On Chern-Heinz inequalities

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Corollary: *If $r=\infty$ and H is constant then $H=0$.*

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(b) If $\quad S \geq c > 0 \quad$ then

$$\sqrt{c} \leq \text{Vol}_{n-1}(\partial\Omega)/\text{Vol}_n(\Omega)$$

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Corollary: *The graph of* $f : R^n \to R$ *has constant mean curvature* H *or nonnegative constant scalar curvature* S *if and only if* $H = 0$ *or* $S = 0$ *respectively.*

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If $G(f)$ has parallel mean curvature vector $H_{G(f)}$

then, for every oriented compact domain $\Omega \subset M$, one has

$$\|H_{G(f)}\| \leq \mathrm{Vol}_{n-1}(\partial\Omega)/\mathrm{Vol}_n(\Omega)$$

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where $H_0^1(\Omega)$ is the completion of C_0^∞ with respect to the metric $\|\varphi\|_\Omega^2 = \int_\Omega \varphi^2 + \int_\Omega |\nabla \varphi|^2$.

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When Ω is bounded with piecewise smooth boundary then $\lambda^*(\Omega)$ coincides with the first eigenvalue of the Laplacian.

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Theorem (Cheeger): *if Ω is bounded with smooth nonempty boundary then*

$$\lambda^*(\Omega) \geq h(\Omega)^2/4 > 0$$

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- *If* $|H| \geq c > 0$ *then* $c \leq h(\Omega) \leq 2\sqrt{\lambda^*(\Omega)}$

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Choose a unit normal vector field η to $G(f)$ so that the mean curvature vector is $H\eta$ with $H > 0$. This is possible since $|H| \geq c > 0$.

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Parallel translate $G(f)$ vertically to produce a smooth codimension 1 foliation $\mathcal{F}$ of $\tilde{\Omega} = \Omega \times R$.

Observe that, according to Cheeger, we have that

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Consider the trunked cylinder $\Omega \times [-a, a]$ and the restriction of the vector field η to it.

Let A be an open connected subset of Ω with compact closure and smooth boundary, and consider $A \times [-a, a] \subset \Omega \times [-a, a]$

By the divergence Theorem we have

$$\int_{A\times[-a,a]} \operatorname{div}\eta = \int_{\partial(A\times[-a,a])} \langle \nu, \eta \rangle$$

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If $\quad S \geq c > 0 \quad$ *then* $\sqrt{c} \leq h(\Omega) \leq 2\sqrt{\lambda^*(\Omega)}$.

Proof

Let $\{e_1, \ldots, e_n\}$ be an orthonormal basis for $T_pG(f)$.

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Gauss equation gives

$$K(e_i, e_j) = \bar{K}(e_i, e_j) + \langle B(e_i, e_i), B(e_j, e_j)\rangle - |B(e_i, e_j)$$

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a) *If $G(f)$ has constant mean curvature H then it is a minimal hypersurface. Furthermore, if M is closed then f is constant.*

b) *If M has non-positive sectional curvatures and $G(f)$ has constant nonnegative scalar curvature S then $S = 0$.*

Remark

The class of Riemannian manifolds M with $\lambda^*(M) = 0$ is quite large. It contains all closed Riemannian manifolds and all complete noncompact Riemannian manifolds with Ricci curvature asymptotically nonnegative.

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Recall that M has Ricci curvature asymptotically nonnegative if $Ric_M(x) \geq -\psi(\text{dist}_M(x_0, x))$, for a continuous function $\psi : [0, \infty) \to [0, \infty)$ with $\lim_{t\to\infty} \psi(t) = 0$, $x_0 \in M$.

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Let $f : M \to R$ *be a* C^2 *function e represent by* H *the mean curvature of* $G(f)$*.*

If $|H| \geq c > 0$ *then* $k = a^2$ *for some* a *satisfying* $(n-1)|a| \geq c$*.*

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Suppose that the mean curvature H_F of each leaf $F \in \mathcal{F}$ computed with respect to η is positive and bounded from zero.

Then

$$2\sqrt{\lambda^*(\Omega)} \geq \inf_{F\in\mathcal{F}} \inf_{x\in F} H_F(x)$$

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where X represents smooth vector field on $\tilde{\Omega}$ such that $\inf_{\tilde{\Omega}} \mathrm{div} X > 0$ and $\sup_{\tilde{\Omega}} |X| < \infty$.

They have shown that $\lambda^*(\tilde{\Omega}) \geq c(\tilde{\Omega})^2/4$

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Changing η by $-\eta$ if necessary we may

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a) *There is no transversely oriented codimension-one C^2 foliation $\mathcal{F}$ of M for which $|H|_F > c_F > 0$ for each $F \in \mathcal{F}$.*

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Final Comments

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Question: *Can one extend this result for R^n with $n < 8$?*

Question: *Would be true that the leaves of such foliations in R^n are always graphs of functions?*

Thank you.

References

Heinz, E.: *Über Flächen mit eindeutiger projektion auf eine ebene, deren krümmungen durc ungleichungen eingschränkt sind.* Math. Ann. 129, 451-454 (1955).

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