On Rating Cash Flow CDO's using BET technique

J. Garcia, T. Dewyspelaere, R. Langendries, L. Leonard and T. Van Gestel

12th Version Oct 8th 2004

This paper begins with a short explanation of what collateralized debt instruments (CDO's) are and how investors might use them. We give detailed descriptions of the waterfall and the binomial expansion technique (BET) first proposed by developed by Moody's. We then test how the model parameters affect the rating of the different tranches of a real contract (a cash flow CDO).

Keywords: Collateralised Debt Obligations, BET, Cash Flow CDO's, Rating Structured deals.

1) Introduction

The market for the Collateralized Debt Obligation (CDO) has grown from around 4 bi USD by 1996 until 137 bi USD by 2001 (see Backman et alli [Backman(1995)] and the references therein). One can see them as belonging to the class of asset backed securities (ABS). They are instruments whose cash flows are backed or collateralized by pools of assets.

There are currently at least six models to deal with a CDO note: the Binomial Expansion Technique (BET) and the Fast Fourier Transform algorithm (FFT) developed by Moody's (see Cifuentes et alli [Cifuentes(1996)] and Debuysscher [Debuysscher(2003)]), a copula approach developed by S&P (see Bergman [Bergman(2001)]), the Fitch approach (see Bund et al [Bund(2003)]), the copula function approach developed by CreditMetrics (see Mina [Mina(2001)]) and the Portfolio Risk tracker (PRT) approach developed by the Risk Group division of S&P (see Perraudin et alli [Perraudin (2004)] and the references therein for details). The main objective of this paper is to give a quite detailed description of the BET approach showing the sensitivity of rating and expected loss of different tranches to its parameters. Additionally all the evaluations are made using a real CDO contract[1].

This paper contributes to the field by giving a detailed description of the BET model which is largelly used by practicioners on the field. The model is shown in an algorithmic way making it easy to be reproduced in academia. Addionally the paper provides data and a structure in which the parameters of the model are changed showing how they impact the rating of the different tranches. The paper will be structured as follows. In section 2 we give a summary of what are CDO's and for what they might be used. In section 3 we describe the waterfall of a typical CDO contract. In section 4 we give a detailed description of the BET methodology. In section 5 we present a contract and its waterfall and the results of the stress tests made showing the sensitivity to the different parameters used in the BET. In section 6 we finish with a conclusion.

2) A Brief Overview of CDO's

In this section we give a brief overview of the CDO structures. We will restrict the overview to the CDO basics that are necessary to understand the BET model to be explained in section 3. Additionally we refer to Lucas [Luca(2001)] for more detail on the structure of the contracts, to Hill and Vacca [Hill(1999)] for arbitrage CDO's, to Goodman [Goodman(2002)] for synthetic CDO's, to Falcone [Falcone(1998)] for a rating approach on market value CDO's.

A CDO as an asset-backed security (ABS) is set up as a special purpose vehicle (SPV) that invests in a pool of securities to be used as collateral. The interest and principal payments from the collateral pool are allocated to the notes following a certain prioritization schedule. The prioritization schedule is known a priori and is detailed in the prospectus of the CDO (see section 5 for more detail). The collateral can be of very different types of securities: high yield bonds and emerging market debts (CBO's), bank loans (CLO's), ABS/MBS's, CDS's (synthetic CDO's), equity funds or (as more recently) notes of other CDO's.

Depending on the purpose of the CDO issuer there are basically two main classes of CDO's: *balance sheet* and *arbitrage*. In a *balance sheet* CDO a financial institution securitizes loans in its balance sheet with four immediate purposes: a) relieve in capital, b) increase in liquidity of the loans, c) improve performance measurement ratios, and d) transfer risk off balance sheet. In an *arbitrage* CDO one intends to get the difference between the cost of funding and the return on high yield investing. This means that in an arbitrage CDO the yield on the assets has to be higher than the total fees and the cost of funding the instrument. The typical collateral is either high yield bonds or loans.

Depending on the way the collateral pool is managed a CDO may be of two types: *cash flow* and *market values*. In a *cash flow* CDO the manager is not supposed to engage on actively trading the assets in the collateral and there are very strict rules on buying and selling of collateral. Uncertainties in the payments of cash flow CDO's are related to the number and the timing of defaults. In a *market value* CDO on the other hand the payments are determined by gains on the marked to market value of the collateral pool. The gains are mainly determined by the trading performance of the manager. An algorithm for a market value CDO would have to take into account the trading behavior of the collateral manager. In this paper we will be dealing with *cash flow* CDO's only.

A typical CDO structure can be seen in fig 1. The notes to be issued by the SPV are tranched into different rating classes. The rating of each class is determined by the seniority of the note in the schedule of receiving principal and interest payments. Each CDO has a prospectus containing the detailed conditions of the contract. The priority in which cash flows are allocated for each tranche holder is described in the *waterfall* section of the prospectus (see section 3 for a more detailed description).

The senior notes are rated from AAA to A, the mezzanine notes from BBB to B being subordinated to the senior notes. Finally the equity holders generally named the subordinated notes receive the residual (what is left) on the CDO cash flow scheme. The equity holders are supposed to absorb the first losses in the whole structure.

[1] The name of the contract will not be mentioned due to confidentiality agreements.

