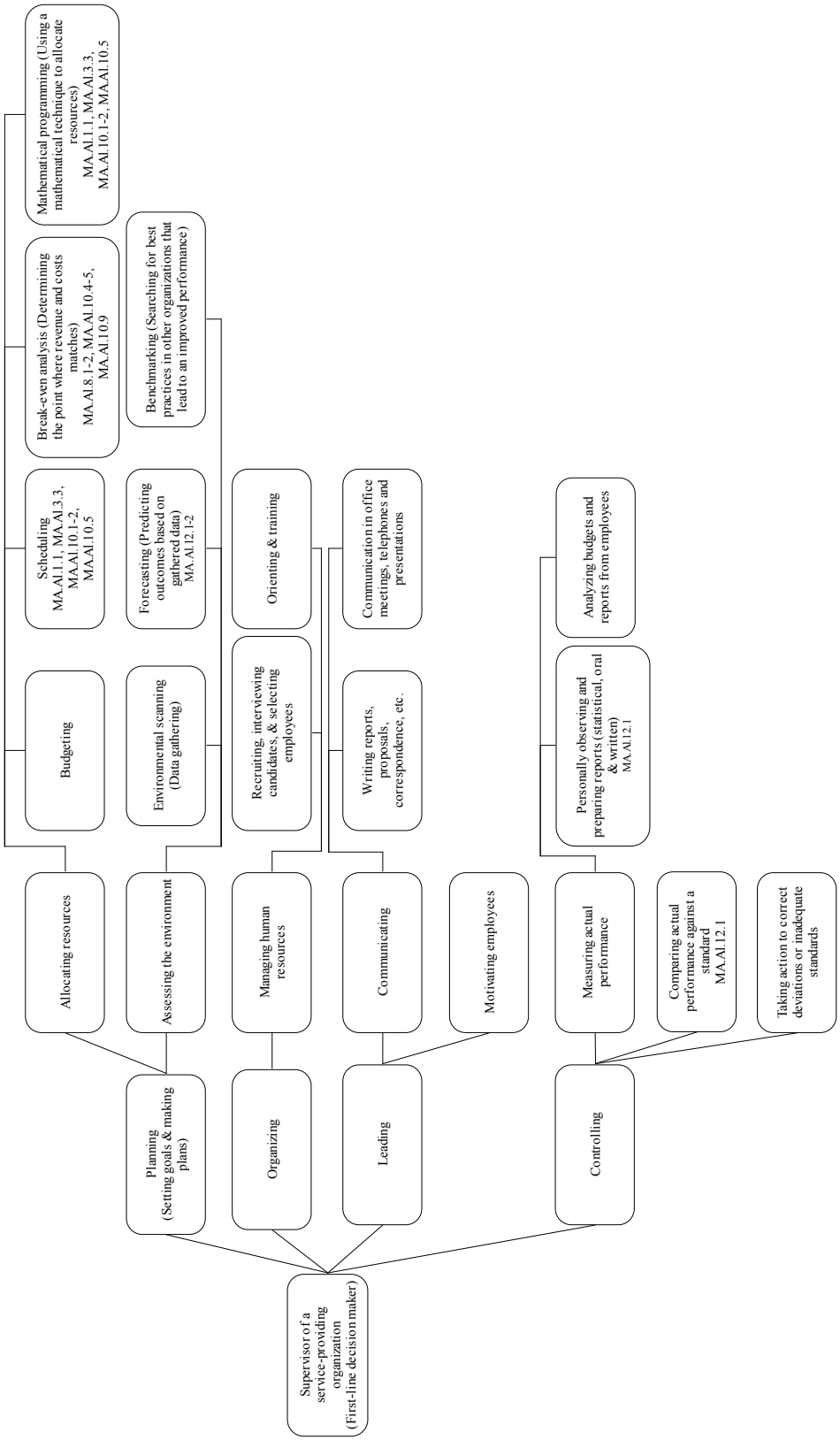
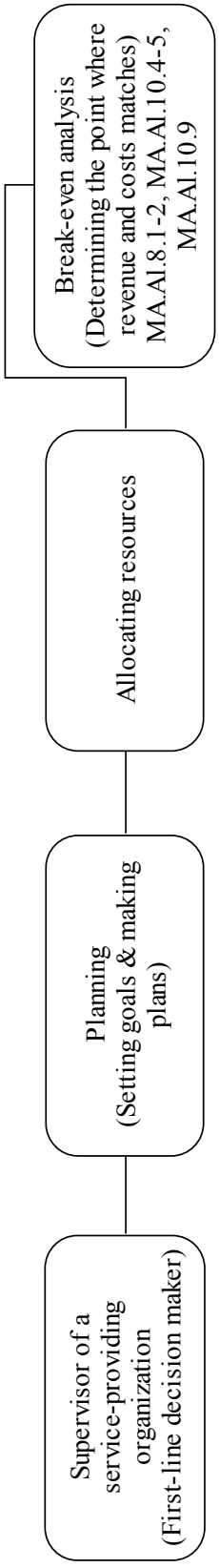
1.1. Exploded-web diagram



