ON THE DIFFERENT APPROACHES FOR CAPITAL ALLOCATION

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Abstract

The concept of economic capital (EC) refers to the amount of capital a financial institution is supposed to set aside in order to prevent that its net asset value fall below a certain 'catastrophic level'. One then associates EC with the idea of a protection buffer for unexpected losses that might be incurred by the conglomerate. Traditionally one defines the EC with a certain confidence level (say 99.95%) of the loss distribution. The problem we will address is the computation of the total credit risk component of the economic capital and how to allocate it among the different entities of a financial conglomerate. The entities might be seen as business lines, portfolios or even whole institutions of a financial conglomerate. The model used to generate the loss distribution uses Monte Carlo (MC) simulation. For the time being, there are several models currently available in the literature for the allocation of the EC of a conglomerate among its different Business Lines. In this presentation we will present numerical results of a comparison between the main approaches, enlightening the drawbacks and advantages of each of them.

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1. INTRODUCTION

The concept of *economic capital* (EC) refers to the amount of capital a financial institution is supposed to set aside in order to prevent its net asset value falling below a certain level that would have an impact on its normal operation. It is supposed to function as a buffer for any unexpected losses (ULs) that might be incurred by the institution.

On the regulatory side the Basel II framework has forced banks to use methodologies that link EC allocation techniques with risk. In addition, supervisors will be closely monitoring the procedures the banks will have put in place to deal with economic capital on Banking Supervision (BCBS). Moreover the increasing competition and its pressure on business margins have brought up the problem of efficient EC allocation among the different entities and business lines of a financial conglomerate. Directly related to the problem of efficient EC allocation is the problem of measuring the diversification benefits and risk adjusted contributions for individual positions and business lines taking into account the whole portfolio.

Several approaches have been proposed to calculate risk contributions at position and entity levels. Tasche (2004) for example shows that the only suitable way to measure performance is by defining a risk contribution as the derivative of the risk measure in the direction of the asset weight. Kalkbrener et al. (2004) on the other hand compares the expected shortfall measure with the classically used $VaR/CoVar$ approach. Recently Govaerts et al. (2005) proposed algorithm in which one uses both the whole loss distribution of the portfolio and the standalone distributions of the individual sub-portfolios to allocate EC. The problem this paper proposes to address is the one of comparing some of the different methodologies largely used in practice by market participants for allocating EC among the different entities of a financial conglomerate. Given the complexity of the task behind this work this paper should be seen as the first on a series of research articles in which the end goal is to present an approach for using such a system for active credit ALM portfolio management. In this scope a full discussion of the methodologies with a detailed explanation of the differences on the portfolios and positions levels is out of the scope of this article.

The paper will be structured as follows. In section 2 and section 3 we describe the portfolio model used and some of the different risk measures used in the market respectively. In section 4 we test those measures in a typical banking portfolio and give comments on the differences. In section 5 we condlude the article with a resume of the differences and give a hind of the results of the forthcoming research.

2. THE PORTFOLIO MODEL

In order to be as realistic as possible with what is done in practice we have made our analysis on a one-period framework using a model (see Gupton et al. (1997) for details) commonly adopted by practitioners that also includes ratings migrations. In such a model the credit portfolio will consist of bonds whose returns Y_i are given by:

$$Y_i = \alpha \cdot \sum_{i=1}^{2} \beta_i \cdot Z_i + \sqrt{1-\alpha^2} \cdot \xi_i \tag{1}$$

with α representing the average correlation between the bonds and the systematic risk factors ((Z_1, Z_2): the market) which we suppose to be two: an industry and a country; and ξ_i being the idiosyncratic risk term ($\mathcal{N}(0,1)$: unidimensional gaussian distributed with mean zero and standard deviation one). The loss distribution of the portfolio will be given by:

$$L = \sum_{i=1}^{n} L_i \tag{2}$$

with the individual losses L_i given by one year forward changes in prices of each position due to rating migrations.

The rating migrations are determined using a standard Gaussian copula algorithm for the systematic factors and a standard uni-dimensional Gaussian for the idiosyncratic factor. The correlation between the market factors have been taken from the equity markets as explained in de Servigny and Renault (2003) [2]. The transition probability matrix (TPM) is a historical one and some standard adjustments have been done in order to compensate for some ratings incoherences.

3. ECONOMIC CAPITAL AND RISK MEASURES

Once the portfolio loss distribution has been determined one then uses risk measures for determining the unexpected loss and the allocation of the economic capital.

Assume a loss distribution defined by L and a certain quantile α. The credit value at risk ($cVaR$) and the expected shortfall (ES) associated with the quantile are defined respectively as:

$$cVaR_\alpha(L) = \inf\{x > 0 | P(L \le x) \ge \alpha\} \tag{3}$$

$$ES_\alpha(L) = \mathbb{E}_P[L | L > cVaR_\alpha(L)] \tag{4}$$

Below we will define approaches using standard risk measures that are largely used in practice for allocating economic capital. Assume the conglomerate is comprised of n subportfolios whose allocations we want to determine. The approaches we will be comparing in this paper are the following:

a) $VaR/Covar$: although largely used by practitioners this approach is typical for the case of Gaussian loss distribution. In this approach the allocated capital of a certain subportfolio will be given by:

$$EC_i(\alpha) = \frac{cov\{L, L_i\}}{\sigma_T^2} \cdot EC_T(\alpha) \tag{5}$$

where L_i and L are the losses of sub and the total portfolio respectively. And σ_T^2 and $EC_T(\alpha)$ are the total portfolio loss variance and the EC for the total portfolio (assumed to be $cVaR_\alpha(L)$).