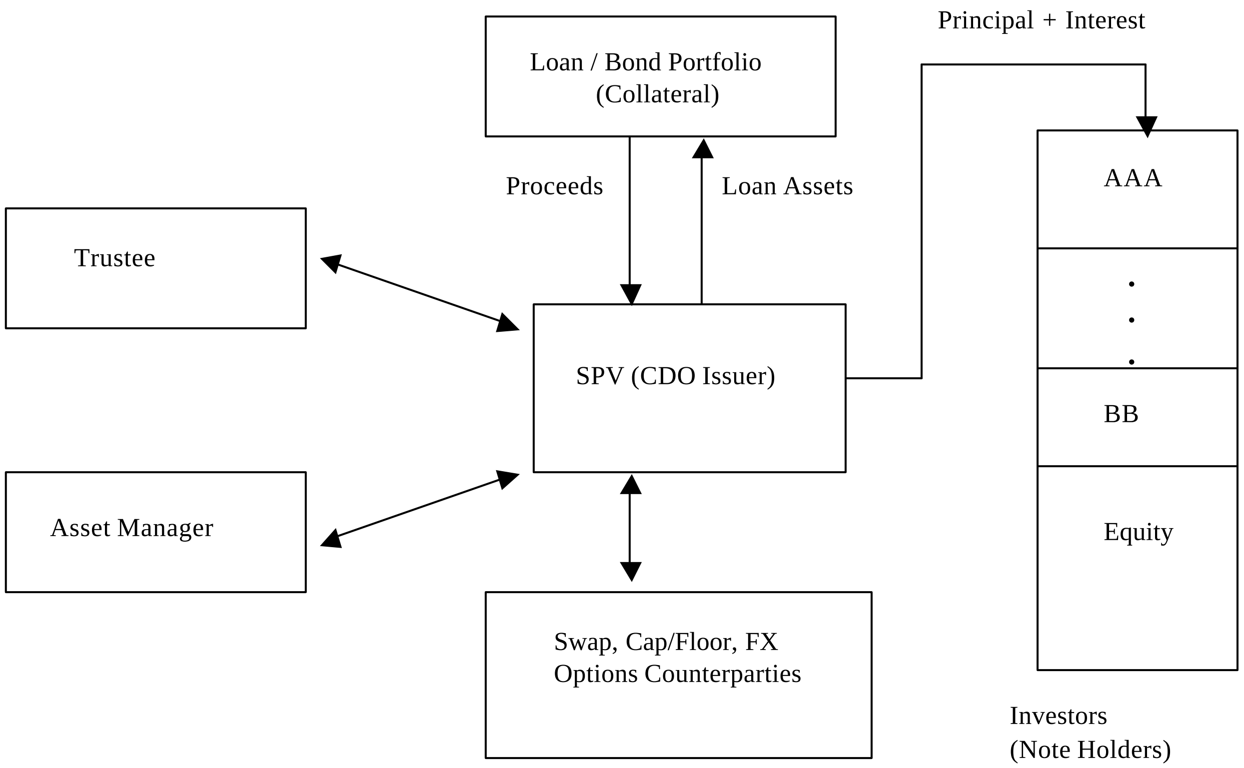


Fig 1 Typical CDO structure

In the next section we give a detailed description of a waterfall structure.

3) The Waterfall of a Cash Flow CDO's

The waterfall shows how the cash coming from the collateral is allocated to the different note holders. We will describe below how a typical cash flow transaction is structured.

Basically a CDO structure consists of a special purpose vehicle (SPV) that issues notes to finance its taking several exposures. The proceeds of the exposures will in turn be used to pay coupons and principal of the notes. The idea behind the SPV, an entity specially created for the purpose of the transaction, is that it is bankruptcy remote, meaning that it is immune to insolvency of the originator.

The SPV can take exposure to the collateral in two ways: a) by completely buying the collateral or; b) by buying one or several credit default swaps (CDS) that give protection to the default of the entities in the collateral. The later is said to be a synthetic structure. In that case the proceeds of the issued notes are used to buy other (almost risk free) securities that will be partly sold if the SPV needs to make a payment on the CDS. Coupons on the notes are paid with interest from the "risk free" assets and premiums received from the CDS. Principal on the notes is redeemed with the proceeds of what is left of the risk-free assets at the end of the transaction. In what follows we will assume the first type of structure: where the SPV owns the "risky" collateral. In that case coupon and principal payments to the notes are made with the interest and principal payments (redemption at maturity or recovery amount in case of default) from the risky collateral.

The notes are tranched in different classes with different risk profiles in receiving interest and principal from the structure and with the accompanying different yields. We show in Tables 1 a typical note structure. For an example of a waterfall see fig. 2 in section 5 (observe that the waterfall can vary in very many ways). One should note that generally most of the notes are concentrated in the senior notes (A's) while the other debt tranches comprise around 5% to 15% while the equity tranche gets 2% to 15%.

Classes	SubClasses	Interest
A	A1A	Fixed
	A1B	Fixed
	A2	Float
B	B1	Fixed
	B2	Float
C	C1	Fixed
	C2	Float
D	D1	Fixed
	D2	Float
E	E	-

Table 1 Tranches in a typical cash flow CDO

The senior tranches (A notes) are the first in line to receive coupon payments (after the payment of fees). The mezzanine notes (B and C) can receive their coupons only if all interests to the senior notes are paid and sufficient protection for these notes remains (senior overcollateralization (O/C) and interest coverage (I/C) tests). The most subordinated note is called the equity tranche or the first loss piece (the tranche E in Table 1). The equity holders will receive the so called excess spread that is what is left after the payment of all the fees and interest to the more senior notes. Normally a large or the whole part of the equity piece is hold by the originator of the CDO. The redemption of the notes follows the same oreder of seniority: first the most senior notes get redeemed, next the mezzanine notes and at the end, if anything remains, the equity notes. Sometimes a structure can have junior notes (the D notes above) between the mezzanine notes and the equity piece.

4) **BET Methodology**

In this section we give a brief description of the BET algorithm. We refer to Cifuents [Cifuentes(1996)] for a first reference on BET. The BET methodology is based on the concept of *diversity score* (DS) and it is an application of the binomial formula from probability theory to a simplified version of the portfolio. The basic idea behind the methodology is to map the portfolio of *heterogeneous correlated* securities with distinct probabilities of defaults into a portfolio of *homogeneous independent* securities each having the same probability of default. The second portfolio is said to be the *idealized* portfolio. Once the DS and the relevant probability of default are determined one applies the binomial formula to give first approximations for the default scenarios.

The first step is the calculation of the diversity score (DS) of the collateral portfolio[2]. The goal of the DS measure is to redefine the pool of correlated heterogeneous assets into a pool of DS independent homogeneous assets. The lower the diversification, i.e. the higher the concentration, the lower should be the DS. Diversification is measured on two levels: first at issuer level by dividing every issuer's exposure by the average issuer exposure and capping the resulting ratio to 1. This gives the Equivalent Unit Score (EUS) per issuer. The more issuers with exposures that are larger than the average (high issuer concentration), the lower will be the average EUS of the issuers. Diversification is also measured at the industry level by summing first all EUS of all issuers in a certain industry (this sum is called the Aggregate Industry Equivalent Unit Score or AIEUS) and then scaling down the AIEUS into an industry diversity score whereby the bigger AIEUS are scaled down more (e.g. if AIEUS(industry i)=1 then DS(industry i)=1, if AIEUS(industry j)=20 then DS(industry j)=5). The DS of the pool is equal to the sum of all industry DS. The higher the average number of issuers per industry (high industry concentration), the lower will be the DS. A detailed computation of the DS of a portfolio is shown in the appendix.

The second step consists in determining the expected cash flows from the collateral for each possible number of homogeneous bonds defaulting. To do this one needs to make some assumptions for:

a) the principal payments from the collateral. All homogeneous bonds are assumed to have the same maturity that is derived from the weighted average life (WAL) of the collateral pool (alternatively one could also take into account the expected principal redemption schedule of the collateral pool);

[2] The Diversity Score algorithm described and used in this work, is the one that, from our experience, is the most widespread. There exists another Diversity Score algorithm that takes into account correlation, face value and default probability (based on rating and maturity). We refer to Cifuents et alli [Cifuents (1998)] for more details on the alternative calculation method and to section 6 of this paper for a brief comment on its usefulness.

b) the interest payments from the collateral. All homogeneous bonds are assumed to have the same coupon, equal to the weighted average coupon (WAC) of the collateral pool;

c) the recovery rate (WAR) in case of default. A standard approach uses one predetermined rate[3] (e.g. 30%). Alternatively one can use a recovery rate that takes into account the types of exposures (secured, unsecured, subordinate, bonds or loans, etc) of every collateral pool. Another common approach is to generate the recovery from a beta distribution.