1.2. Cut-down diagram



2. Rationale

Though most teachers and parents aspire that their students and children finish college, not all of them will. Some will drop-out from college, and some will drop-out even before they reach college. When they drop-out from school, eventually they need to find a job. They can find work in service-related industries, where they can be trained to perform the job.

Organizations promote from their employees to fill supervisory roles. Supervisors are first-line decision makers. They make decisions that relate to everyday operations of the organization. Organizations believe that though new supervisors do not have management experience, they are knowledgeable enough to make sound decisions on operations. As new supervisors fulfill their roles, they learn that there are tools to ensure that their decisions are sound. One of those tools is Break-Even Analysis.

Break-Even Analysis is a mathematical tool that finds the point where revenues equal costs. This point is important because it tells how much an organization needs to produce and sell to break-even. If the revenues and costs were linear functions, a student with a course in Algebra can perform break-even analyses.

The Algebra concepts that a student has to know to perform a break-even analysis are the slope of a line, the equation of a line, and solutions of systems of two linear equations in two variables. Studying these concepts, the students can meet Benchmarks MA.Al.8.1, MA.Al.10.4 and MA.Al.10.5.

Described in the following pages are outcomes, lessons and assessment that produce evidence of meeting those benchmarks. Students will be taught concepts relating to the slope of a line, the equation of a line, and solutions of systems of two linear equations in two

variables. Applications of these concepts will be modeled in a break-even analysis I will demonstrate in the classroom. After the lessons, the students will choose the service they want to provide for a fee, decide on a price for that service, and determine costs. They will then perform a break-even analysis on their own. They will present their break-even analysis to me in a scheduled personal conference. They will prepare visual aids for this presentation. During the presentation, they will perform the procedures of a break-even analysis.

Before this unit, I have taught the students addition, subtraction, multiplication, and division equations. They know how to manipulate equations to isolate variables, and have met Benchmarks MA.Al.3.1-2 and MA.Al.10.3. We will review plotting ordered pairs in a Cartesian plane in a lesson. They have learned this in their previous mathematics course. After this unit, I will teach the students graphing of linear inequalities, solving systems of linear inequalities, and word problem applications of solving systems of linear equalities and inequalities. The skills and concepts the students will learn in this unit will easily transfer to the next unit described in this paragraph. In the next unit, the students will meet Benchmarks MA.Al.4.1, 10.1-2, and 10.9.

3. Standards, Benchmarks, and Outcomes

The first two chosen benchmarks deal with slope of a line, and the equation of a line. When the students have an understanding of those, then they can graph lines, find the intersection of the lines algebraically and graphically, which is essentially solving the system of two linear equations in two variables or the third chosen benchmark.

Standard MA.Al.8. Geometry and Spatial Sense: REPRESENTATIONAL SYSTEMS: Select and use different representational systems, including coordinate geometry**.**

Benchmark MA.Al.8.1. Graph linear equations using slope-intercept, point-slope, and *x*- and *y*-intercept techniques.

Outcome: Given at least either a slope and a point, or two points, the students can

- find other points on the same line,
- plot those points in a Cartesian plane, and
- graph the linear equation

To graph linear equations, one must be given at least either a point and a slope, or two points. From either one of those, the *x* and *y*-intercepts can be solved using the slope formula. The *x* and *y*-intercepts are points on the same line. Thus, the students can plot these points in a Cartesian plane, connect them with a straight line segment, and draw arrowheads at the end of the line segment. This represents the graph of the linear equation.

Standard MA.Al.10. Patterns, Functions, and Algebra: SYMBOLIC REPRESENTATION: Use symbolic forms to represent, model, and analyze mathematical situations.

Benchmark MA.Al.10.4. Determine the equation of a line when given the graph of the line, the slope and a point on the line, or two points on the line.