[2] In our case we have used the equity correlations given by Portfolio Risk Tracker (PRT) from S&P.

b) Pro-Rata $cVaR$: In this approach one uses the standalone $cVaR_\alpha$ of each sub-portfolio as a weight in the allocation of the total risk[3]:

$$EC_i(\alpha) = \frac{cVaR_\alpha(L_i)}{\sum cVaR_\alpha(L_i)} cVaR_\alpha(L) \tag{6}$$

c) Basel II: in this approach we use the relative proportions resulted from the Basel II formulas to allocate $cVaR$. Assume for example that Bsl_i is the regulatory capital of portfolio i then the allocated capital for portfolio i will be given by:

$$EC_i(\alpha) = cVaR_T * Bsl_i / (\sum_{i=1}^{n} Bsl_i) \tag{7}$$

The principle behind this approach is to keep Basel II proportions for EC allocation.

d) Marginal Optimization of Total $cVaR_\alpha$ (see Govaerts et al. (2005)): the idea is to search on the standalone loss distribution of each subportfolio the quantile for which the addition of the $cVaR_\alpha$ of the subportfolios would equal the total $cVaR$ of the whole portfolio. One then searches the quantile β on the standalone loss distribution of the subportfolios such that:

$$\beta = \inf\{\beta' \in [0,1] : \sum_{i=1}^{n} cVaR_{\beta'}(L_i) \geq cVaR_\alpha(L)\} \tag{8}$$

then $EC_i(\alpha)$ is defined as

$$EC_i(\alpha) = cVaR_\beta(L_i) \tag{9}$$

e) Credit VaR Contribution via Expected Shortfall: In this approach $cVaR$ is allocated using the concept of Expected Shortfall contribution. The Expected shortfall contribution is defined by:

$$ES_\beta(L_i) = E_P[L_i | L_i > cVaR_\beta(L_i)] \tag{10}$$

The allocation is then given by:

$$EC_i(\alpha) = \frac{ES_\beta(L_i)}{ES_\beta(L)} cVaR_\alpha(L) \tag{11}$$

Observe that the quantiles for the $cVaR_\alpha$ and for the ES_β do not need to be the same. For example a bank might have its $cVaR_\alpha$ depending on a quantile α of (say) 99.97% while allocating it following a quantile β of 99%. I.e. portfolios that are more risky would need more capital. Such decisions are strategic and depend on the policy of the bank.

f) Expected Shortfall that equals $cVaR_\alpha$: in this approach we will be looking to the ES quantile that equals the $cVaR$. Then the allocation will be done using the ES. Assume for example that:

$$\beta = \inf\{\beta' \in [0,1] : ES_{\beta'}(L) \geq cVaR_\alpha(L)\} \tag{12}$$

[3]As measured by the total $cVaR$ that takes into account the whole correlation structure of the portfolio

In this way:

$$EC_i(\alpha) = ES_\beta(L_i) \tag{13}$$

Observe that the main objective of this approach is to eliminate the problem that $cVaR$ is a non-additive measure (see Artzner et al Artzner et al. (1999) for details).

The results of the experiment will be givin in function of the *diversification benefit (DB)* of a portfolio and it is defined as:

$$DB_T = 1 - EC_T / (\sum_{i=1}^{n} EC_i) \tag{14}$$

where EC_T is the total economic capital for the whole portfolio and $cVaR_\alpha(L_i)$ is the stand alone $cVaR$ of the portfolio i composed by n subportfolios. The DB is a measure of the diversification gain one has when the subportfolios are put together in one large portfolio.

4. THE RESULTS

For the tests that follow we have selected 5 portfolios of different sizes, compositions and concentrations. The portfolios chosen have in general good quality and we have made them quite concentrated to show problems practicioners may face. The compositions of the different portfolios in terms of average rating, average maturity and concentration factor (defined as the percentage of issuers with 50% of the portfolio) are shown in table 1. We show the standalone $cVaR_{99.97\%}$ of each portfolio (it is given as a percentage of the total $cVaR_{99.97\%}$ of the whole portfolio).

In terms of sector concentrations we have build the portfolios P1 up to P5 with securities from mainly four sectors (financials, sovereigns, utilities, and some ABS's (not more than 10% of this class)) while portfolio P6 contains ABS's only. When comparing the sub-portfolios, P6 is the most diversified sub-portfolio.

	Avg Dur(yr)	Rating	CF	Amount(%)	Std-Alone VaR
P1	13	AA	5	43.3	36.5
P2	7	A+	7	35.4	56.3
P3	11	A+	11	5.4	13.3
P4	1	AA-	9	10.3	15.6
P5	30	AA	3	5.1	8.7
P6	5	AAA	30	0.5	0.4

Table 1: Composition of the Different Portfolios.