Assume that N_T is the total notional of the collateral portfolio. The notional involved in the case of j defaults is given by:

$$D_j = \frac{N_T}{DS} \cdot j \qquad (1)$$

and the loss on Dj is given by:

$$Loss_j = D_j \cdot (1 - WAR) \qquad (2)$$

d) the distribution of the losses in time. A standard approach assumes six default patterns:

$$\begin{array}{ll} i. & 50\,10\,10\,10\,10\,10 \\ ii & 10\,50\,10\,10\,10\,10 \\ iii & 10\,10\,50\,10\,10\,10 \\ iv & 10\,10\,10\,50\,10\,10 \\ v & 10\,10\,10\,10\,50\,10 \\ vi & 10\,10\,10\,10\,10\,50 \end{array} \qquad (3)$$

In each of the scenarios above one distributes the losses along 6 years. In i) 50% is lost in the first and 10% in the remaining years. In ii), 10% of the losses occur in the first year, 50% in the second and 10% in the remaining and analogously for iii) up to vi). With the scenarios above one can determine the amount of money left in the collateral.

In the third step one goes through the waterfall and determines the amounts that each note holder will receive (see table 9 in section 5 for an example of a waterfall) and how large is the

[3] This approach uses issue ratings (as opposed to issuer ratings) for determining default probabilities and in doing so different recovery levels for assets with different seniorities are already taken into account.

"present value" (see section 5.1) of their losses in each of the collateral cash flow scenario's generated in step 2. In order to count for interest rate risk a common market practice is to make parallel shifts on the levels of the yield curve. In here we will be considering three possible scenarios: flat; upper 1% and upper 2%.

In the fourth step one uses the weighted average rating factor (WARF) and the weighted average life (WAL) of the collateral pool to determine the probability of default p of one idealized bond[4]. With this probability and the binomial formula one can calculate the probability of each of the collateral cash flow scenario's generated in step 2 (0 up to DS idealized bonds defaulting). Assume for example that in one scenario we would have n defaults (where $n <= DS$). The probability of n defaults (P_n) is given by:

$$P_n = \binom{DS}{n} p^n \cdot (1-p)^{DS-n} \qquad (4)$$

The impact on the senior tranches depends on the tail of the loss distribution of the collateral portfolio. In order to emulate a fatter tail it is a common market practice to stress the probabilities in eq. 4 by multiplying the probability of default p and the recovery rates by a stress factors (> 1 for defaults, < 1 for recoveries). The size of the stress factor depends on target rating (the larger the target rating the larger the default stress factor and the lower the recovery rate stress factor). Suppose e.g. that one is targeting a note to have AA rating. The probability (to be used in eq. 4) and the recovery rate would then be multiplied by 1.4 and 0.67 respectively (see Table 8 in section 5 for more details).

The fifth step consists in bringing together the results for the different default scenario's of step 3 with the probabilities for each of those scenario's calculated in step 4. This leads to a loss distribution and an average ("expected") loss for every scenario.

In the sixth step one compares the expected loss obtained in the step 5 with a "target" loss (this is the idealized loss for a bond with the maturity equal to the average life of the tranche and a rating equal to the target rating), and decides whether or not the tranches passes for the target rating. The rating to be assigned to the tranche is then equal to the highest possible rating for which the tranche passes the test.

The Algorithm above is conceived to determine a rating and not to give one ("the") expected loss percentage or "the" price for a certain tranche. There are at least two reasons for it:

a) the level of losses is impacted by the target rating one wishes to test (through the use of different stress factors): expected losses are higher for a higher target rating.

[4] This is done by using a table which maps rating factors to default probabilities (see Table 17 in the Appendix 2).

b) As seen in (3) the above algorithm will give expected losses for different scenarios. But which one of them (if any) is the “expected” one?

In the next section we give the results of the experiments.

5) **Results**

We will divide this section in two subsections. In section 5.1 we give a detailed description of the CDO contract being rated and in section 5.2 we give the results of the tests made in the contract.

5.1) Contract Description

The data given in here is for a structure that has already started. We have used a real contract in which we have changed the collateral amounts.

In what follows the expected loss (EL) will be evaluated using the following formula:

$$EL = \sum_i CF_i \cdot DF_i \qquad (5)$$

and:

$$DF_i = e^{-(r+s)\cdot t_i} \qquad (6)$$

where *r* and *s* are the risk free and the spread rates respectively. For our case the notes A (see table 3) will be discounted at Libor + 0.65 while the notes D1 will discounted at 11.875% (this means that notes with the same seniority might have different expected losses (see e.g. the results for notes D1 and D2 in section 5). The notes issued in the structure with the current amount still outstanding are shown in Table 2.

Notes	Notional	Coupon Type	Coupon Spread (%)
A	167,494,728	Floating	0.65
B	37,000,000	Floating	1.0
C	42,623,383	Fixed	8.625
D1	8,005,709	Fixed	11.875
D2	17,632,067	Fixed	12.57
E	28,009,384	Fixed	3.0
Total	293,765,271	-	-

Table 2 Tranche notes of the CDO structure

The fees to be paid in the structure are described in Table 3.

Name	Fixed Amount	Fixed Rate (%)	Calculation Basis
Adm. Expenses Flt	-	0.0175	Collateral Balance
Adm. Expenses Fxd	40,000	-	-
Snr. Coll. Mgmt. Fee	-	0.05	Collateral Balance
Sub Col. Mgmt Fee	-	0.45	Collateral Balance

Table 3 Fees to be paid during the lifetime of the CDO

The hedges existent in the structure are shown in table 4.

Type	Notional	Strike (%)	Start Date	Expiry Date	Type
Swap	269,000,000	6.265	23/04/1999	23/05/2004	Payer
Cap	269,000,000	6.25	23/11/2004	23/11/2004	Payer
Cap	266,210,587	6.25	23/05/2005	23/05/2005	Payer
Cap	245,597,044	6.25	23/11/2005	23/11/2005	Payer
Cap	227,863,689	6.25	23/05/2006	23/05/2006	Payer
Cap	182,836,154	6.25	23/11/2006	23/11/2006	Payer
Cap	162,161,261	6.25	23/05/2007	23/05/2007	Payer
Cap	137,639,658	6.25	23/11/2007	23/11/2007	Payer
Cap	105,362,505	6.25	23/05/2008	23/05/2008	Payer
Cap	60,947,263	6.25	23/11/2008	23/11/2008	Payer
Cap	45,039,112	6.25	23/05/2009	23/05/2009	Payer
Cap	29,500,000	6.25	23/11/2009	23/11/2009	Payer
Cap	28,929,107	6.25	23/05/2010	23/05/2010	Payer
Cap	27,041,995	6.25	23/11/2010	23/11/2010	Payer
Cap	26,475,951	6.25	23/05/2011	23/05/2011	Payer

Table 4 Hedge Fees to be paid during the lifetime of the CDO

The ratios used for the OC ratio tests are shown in Table 5. Observe that although we have kept the trigger of the tests as being described by ratios it is also common that besides ratios there might be amounts involved (especially for the senior tranches).