Outcomes:

- When given one of either the graph of a line, or two points on the line, the students can determine or solve for the slope and the *y*-intercept.
- When given the slope and a point on the line, the students can solve for the *y*-intercept.
- Then, the students can use the slope and the *y*-intercept to formulate the equation of the line.

To determine the equation of a line, the students must be given the graph of the line on a coordinate system with equal scales. From the graph of the line, the students can determine the slope by estimating two points on the line, and using the slope formula to solve for the slope. With a value of the slope and a point, the students can then solve for the *y*-intercept. The students can then formulate the equation of the line in the slope-intercept form.

Standard MA.Al.10. Patterns, Functions, and Algebra: SYMBOLIC REPRESENTATION: Use symbolic forms to represent, model, and analyze mathematical situations.

Benchmark MA.Al.10.5. Solve systems of two linear equations in two variables algebraically and graphically.

Outcome: Given two linear equations, the students can

- graph those equations in the same Cartesian plane,
- locate the point of intersection on the graph,
- estimate the coordinates of the point on the graph, and
- confirm the estimated coordinates by solving for the coordinates of the point of intersection of the lines

To solve for systems of two linear equations in two variables, the students need to find a point that satisfies both equations. This point is an intersection. Thus, to find the solution for systems of two linear equations in two variables, the students need to find the point of intersection on the graph. To algebraically solve for the point of intersection, the students need to equate both equations, solve for the value of the independent variable, and then substitute the value of the independent variable in one of the equations to solve for the value of the dependent variable.

4. Lessons (HTS.6.1, HTS.7.4)

The lessons build on the formula for the slope of a line. From the slope of a line, the lesson proceeds to building the equation of a line in two variables, the *x*- and *y*-intercepts, and graphing the equation of the line. Lines in the same Cartesian plane can intersect. Finding the points of intersection is essentially solving for the systems of linear equations.

4.1. Graph of lines, the slope of a line

I will review plotting points on a Cartesian plane and estimating ordered pairs given a point on a Cartesian plane. I will show them graphs of lines on a Cartesian plane. The class will compare the "steepness" of a line from another line. I will ask them to formulate for me a ratio that will tell us numerically that a line is "steeper" than another line (HTS.7.2, HTS.7.3). The class will discuss the mathematical definition of a slope, which is

> If $P_1\ (x_1,y_1)$ and $P_2\ (x_2,y_2)$ are any two distinct points on line *l*, which is not parallel to the vertical axis, then the slope of *l*, denoted by *m* is given by
>
> $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

4.2. Equation of a line

The class will review the equation of the slope. The equation of a graph, which is an equation that is satisfied by the coordinates of those, and only those points on the graph, will be defined. I will ask the students to find the slope and other points on the line, given two points on the line. The class will list their points as ordered pairs on the board (HTS.2.7, HTS.7.6). I will then demonstrate that given any point they have listed and the slope of a line, I can find other points using the formula for the slope. I will then define that a line in a Cartesian plane is all the points satisfying the equation $y-y_1=m(x-x_1)$, where *m* is the slope of the line, (x_1,y_1) is a point on the line. I will further demonstrate that if the given point is (0,*b*),

then the equation of the line becomes $y-b=m(x-0)$ or $y=mx+b$. The y-intercept b can now be defined as the point $(0,b)$ where the line intersects the y or vertical axis.

4.3. Graphing linear equations in two variables

The class will review the equation of the line, and the definition of the y-intercept. I will demonstrate how to graph a line given the equation of the line and the y-intercept. I will find another point on the line by using the equation of the line. I will then demonstrate that if I connect the point I found and the y-intercept, and draw arrowheads on the ends of the line segment, then I have a graph of a line. Given two points on the line, I will then demonstrate how to find the slope of the line, the y-intercept, the equation of the line, and the graph of the line. The class will then do exercises graphing lines when they are given two points, a value of the slope and a point, or a value of the slope and the y-intercept.

4.4. Solving systems of linear equations graphically and algebraically

To review the concepts learned from the previous lesson, students will work on groups and demonstrate, given two points on the line, how to solve for the slope, the y-intercept, the equation of the line, and then graph the line (HTS.2.5, HTS.7.6). They will present their work to the class (HTS.2.7), and I will give corrections and further explanations when needed. The solution of systems of linear equations will be first defined graphically as the point where all the lines intersect. Because the point where all the lines intersect can be found in all the lines, then the point of intersection must satisfy all the linear equations. Therefore, to solve for the point where all the lines intersect, I have to find a value of the independent variable which will result to the same value for all the dependent variables. I will then demonstrate how to find the intersection algebraically by equating all the equations

and solving for the value of the independent variable. Thus, the solution of linear equations is the ordered pair which satisfies all the linear equations.