In the present analysis, One used the Moody's transition probability adjusted for some rating imperfections, the correlation function is the one equity markets that comes in PRT (Portfolio Risk Tracker) the credit risk system of S&P (see de Servigny and Renault (2003) for more details). The systematic factor used was calculated via regression using equity data and in this study we will be using 50% for that factor (although the market factor has proved to be quite lower than 45% we have been using 50% for a question of prudence). The result of the tests (in terms of DB) for the different methodologies is shown in table 2.

	P1	P2	P3	P4	P5	P6
a) $VaR/CoVaR$	1.8	19.0	48.9	82.8	-0.10	83.3
b) PRata VaR	23.5	23.5	23.5	23.5	23.5	23.5
c) Basel II	27.0	7.1	36.7	72.7	6.0	64.1
d) Marg. Opt.	15.0	18.7	28.9	46.1	15.7	20.7
e) $cVaR$ contr	11.6	13.3	48.7	75.6	6.1	76.2
f) ES contr	20.2	10.5	48.7	62.0	14.1	80.0

Table 2: Diversification benefit of EC allocation using different measures.

A first observation is about the $Var/Covar$ approach. As already reported elsewhere it can lead to a capital allocation that is higher than its stand alone VaR and in some very special cases even higher than the whole amount of the portfolio. An example of it can be seen in the case of subportfolio P5.

The simple ProRata approach has the characteristic of deviding equally the DB among the subportfolios independent of the correlation within the subportfolios. As it is seen for subportfolio P6 this approach can have a negative impact on small subportfolios that would present ideal diversification characteristics with respect to the remaining portfolio. The Basel II approach has also the charateristic of simplicity (as the numbers are anyway available in the internal systems of most banks). The problem with this approach is that the correlation underlying Basel II formulas does not necessarily represent the correlation among the sub-portfolios under study (see e.g. the allocation given by the ES contrib approach).

The Marginal Optimization uses the stand alone loss distribution for allocation and that distribution does not take into account the correlation among the subportfolios. This is again evidenced by the allocation given to subportfolio P6. The $cVar$ and the ES contribution approaches account quite well the DB brought by subportfolio P6.

The $cVaR$ contribution methodology permits one to transfer risk among the subportfolios in a way that risk generated in low risk subportfolio is allocated to a higher risk one. Although it brings up the issue that the quantile used for allocation (99% in our case) is certainly arbitrary and certainly depends on management decisions. The ES contribution has the problem that one does not know in advance which quantile (β on the equation 12) one will need to take, implying that one will need to make a couple of simulations to determine it (what can be time consuming).

The first three approaches have the advantage of simplicity at the cost of loosing important insights when allocating the DB. The Marginal Optimization method represents an increase in mathematic complexity. For communication purposes within subsidiaries and business lines it can be quite convinient: the standalone VaR is certainly available at the subportfolio levels and the holding would only need to pass the information on the specific quantile for allocation purpose. The disadvantage is the loss in correlation among the subportfolio when deciding the allocation. The $cVaR$ and the ES contributions both use ES factors for allocation purposes. The $cVar$ contribution brings up the quite (politically) sensitive issue of determining the quantile for allocation purpose. The ES contribution has the additional complexity of needing preliminary simulations to determine the allocation quantile.

5. CONCLUSIONS

In this paper we have shown the impact of different capital allocation methodologies for subportfolios of a large conglomerate and the for individual positions. We have discussed six methodologies, three quite simple and streight forward: the $VaR/CoVar$, the ProRata and the Basell II (factors); and three more complex ones: Marginal Optimization, $cVaR$ contribution and ES contribution. The methodologies were tested on the problem of allocating risk as measured by $cVaR$ at the 99.97% quantile on six subportfolios.

The first three approaches have the advantage of simplicity at the cost of loosing important insights when allocating the DB. The Marginal Optimization method represents an increase in mathematical complexity. For communication purposes within subsidiaries and business lines it can be quite convenient: the standalone $cVaR$ is certainly available at the subportfolio level and the holding would only need to pass the information on the specific quantile for allocation purpose. The disadvantage is the loss in correlation among the subportfolio when deciding the allocation. The $cVaR$ and the ES contributions both use ES factors for allocation purposes. The $cVar$ contribution brings up the quite (politically) sensitive issue of determining the quantile for allocation purpose. The ES contribution has the additional complexity of needing preliminary simulations to determine the allocation quantile.

A continuation of this study includes building tables showing economic capital consumptions per position, rating, and sector. Those tables are put in the context of a portfolio management approach to an ALM credit desk. In order to avoid loosing resolution on a position level and being able to build a scenario analysis framework for the whole portfolio of the financial conglomerate a paralel system (with up to 25 machines) has been put in place being able to handle large amounts of positions in very short time. Additionally an innovative importance sampling algorithm has been implemented to improve the performance of the system. This study which represents the continuation of what has been shown in this paper will be published in brief.

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