Class	Trigger
A/B	1.2
C	1.07
D	1.03

Table 5 Ratios used in the OC Ratio tests.

The OC tests for the A/B, C and D tranches are given by:

$$OC_{A/B} = \frac{NumeratorOC}{\sum_{i}^{A,B} \Pr incipalNote(i)} \tag{7}$$

$$OC_{C} = \frac{NumeratorOC}{\sum_{i}^{A,B,C} \Pr incipalNote(i)} \tag{8}$$

$$OC_{D} = \frac{NumeratorOC}{\sum_{i}^{A,B,C,D} \Pr incipalNote(i)} \tag{9}$$

where:

$$NumeratorOC = ColBalance + Col\Pr incipal - MarketValueCurrentDefaults \tag{10}$$

and ColBalance means the total notional amount of the performing collateral bonds.

The values for the IC ratio tests we show in table 6 below.

Class	Trigger
A/B	1.2
C	1.07
D	1.03

Table 6 Ratios used in the IC tests.

The IC tests for the A/B, C and D tranches are given by:

$$IC_{A/B} = \frac{NumeratorIC}{\sum_{i}^{A,B} (Interest(i) + DeferredInterest(i))} \quad (11)$$

$$IC_{C} = \frac{NumeratorIC}{\sum_{i}^{A,B,C} (Interest(i) + DeferredInterest(i))} \quad (12)$$

$$IC_{D} = \frac{NumeratorIC}{\sum_{i}^{A,B,C,D} (Interest(i) + DeferredInterest(i))} \quad (13)$$

where:

$$NumeratorIC = CollateralInterest + Fees \quad (14)$$

The characteristics of the pool data are given in tables 7.a and 7.b.

Collateral Amount[5]	257,136,962
DS	34
WARF	2081 (B1)
WAL	5.00
WAC	8.87%
Payment Freq.	Semi-Ann.
Fixed Rate(%)	100
Interest Acc.	8,437,169
Principal Acc.	4,499,582

Table 7a Collateral Information

Rating	Amount	Rating	Amount
Baa2	3	**B1**	11
Baa3	3	**B2**	11
Ba1	12	**B3**	6
Ba2	3	**Caa1**	4
Ba3	5	-	

Table 7b Distribution of bonds in the pool (*all the bonds are Fixed Rate*)

The waterfall structure in the contract is shown in fig. 2.

[5] All the bonds in the collateral are supposed to be fixed rate bonds.