4.5. Application: Break-even analysis (HTS.5.4) and discussion of the performance assessment

To review the concepts learned from previous lessons, two students, given two points each which are assumed to lie on the same line, will demonstrate how to find the slopes of the line, the *y*-intercepts, and the equations of the lines. The students will graph their lines in the same Cartesian plane. The students will find the solution of the linear equations or the intersection of the lines in groups, and they will present their solutions to the class (HTS.2.5, HTS.2.7, HTS.7.6). I will give them two equations where the independent variable is in terms of the volume of sales and the dependent variable is in terms of currency. One equation gives the cost of producing a product, while the other equation gives the revenue of selling the product. The break-even point is the point where the revenue equals the cost. Fixed cost, variable costs, selling price per unit, revenue and profit will be defined. Fixed cost is the cost that remains constant even if the company doesn't produce any products. Variable cost is the cost that depends on volume of the product the company produces. The selling price per unit is the price of a unit of product the company is selling. The revenue is the amount of income the company earns for selling a volume of a product at the selling price per unit. On the cost equation, the fixed cost is the *y*-intercept, and the variable cost is the slope of the equation. On the revenue equation, the selling price per unit is the slope of the equation. Its *y*-intercept is zero because the company will never earn revenue without selling a product. Profit is the revenue less cost.

During this lesson, I will discuss with the students the performance assessment they will do and how they will be graded. As I discuss the instructions with them on section 5.1, I will perform the example given in the section on the board. I will also demonstrate to them how complete their solutions should be of the break-even analysis. I will also discuss with them checklists and rubrics explained in section 6.

4.6. Oral presentations and visual aids (HTS 9.9)

I will invite the students' language arts and fine arts teacher to teach a lesson either on my class or theirs. The language arts teacher will teach the students some guidelines on giving oral presentations in business settings. The fine arts teacher will teach the students some guidelines on creating visual aids for business presentations.

STUDENT PACK

5. The Performance Assessment: A Presentation of a Break-Even Analysis (GLO.4.3)

5.1. Break-even analysis

Please perform the following instructions outside of class time (GLO.1.2). However, if you have any questions and clarifications, you can raise your question in class before reviewing the concepts from the previous lesson begins.

Decide on a service that you can provide and charge other people for. Decide also how long you will offer your service (GLO.3.3). For example, I decided to offer tutorial services for three months. You should be able to provide the service on your own with minimal help from other people. You cannot use any of the values or their multiples in the examples, and submit them as your own work.

Determine the costs you might incur in providing your service. You also need to determine whether each cost activity or item incurs a fixed cost or a variable cost. For example, I decided to spend US$14 for advertisements and US$26 for a separate phone line every month. I also decided that each tutorial session is an hour and a half long. Every time I have a tutorial session, I spend US$4.00 for bus fare. Thus, the fixed cost for the whole life of my business is US$120, and my variable cost is US$4 per tutorial session. Tabulate your fixed costs and variable costs in a table similar to *Table 5.1*.

Table 5.1 Fixed Costs and Variable costs

FIXED COSTS		VARIABLE COST	
DESCRIPTION	COST	DESCRIPTION	COST
Advertisements (US$14 x 3 mo)	US$ 42.	Bus fare per tutorial session	US$ 4.
Phone bill (US$ 26 x 3 mo)	78.		
TOTAL FIXED COST	US$ 120.	TOTAL VARIABLE COST	US$ 4.

The reason why I want you to choose a service that you can provide on your own is because I want to make it easy for you to figure out your costs. If my father were to drive me to the place where I would tutor, and if he did not want me to pay him because he loves me, how would I assign a cost to my father driving me? In business settings, when a service is rendered, a cost is always incurred and the cost must be accounted for. Thus, if the service you chose requires extensive help from other people and they refuse to be paid because they love you, then you cannot possibly account for the cost of their service. If help from other people is essential for you to perform your service, then you must assign a cost for the help you receive.

Decide a price that you want to charge your customers. Ask people who provide a similar service their rates. You may also ask possible customers how much they are willing to pay you if they were to hire you (GLO.2.1). Your price must be reasonable; otherwise no one will hire you (GLO.2.5). Furthermore, your price must also be higher than your variable cost, because it is not profitable to provide a service where the cost is higher than the revenue. It must be expressed per unit, and in the same unit as your variable cost. For example, I decided to charge my customers US$25/session.

Formulate your cost and your revenue functions (Benchmark MA.Al.10.4).

Graph your cost and your revenue functions in the same Cartesian plane (Benchmark MA.Al.8.1). The units of service performed must be plotted on the horizontal axis, while the vertical axis is in currency units. Properly label which line represents which relationship, and the location of the vertical line-intercepts and their values. For example, see *Figure 5.1*.

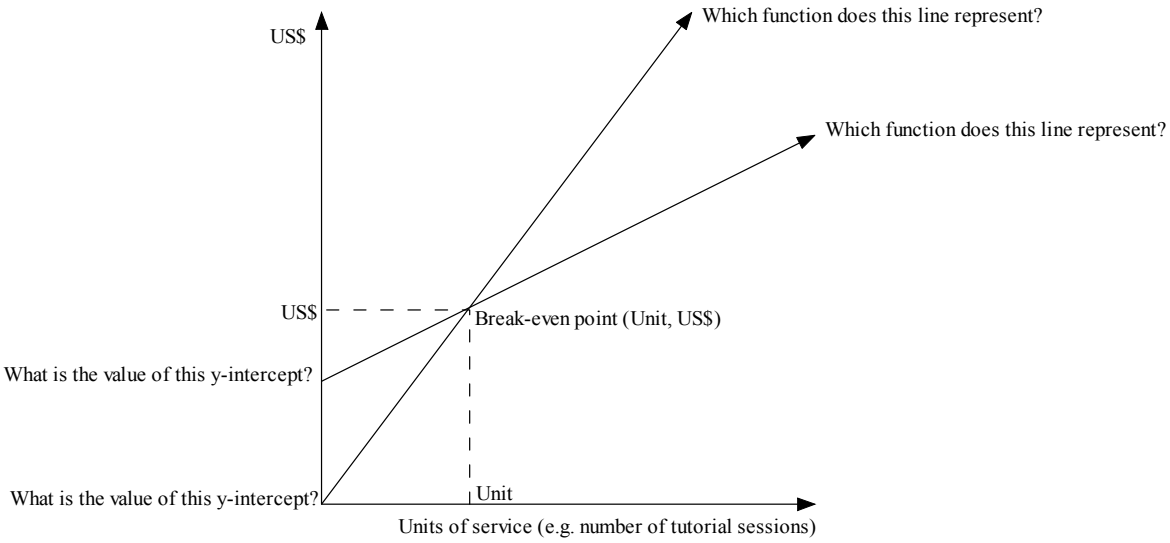


Figure 5.1

How many units of service do you need to break-even? In the example given, this question translates to, "How many tutorial sessions do I need to break-even?" Label the break-even point on the graph, the break-even unit of service on the horizontal axis and the amount of currency the break-even occurs on the vertical axis (Benchmark MA.Al.10.5). Decide on a target profit for the duration of the business (e.g. target profit = US$300). How many units of service do you need to reach that target profit? This question translates to, "How many tutorial sessions do I need to have a profit of US$300?" Decide on a target unit of service for the duration of the business (e.g. target tutorial session = 20). How much profit will you have at this target? If we were to use the example given, this translates to, "How much profit will I have if I were to do 20 tutorial sessions?"

You will write your solutions on paper, prepare visual aids, and then orally present the results of your break-even analysis to me in a scheduled personal conference.

5.2. Written solutions

Graph the functions as complete as Figure 5.1, and write your solutions to those questions in the paragraph two paragraphs above on 8.5" x 11" unlined white papers. Divide

your paper into two columns, and write within a column from left to right, then top to bottom. Use only one side of the sheet (i.e. do not write on the paper front-to-back). Your solutions are complete if you showed how you operated on the functions (i.e. added, subtracted, multiplied, divided, equated, etc.), how you isolated and solved for the independent variable, and how you solved for the dependent variable. Write your names and your schedule for this class on the top-right corner of each sheet of paper you used.

5.3. Outline of your visual aids

Your visual aids can be a slideshow presentation or display boards. The first part of your visual aids should contain which service you will provide and for how long (e.g. A tutorial service with a business life of three months). It also contains your fixed cost and variable cost tables (e.g. *Table 5.1*). Include your price per unit of service in this part of your visual aids as well (e.g. US$25 per tutorial session). The second part of your visual aid must contain your cost and your revenue equations. Make sure you label which equation is which. The graph of your cost and your revenue equations should also be in this part. This graph should be as complete as *Figure 5.1*. The last part of your visual aids should contain your answers to the questions on the last paragraph of section 5.1.