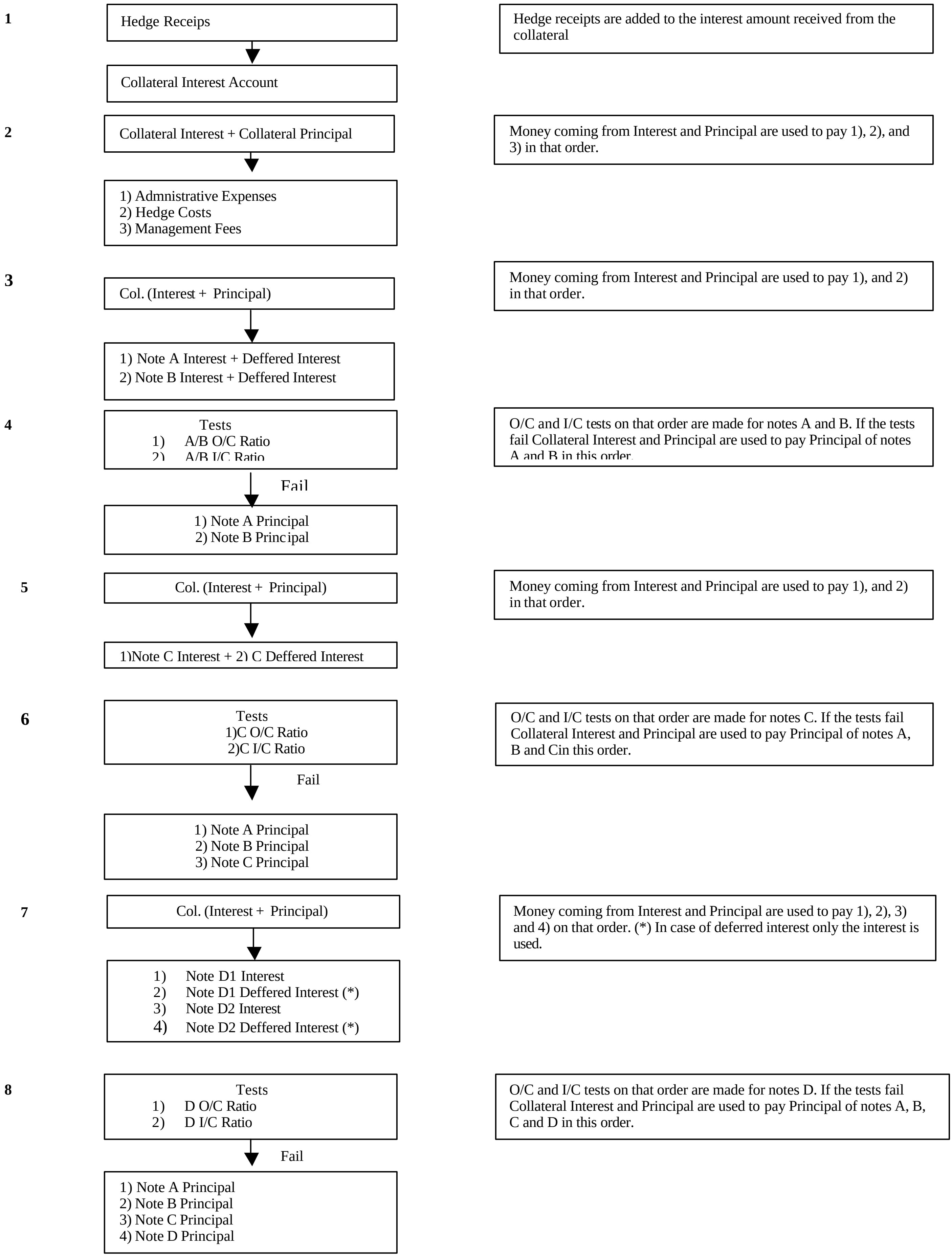


Fig 2 Waterfall structure used in the tests

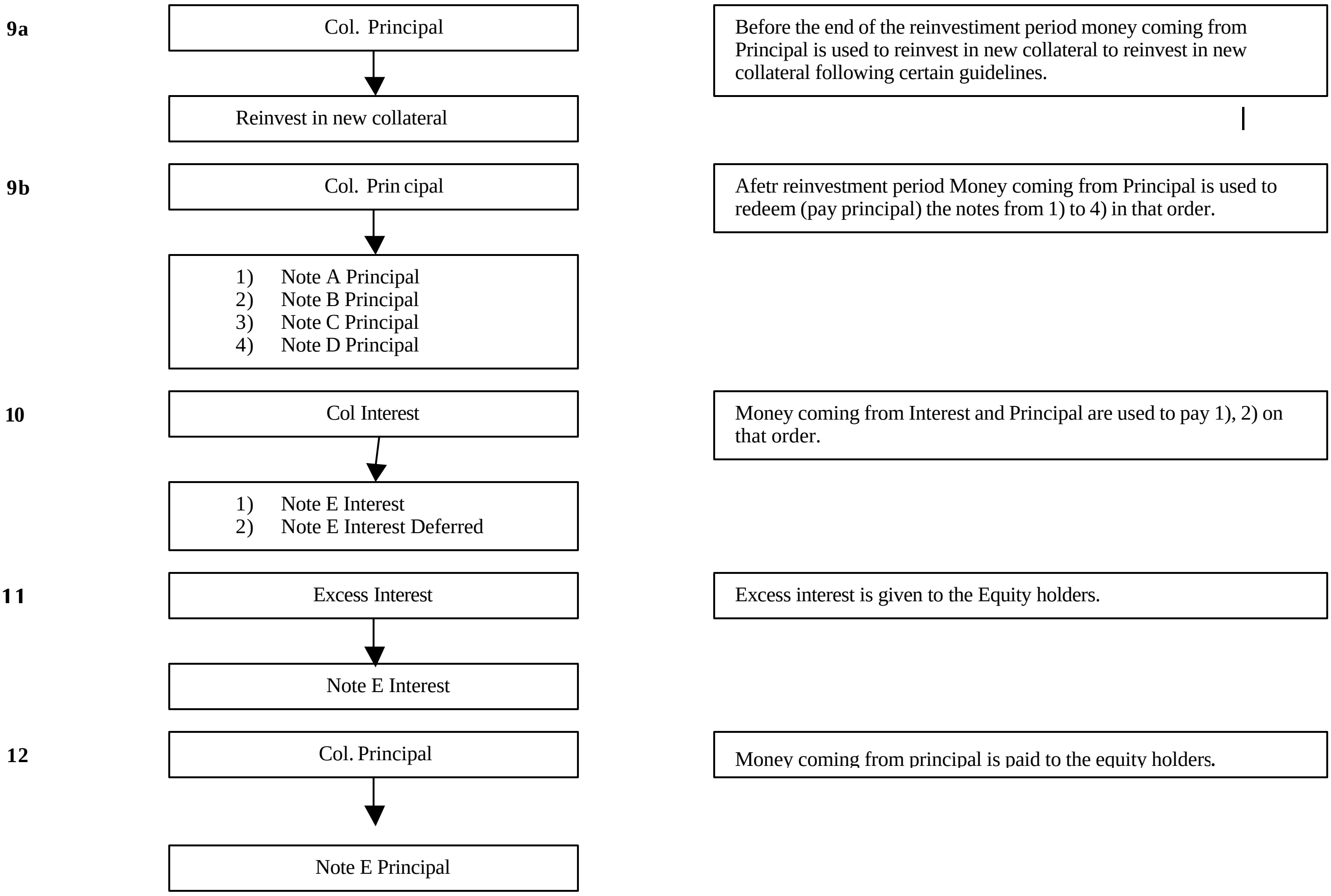


Fig 2 (cont.) Waterfall structure used in the tests

In table 8 we give the stress factors we have used for the probabilities of default and for the recovery rates. As already mentioned the higher the rating the higher the stress factor used.

	Stress Factor	
Rating	**PD**	**Recovery Rate**
AAA	1.50	0.67
AA	1.40	0.67
A	1.31	0.73
BBB	1.23	0.81
BB	1.15	0.89
B	1.00	0.98
CCC	1.00	1.00

Table 8 Stress factors used on the PD's (probabilities of default) and on the RR's (recovery rates) for each target rating.

The yield curve used is shown in table 9.

Time	Rate (%)	Time	Rate (%)
1 W	0.021	**2 Y**	1.454
1 M	0.091	**3 Y**	2.073
2 M	0.182	**4 Y**	2.660
3 M	0.272	**5 Y**	3.079
6 M	0.545	**7 Y**	3.759
9 M	0.817	**10 Y**	4.393
1 Y	1.089	**15 Y**	4.988

Table 9 Yield curve used in the experiment

5.2) The Tests and their Results

In this section we see how changes in different parameters of the model might affect the results. In our results we will show how the rating and the EL is affected under different parametrizations. The EL is calculated using the PV of the coupons and principals as shown in eq. 5.

Our basic scenario with which we will compare the results on the variations of the parameters is the following:

a) flat yield curve;
b) 50% recovery rate;
c) assume that at the end of the CDO the collateral is completely liquidated.

The rating of a tranche will be the one for which it first passes all the default scenarios shown in eq. 3. In what follows (unless otherwise mentioned) the EL shown in the tables will be the one of the 50-10-10-10-10-10 scenario.

5.2.1 Parallel Shifts on the Interest Rate Curve

The impact of the discount curve will be seen by how parallel shifts in the curve can affect the rating and as such the EL of the tranche. This is shown in table 10. Generally speaking, and depending on the hedges (swaps and caps) in the structure, the impact of moves on the yield curve can be quite considerable. Observe that all the bonds in the collateral are fixed rate while the lower tranches are fixed and the upper floating. This means that if rates go up more interest is paid to the upper tranches letting less to the lower tranches. Additionally the hedges via the caps and the swaps (see Table 5) are such to limit the impact of increases in interest rates. As an end result we have that the lower the tranche the more pronounced is the impact of higher rates, while the losses on the senior tranches will be quite limited. This is indeed what we see in Table 10 where we show the results of shifts in the interest rates from –1% up to 2% in steps of 1%.

One should still be aware that due to the structure of CDO contracts depending on the outstanding collateral moves on the yield curve can certainly act as a step function on the EL of a senior tranche.

	IR – 1%		**IR**		**IR + 1%**		**IR + 2%**	
Tranch	**Rating**	**EL(%)**	**Rating**	**EL(%)**	**Rating**	**EL(%)**	**Rating**	**EL(%)**
A	Aaa	0	Aaa	0	Aaa	0	Aaa	0
B	Aaa	0.0	Aaa	0	Aaa	0	Aaa	0
C	Aa3	0.06	A1	0.07	A3	0.27	Baa1	0.28
D1	Baa1	0.34	Baa3	1.49	Ba1	2.25	B1	3.17
D2	Ba3	5.48	B1	5.78	B3	16.29	Caa2	32.93
E	Caa3	65.56	D	84.45	D	95.56	D	98.04

Table 10 Impact of different moves on the discount curve on the rating and EL for the different tranches.

5.2.2) Impact of Changes on the Recovery Rate

The impact of the RR on the EL of the structure for the case of the basic scenario is shown in Table 11. We present the results for the following three cases of RR's: a) 30%; b) 50% (basic scenario); and c) 70%. The lower the RR the higher the losses (one gets in case of default of collateral). As it would be expected the lower the recovery rate the higher the EL. Moreover the lower the seniority the higher the proportional impact (normally the lower the seniority the lower the amounts of issued notes).