Make your visual aids neat, business-like and simple. Your background should be of a light and solid color. Your letters and numbers should be approximately an inch tall, and should be of dark and solid colors, including the figures. Do not use more than eight colors in your visual aids. Margins should be proportional. Your visual aids will not affect my judgment on your competency, but it can increase your grade for this assessment.

5.4. Description of the presentation (Benchmark LA.9.6.2, GLO.5.2)

On your presentation, you will explain to me why you chose your service, how you came up with your fixed and variable costs, and why you are charging your price. You will also explain to me how you formulated your cost and revenue equations. I will also ask you to recreate your solutions for the break-even point and answers on the last part of your visual aids. You will be allowed to use your written solutions as notes during the presentation. Study your material thoroughly before presenting. I will ask you questions.

6. Grading

You will be graded according to the checklists and rubrics below. You must get at least a "1" in each item in the checklists below to be competent.

During the oral presentation, did you

Correctly formulate the linear functions?	No (0)	Yes (1)
Were *y*-intercepts correctly identified from the costs?		
Were slopes correctly identified from the costs & price?		
Were the cost and the revenue functions correctly formulated?		

Correctly identify the parts of your graph, or correctly recreated your graph?	No (0)	Yes (1)
Were axes correctly assigned and identified?		
Were the graphs of the functions correctly identified or recreated on the Cartesian plane?		
Were *y*-intercepts of the cost and the revenue functions correctly pointed out on the graph?		

Correctly explained how to solve the system of linear equations?	No (0)	Yes (1)
Were the functions operated correctly? (i.e. To solve the system of linear equations, will you add, multiply, etc. the functions?)		
Was the process of solving for the independent variable correctly outlined?		
Was the process of solving for the dependent variable correctly outlined?		
Was the coordinate of the solution to the system correctly pointed out on the graph?		

The rubrics and checklists below will increase your grade only after you got a "1" on each item in the checklists above.

	(1)	(2)
Visual aids	Visual aids are complete	Visual aids are complete and follow the correct sequence
	Background is of a light and solid color; letters and numbers are approximately an inch tall, and are of dark and solid colors, including the	Background is of a light and solid color; letters and numbers are approximately an inch tall, and are of dark and solid colors, including the figures; less than nine colors;

	figures; less than nine colors; margins are proportional	margins are proportional; theme was selected to unify the visual aids

Graph of linear functions	No (0)	Yes (1)
Were *y*-intercepts correctly labeled?		
Were the functions correctly labeled?		
Were the axes correctly labeled?		
Was the solution to the system correctly labeled and its coordinates attached?		

Written solution	No (0)	Yes (1)
Are all the components (the graph, and the solutions to the break-even point, the units of service given a target profit, and the profit given the target units of service) there?		
Is the formulation for the solution of the units of service given a target profit correct?		
Is the formulation for the solution of the profit given the target units of service correct?		
Is the solution of the units of service given a target profit correct?		
Is the solution of the profit given the target units of service correct?		
Is the units of service given a target profit correct?		
Is the profit given the target units of service correct?		

Appendix

1. General Learner Outcomes

The students will meet these General Learner Outcomes in this performance assessment:

GLO.1.2. Plans and manages time and resources to achieve goals.

The preparation for the presentation is outside of class hours. Thus, the students learn to plan their preparation and manage their time.

GLO.2.1. Respects people's feelings, ideas, abilities and cultural diversity.

The students ask other people so that they can come up with a price for their service. As they ask various people, they will learn to respect people's ideas and cultural beliefs about pricing.

GLO.2.5. Demonstrates responsible and ethical behavior in decision making

The price that the students choose will reflect their ethical behavior. Quality of service, price of competition, and demand for service will influence their decision on price. They may set a higher price because they believe that they can provide a higher quality of service, or they may set a price well below competition to gain market dominance. These are some examples of how their pricing decision reflects ethical behavior.

GLO.3.3 Generates new and creative ideas and approaches to developing solutions.

The students generate new and creative ideas and approaches as they think of services they can provide.

GLO.4.3. Produces evidence that meets or exceeds Hawaii Content and Performance Standards

This assessment was designed so that students meet the content and performance standards when they perform the minimum requirements of this assessment.

GLO.5.2. Communicates effectively and clearly through speaking, using appropriate forms, conventions, and styles to convey ideas and information for a variety of audiences and purposes.

The student presentation meets this learner outcome.

2. Teacher Standards

These Hawaii Teacher Standards were met in the execution of the lessons:

HTS.2.5. Provides learning experiences which actively engage students as individuals and as member of collaborative groups.

When students work in groups, the members of the group help each other remember the concepts previously learned, and help members of the group who had difficulty understanding the concepts.

HTS.2.7. Engages students in different modes of communication.

The students explain their work on the board and work in groups. When they engage in these activities, they switch between a dialogue and a monologue.

HTS.5.4. Connects knowledge of content area(s) to students' prior experiences, personal interests and real-life situations.

The concepts of slope, *y*-intercept, equation of lines, graph of lines, solution of systems of linear equalities, and intersection of lines are applied to break-even analysis, which is one of a supervisor's tasks.

HTS.6.1. Plans and implements logical, sequenced instruction and continually adjusts plans based on learner needs.

The lessons are sequenced in such a way that previous concepts learned are used to develop new concepts. Reviews are done at the beginning of the lesson to informally assess if concepts need to be retaught or clarified.

HTS.7.2. Helps students to question, problem-solve, access resources, use information to reach meaningful conclusions and develop responsibility for their own learning.

I help students to question their concept of "steepness" of a line by finding a ratio that would compare the "steepness" of a line to another line. When they find out how to properly represent the ratio, then they have also developed on their own the concept of the slope of a line.

HTS.7.3. Provides challenging learning experiences which develop higher order thinking skills.

Developing the ratio of the slope correctly is challenging for students, which develops their analysis, evaluation and synthesis skills.

HTS.7.4. Varies instructional roles (e.g., instructor, facilitator, coach, co-learner, audience) in relation to the content and purpose of instruction and students' needs.

Throughout the lessons, I switch between the roles of an instructor, a facilitator and a member of the audience. I become an instructor when I demonstrate. I facilitate the development of concepts when I ask them to develop the ratio of the slope. I become a member of the audience when they present their work on the board.

HTS.7.6. Provides opportunities for students to apply and practice what is learned.

Every time I ask the students to demonstrate how to solve for or find the slope of the line, the *y*-intercept, the equation of the line, and the graph of the line when given two points, the students have opportunities to apply and practice what they learned. They have another opportunity when they are asked to solve for the intersection of two lines.

HTS.9.9. Works collaboratively with other professionals.