	30%		50%		70%	
Tranch	**Rating**	**EL(%)**	**Rating**	**EL(%)**	**Rating**	**EL(%)**
A	Aaa	0	Aaa	0	Aaa	0
B	Aaa	0	Aaa	0	Aaa	0
C	Baa1	0.26	A1	0.07	Aa1	0.01
D1	Ba2	4.60	Baa3	1.49	A3	0.35
D2	Caa1	18.07	B1	5.78	Ba1	2.68
E	D	91.59	D	84.45	D	70.55

Table 11 Impact of changes in the assumptions on the RR for the case of flat yield curve.

5.2.3) Impact of Changes on the Diversity Score (DS)

Another important question is how the results of the rating are affected by changes in the diversity score (DS). Observe that changes in the DS are equivalent to changes on the correlation of the collateral pool. In general increasing correlation should impact negatively the higher tranches and positively the very low ones. The reason for it is that high correlation creates extreme scenarios like every name going in default or no name defaulting. In the first case the very senior tranches are negatively affected while in the second ones the very low ones are positively affected. As we wee in Table 12 we have varied the DS by about 10% up and down. Observe that the changes have not been enough to affect the A and B notes. For the C up to D2 notes the decrease in correlation has decreased the losses while for the E notes it has caused an increase. One should be aware that the sensitivity to changes will certainly depend on the state of the collateral.

	30		34		38	
Tranch	**Rating**	**EL(%)**	**Rating**	**EL(%)**	**Rating**	**EL(%)**
A	Aaa	0	Aaa	0	Aaa	0
B	Aaa	0	Aaa	0	Aaa	0
C	A1	0.10	A1	0.07	A1	0.04
D1	Baa3	1.86	Baa3	1.49	Baa2	1.12
D2	B1	6.59	B1	5.78	B1	5.01
E	D	83.66	D	84.45	D	85.30

Table 12 Impact of the DS in the rating for the case of flat yield curve and RR at 50%.

5.2.4) Impact of Overcollateralization

Variations on the collateral pool affect dramatically the rating of a tranche. The higher the amounts in the collateral the more is available to generate cash for the notes and one should expect a sensible improvement of the ratings of all the tranches (see the column +10%). The falling in the collateral will additionally trigger failures of O/C and I/C tests causing early redemptions of the senior notes. In general then one would have that the impacts are higher for the lower tranches than for the senior ones. This is indeed what is seen in Table 13.

	-20%		-10%		0%		+10%	
Tranch	**Rating**	**EL(1)**	**Rating**	**EL(1)**	**Rating**	**EL(1)**	**Rating**	**EL(1)**
A	Aaa	0	Aaa	0	Aaa	0	Aaa	0
B	A2	0.25	Aaa	0	Aaa	0	Aaa	0
C	Caa3	37.35	Ba1	1.57	A1	0.07	Aa3	0
D1	D	100	Caa2	24.52	Baa3	1.49	Baa1	0.02
D2	D	100	D	81.33	B1	5.78	Ba3	0.13
E	D	100	D	100	D	84.45	B2	12.26

(1) EL in %.

Table 13 Impact on the tranche ratings due to variations on the Collateral Pool amount.

5.2.5) Impact of Changes on the Average Rating of the Collateral Portfolio

Changes in the ratings of the collateral pool will affect the WARF of the pool and as such the probability of default of the idealized bond. Higher PD implies higher expected losses and lower rating with the junior tranches being the first to be impacted. This is exactly the behaviour we observe in table 14.

	-2 notches		-1 notch		0	
Tranch	**Rating**	**EL(%)**	**Rating**	**EL(%)**	**Rating**	**EL(%)**
A	Aaa	0	Aaa	0	Aaa	0
B	Aaa	0	Aaa	0	Aaa	0
C	Baa2	0.55	Baa1	0.16	A1	0.07
D1	Ba3	6.57	Ba1	2.58	Baa3	1.49
D2	Caa1	22.82	B3	15.02	B1	5.78
E	D	95.70	D	92.44	D	84.45

Table 14 Impact on the tranche ratings due to notching down the collateral portfolio.

5.2.6) Impact of Changes on the WAL of the Collateral Pool

The weighted average life of the collateral is 5.00 year. The WAL is used in BET in the determination of the PD of the independent bond. As explained in section 2 the PD is determined by seeing the PD associated with the WARF and the WAL. In this way the higher the WAL (everything being kept constant) the higher the PD, the higher the EL. This results is shown in the Table 15 below.

	7.00		6.00		5.00		4.00	
Tranch	**Rating**	**EL(%)**	**Rating**	**EL(%)**	**Rating**	**EL(%)**	**Rating**	**EL(%)**
A	Aaa	0	Aaa	0	Aaa	0	Aaa	0
B	Aaa	0	Aaa	0	Aaa	0	Aaa	0
C	A3	0.46	A3	0.31	A1	0.07	Aa2	0.03
D1	Ba1	3.26	Ba1	1.53	Baa3	1.49	Baa1	0.32
D2	B1	10.91	B1	9.98	B1	5.78	B1	2.80
E	D	88.28	D	87.89	D	84.45	D	81.07

Table 15 Impact on the tranche ratings due to variations on the WAL of the collateral.

6) Conclusions

In this article we have described the BET methodology and we have shown how changes in several "parameters" of the model impact the rating and expected loss for the different tranches of a cash flow CDO. The parameters we have varied are a) interest rate curve (parallel shifts); b) recovery rates; c) diversity score (of the collateral portfolio); d) overcollateralization; e) average rating of the collateral; and f) maturities of the collateral. In general we have seen that the lower the tranche the higher the proportional impact of such changes.

Generally speaking the strength of the BET model resides in its simplicity. The problem is that simplicity comes at a cost. The concept of correlation for example is hidden in the calculation of the DS. As already mentioned in the section 3 there are two approaches for calculating the DS and in this article we have used the simplest of them (as is standard in the market for the sort of CDO here analyzed). One should be aware that for the case of CDO's on more complex collateral (composed e.g. of ABS/MBS notes) one will need a more sophisticated formulation (of DS) that takes into account parameters such as correlation among the underlying assets (see e.g. Witt [Witt(2004)] for more details).

A second aspect of the BET model is that one does not have access to the exact timing of default as happened in the copula model (see Garcia et alli [Garcia(2004)] for a detailed comparison between MC copulas and BET or Renault et alli[6] [Renault(2004)] for a study involving copulas, BET and the PRT (from S&P) model). This means that the process of rating a note can become a bit more arbitrary than one would like. In our case for example we have given to a note the rating for which the note survives every default scenario (see eq. 6) assuming the interest rate curve stays like it is today. A more conservative analyst could request the rating for which the note passes the same default scenarios but assuming a +1% (or even a +2%) shift in the interest rate curve.

Despite its apparent simplicity it is our experience that BET algorithm is a very reliable tool when properly used. But given the complexity of the instruments being priced one would better have more than one model to compare alternative views. This is the object of our next studies.

[6] The interested reader should be aware that in the referred article the results for the BET methodology has been misprinted: on Table 2 (of the article) the PD's 0.00%, 0.20% and 29,44% should have been 0.00%, 10.75% and 74.86%.

Acknowledgements

João Garcia would like to express his deeply gratitude to Eric Hermann (Head of Global Risk Management at Dexia Group) for his patience, trust, support and numerous incentives when in the creation of the Credit Methodology in Dexia Group. Without his commitments and management skills none of the codes here used would have been implemented. He would like to thank Ronny Langendries (from the Credit Methodology Dexia Group), Guy Raeves (credit derivatives trader at Dexia Bank Belgium) and Geert Gielens (Senior Quantitative Dealer at Dexia Bank) for numerous practical discussions. Finally he would like to thank for the market makers who provided us with their CDO data used in this article (the data was provided on the condition of anonymity).

Appendix 1: Calculation of the DS of a Portfolio

In this section we give a detailed description of the algorithm used for the DS. Consider a portfolio with M exposures X_m (m = 1 to M) to N different issuers that belong to Q different sectors.

1. Calculate Equivalent Unit Scores (EUS_n) for every issuer n:

a. calculate total exposure per issuer I_n, with n = 1 to N

$$I_n = \sum_{m=1}^{J(n)} X^n{}_m$$

where $X^n{}_m$ is the amount of exposure m from issuer n and J(n) is the number of exposure of issuer n. In this way:

$$M = \sum_{n=1}^{N} J(n)$$

b. calculate average issuer exposure AI

$$AI = \frac{\sum_{m=1}^{M} X_m}{N} = \frac{\sum_{n=1}^{N} I_n}{N}$$

c. calculate EUS_n for every issuer n

$$EUS_n = \min\left(1, \frac{I_n}{AI}\right)$$

Note that, because the EUS are capped at 1, this step penalizes portfolios with issuers with exposures bigger than the average (high issuer concentration).

2. Calculate the Sector Diversity Scores SDS_q for every sector q

$$SDS_q = f\left(A_q\right),$$

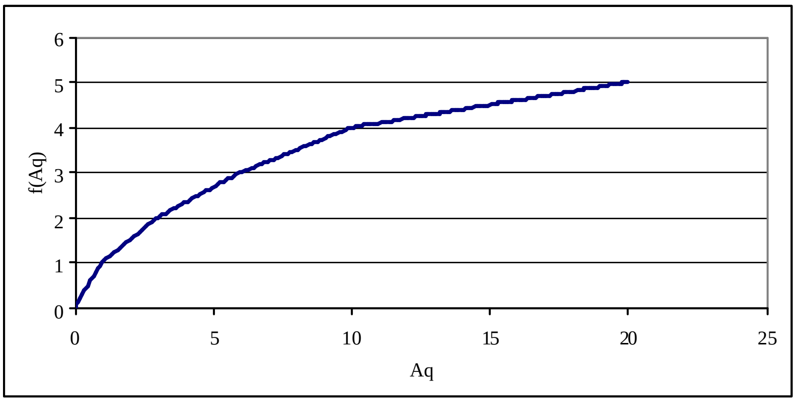
with:

$$A_q = \sum_{n=1}^{J(q)} EUS_n$$

where J(q) represents the number of issuers that belongs to sector q. I.e.:

$$N = \sum_{q=1}^{Q} J(q)$$

$f(\ )$ is the scaling function that converts the numbers in column 1 in Table 16 to the numbers in column 2 (see graph below)



Graphic 1 Scaling function used in the link between DS and EUS

Note that this step penalizes portfolios with high sector concentration.

3) Calculate the Total Diversity Score DS

The DS is then given by:

$$DS = \sum_{q=1}^{Q} SDS_q$$

Note that there is no scaling at this point: all sectors are assumed to be independent.

EUS	DS	EUS	DS	EUS	DS	EUS	DS	EUS	DS	EUS	DS
0.00	0.00	1.15	1.10	2.35	1.70	3.55	2.20	4.75	2.60	6.15	3.05
0.05	0.10	1.25	1.15	2.45	1.75	3.65	2.23	4.85	2.63	6.25	3.08
0.15	0.20	1.35	1.20	2.55	1.80	3.75	2.27	4.95	2.67	6.35	3.10
0.25	0.30	1.45	1.25	2.65	1.85	3.85	2.30	5.05	2.70	6.45	3.13
0.35	0.40	1.55	1.30	2.75	1.90	3.95	2.33	5.15	2.73	6.55	3.15
0.45	0.50	1.65	1.35	2.85	1.95	4.05	2.37	5.25	2.77	6.65	3.18
0.55	0.60	1.75	1.40	2.95	2.00	4.15	2.40	5.55	2.87	6.75	3.20
0.65	0.70	1.85	1.45	3.05	2.03	4.25	2.43	5.65	2.90	6.85	3.23
0.75	0.80	1.95	1.50	3.15	2.07	4.35	2.47	5.75	2.93	6.95	3.25
0.85	0.90	2.05	1.55	3.25	2.10	4.45	2.50	5.85	2.97	7.05	3.28
0.95	1.00	2.15	1.60	3.35	2.13	4.55	2.53	5.95	3.00	7.15	3.30
1.05	1.05	2.25	1.65	3.45	2.17	4.65	2.57	6.05	3.03	7.25	3.33

Table 16 Relation between EUS and DS (this data has been taken from the CDO structure)

Appendix 2 Table to find the default probability given the WAL and WARF

In order to calculate the WARF of a pool one needs a table linking ratings to rating factors. Additionally in order to determine the default probability of the idealized bond one needs a table linking WAL and WARF to default rates. Such relations can be taken from Table 17.

Rating	RF	Default Rates[1]									
		1	2	3	4	5	6	7	8	9	10
Aaa	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
Aa1	10	0.0000	0.0000	0.0001	0.0002	0.0003	0.0004	0.0005	0.0007	0.0008	0.0010
Aa2	20	0.0000	0.0001	0.0003	0.0005	0.0007	0.0009	0.0011	0.0014	0.0016	0.0020
Aa3	40	0.0000	0.0002	0.0006	0.0010	0.0014	0.0018	0.0023	0.0027	0.0033	0.0040
A1	70	0.0001	0.0004	0.0012	0.0019	0.0026	0.0033	0.0041	0.0048	0.0057	0.0070
A2	120	0.0001	0.0007	0.0022	0.0035	0.0047	0.0058	0.0071	0.0083	0.0098	0.0120
A3	180	0.0004	0.0015	0.0036	0.0054	0.0073	0.0091	0.0111	0.0130	0.0152	0.0180
Baa1	260	0.0009	0.0028	0.0056	0.0083	0.0110	0.0137	0.0167	0.0197	0.0227	0.0260
Baa2	360	0.0017	0.0047	0.0083	0.0158	0.0197	0.0241	0.0285	0.0324	0.0360	0.0413
Baa3	610	0.0042	0.0105	0.0171	0.0238	0.0305	0.0370	0.0433	0.0497	0.0557	0.0610
Ba1	940	0.0087	0.0202	0.0313	0.0420	0.0528	0.0625	0.0706	0.0789	0.0869	0.0940
Ba2	1350	0.0156	0.0347	0.0518	0.0680	0.0841	0.0977	0.1070	0.1166	0.1265	0.1350
Ba3	1780	0.0281	0.0551	0.0787	0.0979	0.1186	0.1349	0.1462	0.1571	0.1671	0.1766
B1	2220	0.0468	0.0838	0.1158	0.1385	0.1612	0.1789	0.1913	0.2023	0.2124	0.2220
B2	2720	0.0716	0.1167	0.1555	0.1813	0.2071	0.2265	0.2401	0.2515	0.2622	0.2720
B3	3490	0.1162	0.1661	0.2103	0.2404	0.2705	0.2902	0.3100	0.3258	0.3378	0.3490
Caa1	4763	0.1738	0.2323	0.2864	0.3248	0.3631	0.3897	0.4139	0.4366	0.4567	0.4770
Caa2	6500	0.2600	0.3250	0.3900	0.4388	0.4875	0.5200	0.5525	0.5850	0.6175	0.6500
Caa3	8062	0.5099	0.5701	0.6245	0.6624	0.6982	0.7211	0.7433	0.7649	0.7858	0.8070
D	10000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