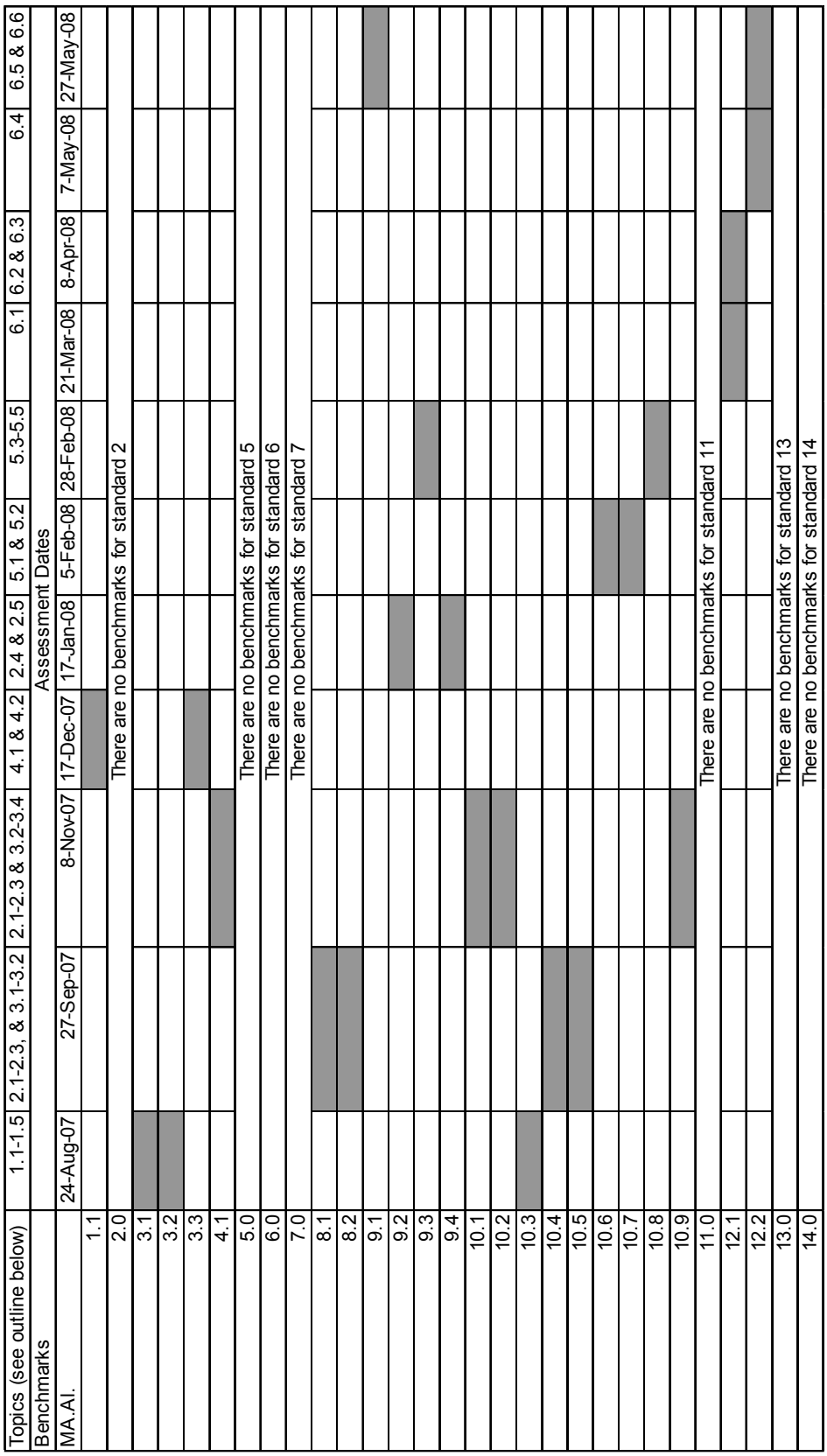
I collaborated with the language arts and fine arts teachers in teaching a lesson on business presentations.

3. Integrated standard

Standard LA.9.6 and Benchmark LA.9.6.2 is also met in this assessment when the students present their break-even analyses.

> Standard LA.9.6. Oral Communication: CONVENTIONS AND SKILLS: Apply knowledge of verbal and nonverbal language to communicate effectively in various situations: interpersonal, group, and public: for a variety of purposes.
>
> Benchmark LA.9.6.2. Give a planned oral presentation highlighting a main idea(s) with support (e.g., statistics, anecdotes, examples).

4. Matrix of Organization



The matrix of organization above organizes which benchmarks the students will be assessed when, and which topics in the content area will meet which benchmarks. The outline below gives the idea of how a topic will meet the indicated benchmarks in the matrix. The matrix also tells which topics an assessment will cover at a specific date.

Unit 1 Foundations of Algebra.
- 1.1. *The real number system and line*
- 1.2. *Introduction to algebraic expression and polynomials*
- 1.3. *Review of radicals*
- 1.4. *Integer and rational exponents*
- 1.5. *Solutions to linear equations and inequalities in one variable*

Unit 2 Functions.
- 2.1. *Review of rectangular coordinate systems*
- 2.2. *Function and function notations*
- 2.3 *Graphs of linear functions*
- 2.4. *Exponential functions and their graphs*
- 2.5. *Direct and inverse variation*

Unit 3 Systems of linear equalities and inequalities.
- 3.1. *Systems of linear equations*
- 3.2. *Solving by elimination*
- 3.3. *Applications*
- 3.4. *Systems of linear inequalities in two variables*

Unit 4 Matrices.
- 1.1. *Matrices and linear systems*
- 1.2. *Matrix operations*

Unit 5 Roots of Polynomials.
- 5.1. *Review of algebraic expressions and polynomials, properties of real numbers, equalities and inequalities, and laws of integral and rational exponents*
- 5.2. *Factoring*
- 5.3. *Polynomial division and synthetic division*
- 5.4. *The remainder and factor theorems*
- 5.5. *Factors and roots*

Unit 6 Probability and Statistics
- 6.1. *Measures of central tendency*
- 6.2. *Standard deviation*
- 6.3. *Range*
- 6.4. *Stem-and-leaf plots and box-and-whisker graphs*
- 6.5. *Scatter plots and correlation*
- 6.6. *Finding lines of best fit*