(1) Default rates for year 1 up to year 10.

Table 17 Relation between rating, rating factors, maturity and default probabilities.

Appendix 3 Table to used to map EL to rating

In the last step of the BET one needs to give the rating of the note. For it one needs a table linking loss with rating (see section 3 for details on how the table is used). This link (for up to 10 years of WAL) can be done using the data in table 18 below.

Rating	Default Rates[1]									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Aaa	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
Aa1	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	0.0003	0.0004	0.0005	0.0006
Aa2	0.0000	0.0000	0.0001	0.0003	0.0004	0.0005	0.0006	0.0007	0.0009	0.0011
Aa3	0.0000	0.0001	0.0003	0.0003	0.0008	0.0010	0.0012	0.0015	0.0018	0.0022
A1	0.0000	0.0002	0.0006	0.0010	0.0014	0.0018	0.0022	0.0026	0.0032	0.0039
A2	0.0001	0.0004	0.0012	0.0019	0.0026	0.0032	0.0039	0.0046	0.0054	0.0066
A3	0.0002	0.0008	0.0020	0.0030	0.0040	0.0050	0.0061	0.0072	0.0084	0.0099
Baa1	0.0005	0.0015	0.0031	0.0046	0.0061	0.0075	0.0092	0.0108	0.0125	0.0143
Baa2	0.0009	0.0026	0.0046	0.0066	0.0087	0.0108	0.0133	0.0157	0.0178	0.0198
Baa3	0.0023	0.0058	0.0094	0.0131	0.0168	0.0204	0.0238	0.0273	0.0306	0.0336
Ba1	0.0048	0.0111	0.0172	0.0231	0.0290	0.0344	0.0388	0.0434	0.0478	0.0517
Ba2	0.0086	0.0191	0.0285	0.0374	0.0463	0.0537	0.0589	0.0641	0.0696	0.0743
Ba3	0.0155	0.0303	0.0433	0.0538	0.0652	0.0742	0.0804	0.0864	0.0919	0.0971
B1	0.0257	0.0461	0.0637	0.0762	0.0887	0.0984	0.1052	0.1113	0.1168	0.1221
B2	0.0394	0.0642	0.0855	0.0997	0.1139	0.1246	0.1321	0.1383	0.1442	0.1496
B3	0.0639	0.0914	0.1157	0.1322	0.1488	0.1606	0.1705	0.1792	0.1858	0.1920
Caa1	0.0956	0.1278	0.1575	0.1786	0.1997	0.2143	0.2276	0.2401	0.2512	0.2624
Caa2	0.1430	0.1788	0.2145	0.2413	0.2681	0.2860	0.3039	0.3218	0.3396	0.3575
Caa3	0.2804	0.3135	0.3435	0.3643	0.3840	0.3966	0.4088	0.4207	0.4322	0.4439
D	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 18 Moody's Idealized Expected Loss

Referrences

[Backman(1995)] Backman A. and G. O'Connor, *Rating Cash Flow Transactions Backed by Corporate Debt 1995 Update*, Moody's Investors Service, Global Credit Research, Apr 7th 1995

[Luca(2001)] Lucas D., CDO Handbook, Global Structured Finance Research, JP Morgan, May 2001.

[Cifuentes(1996)] Cifuentes A., and G. O'Connor, *The Binomial Expension Method Applied to CBO/CLO Analysis*, Moody's Special Report, Dec 13th 1996

[Debuysscher(2003)] Debuysscher A., M. Szego, M. Freydefront, and H. Tabe, *The Fourier Transform Method – Technical Document*, Working Paper, International Structured Finance, Moody's, Jan 2003.

[Bergman(2001)] Bergman S., *CDO Evaluator Applies Correlation and Monte Carlo Simulation to the Art of Determining Portfolio Quality*, Structured Finance, Standard & Poor's, Nov 12th 2001.

[Bund(2003)] Bund S., S. Neugebauer, K. Gill, R.V. Hrvatin, J. Zelter, J. Schiavetta, *Global Rating Criteria for Collateralised Debt Obligations*, Structured Finance, FitchRatings, 1st Aug 2003.

[Mina (2001)] Mina J, Mark-to-Market, Oversight, and Sensitivity Analysis of CDO's, Working Paper Number 01-02, RiskMetrics Group Dec 2001.

[Perraudin (2004)] Perraudin W. and O. Renault, *Standard & Poor's Portfolio Risk Tracker Technical Document*, S&P, 2004

[Hill(1999)] Hill, M.C. and L. Vacca, *Collateralized Debt Obligations: An Introduction to Arbitrage CBOs and CLOs*, Bank of America, 1999

[Goodman (2002)] Goodman, L., *Synthetic CDOs: An Introduction*, J. Der. (Spring), pag: 60-72, 2002

[Falcone(1998)] Falcone, Y.F., and J. Gluck, *Moody's Aproach to Rating Market Value CDO's*, Structured Finance, Moodys, Apr 1998

[Cifuents (1998)] Cifuents A., I. Efrat, J. Gluck, and Eileen Murphy, *Buying and Selling Credit Risk : a Perspective on Credit-linked Obligations*, pag 105-116, In Credit Derivatives: Applications for Risk Management, Investment and Portfolio Optimisation, Risk Books, 1998

[Garcia(2004)] Garcia J., T. Dewyspelaere, R. Langendries, and L Leonard, *Comparing BET and Copulas for CDO Instruments*, Working Paper (Version 7, Dec 8th 2003), Credit Risk Methodology, Dexia Group.

[Renault (2004)] Renault, O., T. Dewyspelaere, and J. Garcia, *A Comparative Analysis of CDO Risk Models*, in Structured Credit Products, Pricing, Rating, Risk Management and Basel II, Ed. By W. Perraudin, Risk Books, Sept. 2004.

[Witt(2004)] Witt G., *Moody's Correlated Binomial Default Distribution*, Structured Finance, Rating Methodology, Moody's Investor Service, Aug 10th 2004.